Weighted Sum-Rate Maximization in Wireless Networks: A Review
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Weighted Sum-Rate Maximization in Wireless Networks: A Review

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Abstract

A wide variety of resource management problems of recent interest, including power/rate control, link scheduling, cross-layer control, network utility maximization, beamformer design of multiple-input multiple-output networks, and many others are directly or indirectly reliant on the weighted sum-rate maximization (WSRMax) problem. In general, this problem is very difficult to solve and is NP-hard. In this review, we provide a cohesive discussion of the existing solution
methods associated with the WSRMax problem, including global, fast local, as well as decentralized methods. We also discuss in depth the applications of general optimization techniques, such as branch and bound methods, homotopy methods, complementary geometric programming, primal decomposition methods, subgradient methods, and sequential approximation strategies, in order to develop algorithms for the WSRMax problem. We show, through a number of numerical examples, the applicability of these algorithms in various application domains.
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Consider a general wireless network with $L$ interfering links. The achievable rate of each link is a scalar function and is denoted by $r_l$. Then the general weighted sum-rate maximization (WSRMax) problem has the form:

$$\max \sum_{l=1}^{L} \beta_l r_l(y)$$
subject to $y \in \mathcal{Y}$.

(1.1)

Here $y = (y_1, \ldots, y_n)$ is the optimization variable of the problem, positive scalar $\beta_l$ is the weight associated with link $l$, and the (possibly nonconvex) set $\mathcal{Y}$ is the feasible set of the problem. In general, $r_l$ is not convex in $y$. Therefore, problem (1.1) is surprisingly difficult to solve, though it appears to be very simple.

In this section, we first provide a discussion that emphasizes the importance of WSRMax problem (1.1) in wireless networks. Next we discuss the importance of global, fast local, as well as distributed solution methods for WSRMax problem. Finally, the existing key literature that address the problem is presented. Applications of optimization techniques for developing algorithms for the problem will be covered in later sections, with more technical detail.
1.1 Motivation

Among various resource management policies, the WSRMax for an arbitrary set of interfering links plays a central role in many network control and optimization methods. For example, the problem is encountered in network utility maximization (NUM) [88], the resource allocation (RA) subproblem in various cross-layer control policies [43 [81], MaxWeight link scheduling in multihop wireless networks [120], power/rate allocation in wireless networks, as well as in wireline networks [115 [124], joint optimization of transmit beamforming patterns, transmit powers, and link activations in multiple-input multiple-output (MIMO) networks [34], and finding achievable rate regions of singlecast/multicast wireless networks [87], among others.

1.1.1 Network Utility Maximization (NUM)

In the late nineties, Kelly et al. [59] introduced the concept of NUM for fairness control in wireline networks. It was shown therein that maximizing the sum-rate under the fairness constraint is equivalent to maximizing certain network utility functions and different network utility functions can be mapped to different fairness criteria. For a useful discussion of many aspects of the NUM concept in the case of wireless network, see [88] and the references therein. In this context, the WSRMax problem appears as a part of the Lagrange dual problem of the overall NUM problem; see [89] and the references therein.

1.1.2 Cross-layer Control Policies for Wireless Networks

For useful discussions of cross-layer control policies, see [40 [43] 68 [71] [81] [90] [142] and the references therein. Many of these policies are essentially identical. It has been shown that an optimal cross-layer control policy, which achieves data rates arbitrarily close to the optimal operating point, can be decomposed into three subproblems that are normally associated with different network layers. Specifically, flow control resides at the transport layer, routing and in-node scheduling\(^1\) resides

\(^1\) in-node scheduling refers to selecting the appropriate commodity and it is not to be confused with the links scheduling mechanism which is handled by the resource allocation subproblem [43].
at the network layer, and resource allocation (or RA) is usually associated with the medium access control and physical layers \cite{43}. The first two subproblems are convex optimization problems and can be solved relatively easily. It turns out that under reasonably mild assumptions, the RA subproblem can be cast as a general WSRMax problem over the instantaneous achievable rate region \cite{43}. The weights of the links are given by the differential backlogs and the policy resembles the well-known backpressure algorithm introduced by Tassiulas and Ephremides in \cite{120,121} and further extended by Neely to dynamic networks with power control; see \cite{81} and the references therein.

1.1.3 MaxWeight Link Scheduling for Wireless Networks

Maximum weighted link scheduling for wireless networks \cite{41,68,105,120,121,138} is a place, in which the problem of WSRMax is directly used. Note that, for networks with fixed link capacities, the maximum weighted link scheduling problem reduces to the classical maximum weighted matching problem and can be solved in polynomial time \cite{68}. However, no solution is known for the general case when the link rates depend on the power allocation of all other links.

1.1.4 Power/rate Control Policies

We see sometimes that the WSRMax problem is directly used as the basis for the power/rate control policy in wireless, as well as in wireline networks \cite{115,124}. For example, in DSL networks, there is considerable research on resource management policies, which rely directly on the WSRMax problem for multiuser spectrum balancing \cite{3,27,28,39,74,94,97,122,128,129,139,140}. Direct application of WSRMax as an optimization criterion can also been seen extensively in joint power control and subcarrier assignment algorithms for OFDMA networks \cite{7,50,54,67,104,106,145}.

1.1.5 Resource Management in MIMO Networks

There are also a number of resource management algorithms in multiuser MIMO networks, which rely on the problem of WSRMax. For example, the methods proposed in \cite{34,85,111,122} rely on WSRMax
for joint design of linear transmit and receive beamformers. In addition, many references have applied WSRMax directly as an optimization criterion for beamformer design in MIMO networks, e.g., [45] [151].

1.1.6 Finding Achievable Rate Regions in Wireless Networks

In multiuser systems many users share the same network resources, for example, time, frequency, codes, space, etc. Thus, there is naturally a tradeoff between the achievable rates of the users. In other words, one may require to reduce its rate if another user wants a higher rate. In such multiuser systems, the achievable rate regions are important since they characterize the tradeoff achievable by any resource management policy [124]. By invoking a time sharing argument, one can always assume that the rate region is convex [124]. Therefore, any boundary point of the rate region can be obtained by using the solution of a WSRMax problem for some weights.

Thus, WSRMax is a central component in many network design problems as we discussed above. Unfortunately, the general WSRMax problem is not yet amendable to a convex formulation [76]. In fact, it is NP-hard [75]. Therefore, we must rely on global optimization approaches [5] [52] for computing an exact solution of the WSRMax problem. Such global solution methods are increasingly important because they can be used to provide performance benchmarks by back-substituting them into any network design method, which relies on WSRMax. They are also very useful for evaluating the performance loss encountered by any heuristic algorithm for the WSRMax problem.

Although global methods find the solution of the WSRMax problem, they are typically slow. Even small problems, with a few tens of variables, can take a very long time to solve WSRMax. Therefore, it is natural to seek suboptimal algorithms for WSRMax that are efficient enough, and still close to optimal; the compromise is optimality [22]. Such algorithms are of central importance since they can be fast and widely applicable in large-scale network control and optimization methods.
Due to the explosion of problem size and the signal overhead required in centralized network control and optimization methods, it is highly desirable to develop decentralized variants of those algorithms. Therefore, finding distributed methods for the WSRMax problem is of crucial importance from a theoretical, as well as from a practical perspective for decentralized implementation of many network control and optimization methods, such as those investigated in [81, 120].

1.2 Global Methods for WSRMax in Wireless Networks

The general WSRMax problem is NP-hard [75]. It is therefore natural to rely on global optimization approaches [5, 52] for computing an exact solution. One straightforward approach is based on exhaustive search in the variable space [28]. The main disadvantage of this approach is the prohibitively expensive computational complexity, even in the case of very small problems. A better approach is to apply branch and bound techniques [52], which essentially implement the exhaustive search in an intelligent manner; see [3, 39, 57, 97, 129, 135, 139]. Branch and bound methods based on difference of convex functions (DC) programming [52] have been proposed in [3, 39, 139] to solve (a subclass of) WSRMax. Although DC programming is the core of their algorithms, it also limits the generality of their method to the problems in which the objective function cannot easily be expressed as a DC [52]. For example, in the case of multicast wireless networks, expressing the objective function as a DC cannot be easily accomplished, even when Shannon’s formula is used to express the achievable link rates. Another branch and bound method has been used in [129] in the context of DSL bit loading, where the search space is discretized in advance. As a result of discretization, this method does not allow a complete control of the accuracy of the solution. An alternative optimal method was proposed in [97], where the WSRMax problem is cast as a generalized linear fractional program [96] and solved via a polyblock algorithm [48]. The method works well for small scale problems, but as pointed out in [5, chap. 2, pp. 40–41] and [96, sec. 6.3], it may show much slower convergence than branch and bound methods as the problem size increases. A special form of the WSRMax problem is presented
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in [23, p. 78] [128], where the problem data and the constraints must obey certain properties and, consequently, the problem can be reduced to a convex formulation. However, these required properties correspond to very unlikely events in wireless/wireline networks, and therefore the method has a very limited applicability.

1.3 Local Methods for WSRMax in Wireless Networks

The worst case computational complexity for solving the general WSRMax problem by applying global optimization approaches can increase more than polynomially with the number of variables. As a result, these methods are prohibitively expensive, even for off line optimization of moderate size networks. Therefore, the problem of WSRMax deserves efficient algorithms, which even though suboptimal, perform well in practice.

Several approximations have been proposed for the case when all links in the network operate in certain signal-to-interference-plus-noise ratio (SINR) regions. For example, the assumption that the achievable rate is a linear function of the SINR (i.e., low SINR region) is widely used in the ultra-wide-band systems, e.g., [98]. Other references, which provide solutions for the power and rate control in low SINR regions include [37, 69, 99]. The high SINR region is treated in [29, 58, 86]. However, at the optimal operating point different links correspond to different SINR regions, which is usually the case with multihop networks. Therefore, all methods mentioned above that are based on either the low or the high SINR assumptions can fail to solve the general problem.

One promising method is to cast the WSRMax problem into a signomial program (SP) formulation [20, sec. 9] or into a complementary geometric program (CGP) [6, 30], where a suboptimal solution can be obtained efficiently; we can readily convert an SP to a CGP and vice versa [30, sec. 2.2.5]. Applications of SP and CGP, or closely related solution methods, have been demonstrated in various signal processing and digital communications problems, e.g., [30, 31, 34, 35, 78, 94, 122, 127]. There are a number of other important papers proposing suboptimal solution methods for the WSRMax
1.3 Local Methods for WSRMax in Wireless Networks

problem, such as [2, 27, 33, 45, 68, 70, 74, 105, 111, 116, 131, 151], among others.

Though the suboptimal methods mentioned above, including SP/CGP based algorithms, can perform reasonably well in many cases, it is worth pointing out that not all of them can handle the general WSRMax problem. The reason is the self-interference problem, which arises when a node transmits and receives simultaneously in the same frequency band. Since there is a huge imbalance between the transmitted signal power and the received signal power of nodes, the transmitted signal strength is typically few orders of magnitude larger than the received signal strength. Thus, when a node transmits and receives simultaneously in the same channel, the useful signal at the receiver of the incoming link is overwhelmed by the transmitted signal of the node itself. As a result, the SINR values at the incoming link of a node that simultaneously transmits in the same channel is very small. Therefore, the self-interference problem plays a central role in WSRMax in general wireless networks [133].

Thus, in the case of general multihop wireless networks, the WSRMax problem must also cope with the self-interference problem. Under such circumstances SP/CGP cannot be directly applicable even to obtain a better suboptimal solution, since initialization of the algorithms is critical. One approach to dealing with self interference consists of adding supplementary combinatorial constraints, which prevent any node in the network from transmitting and receiving simultaneously [13, 14, 24, 38, 46, 61, 71, 138]. This is sometimes called the node-exclusive interference model; only subsets of mutually exclusive links can simultaneously be activated in order to avoid the large self interference encountered if a node transmits and receives in the same frequency band. Of course, such approaches induce a combinatorial nature for the WSRMax problem in general. The combinatorial nature is circumvented in [134], where homotopy methods (or continuation methods) [4] together with complementary geometric programming [6] are adopted to derive efficient algorithms for the general WSRMax problem.

\begin{footnote}
For example, when a node does not transmits and receives simultaneously in the same frequency band.
\end{footnote}
problem. Here, the term “efficient” can mean faster convergence, or convergence to a point with better objective value.

1.4 Distributed WSRMax in Wireless Networks

The emergence of large scale communication networks, as well as accompanying network control and optimization methods with huge signalling overheads triggered a considerable body of recent research on developing distributed algorithms for resource management, see [21, 79, 141] and the references therein. Such distributed algorithms rely only on local observations and are carried out with limited access to global information. These algorithms essentially involve coordinating many local subproblems to find a solution to a large global problem. It is worth emphasizing that the convexity of the problems is crucial in determining the behavior of the distributed algorithms [21, chap. 9]. For example, in the case of nonconvex problems such algorithms need not converge, and if they do converge, they need not converge to an optimal point, which is the case with the WSRMax problem. Nevertheless, finding even a suboptimal yet distributed method is crucial for deploying distributively many network control and optimization methods, e.g., [41, 68, 81, 83, 84, 105, 118, 120, 138], which rely on WSRMax.

Distributed implementation of the WSRMax problem has been investigated in [27, 93, 94, 127, 144] in the context of digital subscriber loops (DSL) networks. Those systems are inherently consisting of single-input and single-output (SISO) links. Related algorithms for SISO wireless ad hoc networks and SISO orthogonal frequency division multiple access cellular systems are found in [53, 117, 147, 146]. However, in the case of multi antenna cellular systems, the decision variables space is, of course, larger, for example, joint optimization of transmit beamforming patterns, transmit powers, and link activations is required. Therefore, designing efficient distributed methods for WSRMax is a more challenging task due to the extensive amount of message passing required to resolve the coupling between variables. In the sequel, we limit ourselves to basic, but still very important, results that develop distributed coordinated algorithms for resource management in networks with multiple antennas.
Several distributed methods for WSRMax in multiple-input and single-output (MISO) cellular networks have been proposed in [10, 11, 72, 95, 130, 132, 136]. Specifically, in [95] a two-user MISO interference channel (IC) is considered and a distributed algorithm is derived by using the commonly used high SINR approximation [29]. Moreover, another approximation, which relies on zero forcing (ZF) beamforming is introduced in [95] to address the problem in the case of multiuser MISO IC.

The methods proposed in [10, 11, 130] derived the necessary (but not sufficient) optimality conditions for the WSRMax problem and used it as the basis for their distributed solution. However, many parameters must be selected heuristically to construct a potential distributed solution and there is, in general no systematic method for finding those parameters. In particular, the algorithms in [10, 11] are designed for systems with very limited backhaul signaling resources and do not consider any iterative base station (BS) coordination mechanism to resolve the out-of-cell interference coupling. Even though the method proposed in [130] relies on stringent requirements on the message passing between BSs during each iteration of the algorithm, their results show that BS coordination can provide considerable gains compared to uncoordinated methods. An inexact cooperate descent algorithm for the case where each BS is serving only one cell edge user has been proposed in [72]. The method proposed in [66] is designed for sum-rate maximization and uses high SINR approximation. A cooperative beamforming algorithm is proposed in [152] for MISO IC, where each BS can transmit only to a single user. Their proposed method employs an iterative BS coordination mechanism to resolve the out-of-cell interference coupling. However, the convexity properties exploited for distribution of the problem are destroyed when more than one user is served by any BS. Thus, their proposed method is not directly applicable to the WSRMax problem. Recently, an interesting distributed algorithm for WSRMax is proposed by Shi et al. [110], which exploits a nontrivial equivalence between the WSRMax problem and a weighted sum mean.

\(^3\) \(K\)-user MISO IC means that there are \(K\) transmitter–receiver pairs, where the transmitters have multiple antennas and the receivers have single antennas.
Introduction

squared error minimization problem. This algorithm relies on user terminal assistance, such as signal covariance estimations, computation and feedback of certain parameters form user terminals to BSs over the air interface. In practice, performing perfect covariance estimation and perfect feedback during each iteration can be very challenging. In the presence of user terminal imperfections, such as estimation and feedback errors, the algorithms performance can degrade and its convergence can be less predictable.

Algorithms based on game theory are found in [49, 55, 65, 107, 108, 109]. Their proposed methods are restricted to interference channels, for example, MISO IC, MIMO IC. The methods often require the coordination between receiver nodes and the transmitter nodes during algorithm’s iterations.

Many optimization criteria other than the weighted sum-rate have been considered in references [12, 113, 123, 148, 149, 150] to distributively optimize the system resources (e.g., beamforming patterns, transmit powers, etc.) in multi antenna cellular networks. In particular, the references [12, 148, 149, 150] used the characterization of the Pareto boundary of the MISO interference channel [56] as the basis for their distributed methods. Their proposed methods do not employ any BS coordination mechanism to resolve the out-of-cell interference coupling. In [113, 123, 148] distributed algorithms have been derived to minimize a total (weighted) transmitted power or the maximum per antenna power across the BSs subject to SINR constraints at the user terminals.

1.5 Outline of the Volume

Section 2 presents a solution method, based on the branch and bound technique, which solves globally the nonconvex WSRMax problem with an optimality certificate. Efficient analytic bounding techniques are introduced and their impact on convergence is numerically evaluated. The considered link-interference model is general enough to model a wide range of network topologies with various node capabilities, for example, single- or multipacket transmission.
(or reception), simultaneous transmission and reception. Diverse application domains of WSRMax are considered in the numerical results, including cross-layer network utility maximization and maximum weighted link scheduling for multihop wireless networks, as well as finding achievable rate regions for singlecast/multicast wireless networks.

Section 3 presents fast suboptimal algorithms for the WSRMax problem in multicommodity, multichannel wireless networks. First, the case where all receivers perform singleuser detection\(^4\) is considered and algorithms are derived by applying complementary geometric programming and homotopy methods. Here we apply the algorithms within a general cross-layer utility maximization framework to examine quantitative impact of gains that can be achieved at the network layer in terms of end-to-end rates and network congestion. In addition, we show, through examples, that the algorithms are well suited for evaluating the gains achievable at the network layer when the network nodes employ self interference cancelation techniques with different degrees of accuracy. Finally, a case where all receivers perform multiuser detection is considered and solutions are presented by imposing additional constraints, such as that only one node can transmit to others at a time or that only one node can receive from others at a time.

Section 4 presents an easy to implement distributed method for the WSRMax problem in a multicell multiple antenna downlink system. The algorithm is based on primal decomposition and subgradient methods, where the original nonconvex problem is split into a master problem and a number of subproblems (one for each base station). A sequential convex approximation strategy is used to address the nonconvex master problem. Unlike the recently proposed minimum weighted mean-squared error based algorithms, the method presented here does not rely on any user terminal assistance. Only base station to base station synchronized signalling via backhaul links. All the necessary computation is performed at the BSs. Numerical experiments

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\(^4\)That is, a receiver decodes each of its intended signals by treating all other interfering signals as noise.
are provided to examine the behavior of the algorithm under different degrees of BS coordination.

Finally, in Section 5 we present our conclusions. The detailed work presented in this volume is based on the research performed by the authors that led to several recent journal and conference publications.
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