Sparse Sensing for Statistical Inference

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Abstract

In today’s society, we are flooded with massive volumes of data in the order of a billion gigabytes on a daily basis from pervasive sensors. It is becoming increasingly challenging to sense, store, transport, or process (i.e., for inference) the acquired data. To alleviate these problems, it is evident that there is an urgent need to significantly reduce the sensing cost (i.e., the number of expensive sensors) as well as the related memory and bandwidth requirements by developing unconventional sensing mechanisms to extract as much information as possible yet collecting fewer data.

The aim of this monograph is therefore to develop theory and algorithms for smart data reduction. We develop a data reduction tool called sparse sensing, which consists of a deterministic and structured sensing function (guided by a sparse vector) that is optimally designed to achieve a desired inference performance with the reduced number of data samples. We develop sparse sensing mechanisms, convex programs, and greedy algorithms to efficiently design sparse sensing functions, where we assume that the data is not yet available and the model information is perfectly known.

Sparse sensing offers a number of advantages over compressed sensing (a state-of-the-art data reduction method for sparse signal recovery). One of the major differences is that in sparse sensing the underlying signals need not be sparse. This allows for general signal processing tasks (not just sparse signal recovery) under the proposed sparse sensing framework. Specifically, we focus on fundamental statistical inference tasks, like estimation, filtering, and detection. In essence, we present topics that transform classical (e.g., random or uniform) sensing methods to low-cost data acquisition mechanisms tailored for specific inference tasks. The developed framework can be applied to sensor selection, sensor placement, or sensor scheduling, for example.

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Introduction

1.1 Pervasive sensors and data deluge

Every day, we are generating data in the order of a billion gigabytes. This massive volume of data comes from omnipresent sensors used in medical imaging (e.g., breast or fetal ultrasound), seismic processing (e.g., for oil or gas field exploration), environmental monitoring (e.g., pollution, temperature, precipitation sensing), radio astronomy (e.g., from radio telescopes like the square kilometre array), power networks (e.g., to monitor wind farms or other distribution grids), smart infrastructures (e.g., to monitor the condition of railway tracks or bridges), localization and surveillance platforms (e.g., security cameras or drones, indoor navigation), and so on.

The acquired data samples are stored locally and then transported to a central location (e.g., a server or cloud) to extract meaningful information (that is, for inference). Due to an unprecedented increase in the volume of the acquired data, it is becoming increasingly challenging to locally store and transport all the data samples to a central location for data/signal processing. This is because the amount of sampled data quickly exceeds the storage and communication capacity by several orders of magnitude. Since the data processing is generally carried out at a central location with ample computing power, mainly the sensing, storage and transportation costs form the main bottleneck. To allevi-
1.1. Pervasive sensors and data deluge

In this era of data deluge, it is of paramount importance to gather only the informative data needed for a specific task. If we had some prior knowledge about the task we want to perform on the data samples, then just a small portion of that data might be sufficient to reach a desired inference accuracy, thereby significantly reducing the amount of sampled, stored and transported data. That is to say, if the inference task is known beforehand, less data needs to be acquired. Thus, the memory and bandwidth requirements can be seriously curtailed. In addition, the cost of data collection (or sensing) can be significantly reduced, where the major factors that determine the sensing cost are the number of physical sensors (and their economical and energy costs) and the physical space they occupy when installed. So, it is evident that there is an urgent need for developing unconventional and innovative sensing mechanisms tailored for specific inference tasks to extract as much information as possible yet collecting fewer data. This leads to the main question:

*How can task-cognition be exploited to reduce the costs of sensing as well as the related storage and communications requirements?*

This is different from the classical big data setting in which the data is already available and the question is how to mine information from that large-scale data. Our problem has close similarities to sampling, and is only related to model information, where the data is not yet available. Given the central role of sampling in engineering sciences, answering this question will impact a wide range of applications. The basic question of interest for such applications is, how to design sensing systems in order to minimize the amount of data acquired yet reach a prescribed inference performance. In particular, the design questions that should be answered are related to the optimal sensor placement in space and/or time, data rate, and sampling density to reduce the sensing cost as well as to reduce the storage and communications requirements. We next illustrate our ideas with two specific examples of sensor placement for indoor localization and temperature sensing.
Introduction

Figure 1.1: Illustration of an indoor localization setup. We show the floor plan of a building (e.g., museum) with candidate locations for installing the access points. The restriction on installing the access points in only certain areas might be for security or ambience purposes.

Example 1.1 (Target localization). Indoor localization is becoming increasingly important in many applications (see [69]). Some examples include: locating people inside a building for rescue operations, monitoring logistics in a production plant, lighting control, and so on. In such environments, global positioning system (GPS) signals are typically unavailable. Thus, other types of measurements such as visual, acoustic or radio waves revealing information about range, bearing, and/or Doppler are used. These measurements are gathered by access points, like cameras, microphones, radars, or wireless transceivers. One such scenario is illustrated in Figure 1.1 where we show an indoor localization setup for navigating a visitor inside a building. An interesting question is, instead of installing many such costly access points randomly, how can we minimize the number of access points (hence, the amount of data), by optimizing their characteristics (e.g., their spatial position, sampling rate) in such a way that a certain localization performance can be guaranteed.
1.1. Pervasive sensors and data deluge

Figure 1.2: Heatmaps of a 32KB data cache (a) without and (b) with a hot spot. Black circles (◦) denote the candidate temperature sensor locations—these are the areas with less or no active logic.

Example 1.2 (Field detection). Consider a multi-core processor with a hot spot. A historical question of interest is to estimate the thermal distribution, for instance, by interpolating noisy measurements. In some applications, though, a precise estimation of the temperature field might not be required. Instead, detecting the hot spots (i.e., the areas where the temperature exceeds a certain threshold) would be sufficient for subsequent control actions. Such a scenario is illustrated in Figure 1.2, where the image on the right (left) shows a 32 KB data cache with (no) hotspots. An important question of interest for such detection problems then is, how to optimally design spatial samplers (i.e., how to optimize the sensor placement [64]) by exploiting the knowledge of the underlying model, physical space and processing limitations.

Such optimally designed sensing systems can be used to perform a number of inference tasks, such as estimation, filtering, and detection.

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1We would like to thank the authors of [57] for the heatmaps.
1.2 Outline

This monograph is organized into three parts. In the first part of this monograph (i.e., in Chapter 2), the theory of sparse sensing is discussed in depth. In the second part of this monograph (i.e., in Chapters 3–6) the developed theory in Chapter 2 is applied to basic statistical signal processing problems. Finally, the monograph concludes with the third part (i.e., Chapter 7), where we pose some interesting open problems for future research.

Chapter 2 on sparse sensing forms the backbone of this monograph. In order to reduce the sensing and other related costs, it is crucial to tailor the sensing mechanism for the specific inference task that will be performed on the acquired data samples. The tool that we will exploit in this monograph to reduce the cost of sensing is sparse sensing, which consists of an optimally designed structured and deterministic sparse (i.e., with many zeros and a few nonzeros) sensing function that is used to acquire the data in order to reach a desired inference performance. Here, the number of nonzeros determines the amount of data samples acquired (thus determines the amount of data reduction). In this chapter, we will model the sparse sensing function as a linear projection operation, where the sensing function is parameterized by a sparse vector. This vector is basically a design parameter that is used as a handle to trade the amount of acquired data samples with the inference performance. We refer to this sparse sensing scheme as discrete sparse sensing, as the continuous observation domain is first discretized into grid points and we select (using the sparse vector) the best subset out of those grid points. To harness the full potential of sparse sensing, we need to sample in between the grid points and take samples anywhere in the continuous observation domain. We refer to such sensing mechanisms as continuous sparse sensing. We will discuss some applications of the proposed sparse sensing mechanisms and also list major differences with the state of the art in data reduction, that is, compressed sensing. Although the specific inference task is kept abstract in this chapter,
1.2. Outline

The obtained novel unifying view allows us to jointly treat sparse sensing mechanisms for the different inference tasks considered in Chapters 3–6.

Chapter 3 focuses on discrete sparse sensing for a general nonlinear estimation problem. In particular, we solve the problem of choosing the best subset of observations that follow known nonlinear models with arbitrary yet independent distributions. We also extend this framework to nonlinear colored Gaussian observations as it occurs frequently when the observations are subject to external noises or interference. The data is acquired using a discrete sparse sensing function, which is guided by a sparse vector. The Cramér-Rao bound (CRB) is used as an inference performance metric and we derive several functions of the CRB that include the sparse vector. To compute the optimal sparse samplers, we propose convex relaxations of the derived inference performance metric and also develop low-complexity solvers. We also discuss greedy algorithms leveraging the submodularity of the inference performance metric. In sum, we can conclude that discrete sparse samplers for nonlinear inverse problems can be computed efficiently (in polynomial or even linear time).

Chapter 4 extends the theory developed in Chapter 3 to nonlinear filtering problems, that is, the focus will be on the design of discrete sparse sensing functions for systems that admit a known nonlinear state-space representation. In particular, we solve the problem of choosing the best subset of time-varying observations based on the entire history of measurements up to that point. The posterior CRB is used as the inference performance metric to decide on the best subset of observations. Although this framework is valid for independent observations that follow arbitrary distributions (e.g., non-Gaussian), we also extend it to colored Gaussian observations. Further, we introduce some additional constraints to obtain smooth sensing patterns over time. Finally, we devise sparse sensing mechanisms for structured time-varying observations (e.g., for time-varying sparse signals). In all these cases, the
discrete sparse samplers can be designed efficiently by solving a convex program or through a greedy algorithm that leverages on submodularity.

Chapter 5 is dedicated to discrete sparse sensing for statistical detection. Specifically, the aim is to choose the best subset of observations that are conditioned on the hypothesis, which belongs to a binary set. Naturally, the best subset of observations is the one that results in a prescribed global error probability. Since the numerical optimization of the error probabilities is difficult, we adopt simpler costs related to distance measures between the conditional distributions of the sensor observations. We design sparse samplers for the Bayesian and Neyman-Pearson setting, where we respectively use the Bhattacharyya distance and Kullback-Leibler distance (and J-divergence) as the inference performance metric. For conditionally independent observations, we give an explicit solution, which is optimal in terms of the error exponents. More specifically, the best subset of observations is the one with the smallest local average root-likelihood ratio and largest local average log-likelihood ratio in the Bayesian and Neyman-Pearson setting, respectively. We supplement the proposed framework with a thorough analysis for Gaussian observations, including the case when the sensors are conditionally dependent, and also provide examples for other observation distributions. One of the results shows that, for nonidentical Gaussian sensor observations with uncommon means and common covariances under both hypotheses, the number of sensors required to achieve a desired detection performance reduces significantly as the sensors become more coherent.

Chapter 6 contrasts with the discrete sparse sensing mechanisms that have been considered in Chapter 5 to Chapter 5, where the sparse sensing functions are parameterized by a discrete sparse vector that needs to be optimally designed. This basically means that the continuous observation domain is first discretized into grid points and we have to select the best subset out of those
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grid points. However, this discretization might be very coarse because of complexity reasons, preventing the system to achieve the best possible compression rates for the considered inference task. Therefore, in this chapter, we introduce continuous sparse sensing (or off-the-grid sparse sensing), where it is possible to sample in between the grid points and take samples anywhere in the continuous observation domain. The basic idea is to start from a discretized sampling space and to model every sampling point in the continuous sampling space as a discrete sampling point plus a perturbation. Then, the smallest set of possible discrete sampling points is searched for, along with the best possible perturbations, in order to reach the prescribed inference performance. We will demonstrate this approach for linear inverse problems, that is, for linear estimation problems with additive Gaussian noise, although it can be extended for other inference problems as well.

Chapter 7 contains the conclusions and outlines a number of directions for future research along with some open problems.
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