Long Range Dependence

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## Long Range Dependence

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## Abstract

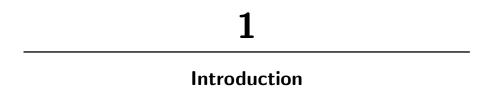
The notion of long range dependence is discussed from a variety of points of view, and a new approach is suggested. A number of related topics is also discussed, including connections with non-stationary processes, with ergodic theory, self-similar processes and fractionally differenced processes, heavy tails and light tails, limit theorems and large deviations.

Keywords: Long range dependence, rare events, large deviations.

Mathematical Subject Classification: Primary 60G10, 62M10; Secondary 60F10

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Long range dependence and long memory are synonymous notions, that are arguably very important. This importance can be judged, for example, by the very large number of publications having one of these notions in the title, in areas such as finance [84], econometrics [115], internet modeling [70], hydrology [109], climate studies [142], linguistics [3] or DNA sequencing [71]. These publications address a great variety of issues: detection of long memory in the data, statistical estimation of parameters of long range dependence, limit theorems under long range dependence, simulation of long memory processes, and many others. Surprisingly, very few of these publications address what long range dependence is. When definitions are given, they vary from author to author (the econometric survey [58] mentions 11 different definitions). The notion of long range dependence can also be applied to different aspects of a given stochastic process [63]. More diverse definitions become possible if, instead of looking at the "usual" stationary processes, one studies stationary point processes, as in [37], or random fields, as in [4].

It is the purpose of this survey to discuss what is meant (often implicitly) by long range dependence, clarify why this notion is

#### 2 Introduction

important, mention different point of views on the topic and, hopefully, remove some of the mystery that surrounds it.

The notion of long range dependence has, clearly, something to do with memory in a stochastic process. Memory is, by definition, something that lasts. It is the requirement that the memory has to be "long" that is special. Why is it important that in one model the memory is "a bit longer" than in another model? The first serious argument that this can be important is in a series of papers of B. Mandelbrot and his coauthors, e.g. [89] and [93]. It is also due to the influence of these early papers and subsequent publications of Mandelbrot (especially [90]) that long range dependence has also become associated with scaling and fractal behavior. We survey some of the early history in Section 2.

The "specialness" of long memory indicates that most stationary stochastic processes do not have it. This also makes it intuitive that non-stationary processes can provide an alternative explanation to the empirical phenomena that the notion of long range dependence is designed to address. This connection between long memory and lack of stationarity is very important. It is related to such well known phenomena as unit root problem [111] and regime switching [42]. We discuss the connections with non-stationary processes in Section 3.

A very attractive point of view on long range dependence is based on ergodic-theoretical properties of the dynamical system on which a stationary stochastic process is constructed. Many features that are intuitively associated with long memory are automatically found in such an approach. For several reasons this approach has not become widely accepted. We discuss this in Section 4.

Most of the definitions of long range dependence appearing in literature are based on the second-order properties of a stochastic process. Such properties include asymptotic behavior of covariances, spectral density, and variances of partial sums. The reasons for popularity of the second-order properties in this context are both historical and practical: second-order properties are relatively simple conceptually and easy to estimate from the data. This approach to the notion of long memory is discussed in Section 5.

The term "fractional" appears very frequently in the context of long range dependence. This usually refers to a model constructed using a generalized operation of a non-integer order, whereas the "usual" order of the operation has to be integer. The examples include differencing or differentiation "non-integral number of times." Certain features often associated with long memory can sometimes be obtained by doing so. Models obtained in this way are discussed in Section 6.

It is, once again, largely due to the early history that the notion of long range dependence has become closely associated with self-similar processes. Self-similar processes are stochastic models with the property that a scaling in time is equivalent to an appropriate scaling in space. The connection between the two types of scaling is determined by a constant often called *the Hurst exponent*, and it has been argued that the value of this exponent determines whether or not the increments of a self-similar process with stationary increments possess long range dependence. We discuss self-similar processes in Section 7.

The final part of this survey, Section 8, introduces a different approach to understanding long memory, a one that is related to the notion of phase transitions. We argue that this approach makes the notion of long range dependence both intuitive and practical. One should hope for major future research effort in this direction.

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