Observability of Hybrid Dynamical Systems

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Contents

Abstract

Hybrid systems, i.e., heterogeneous systems that include discrete and continuous time subsystems, have been used to model applications in automotive such as engine, brake, and stability control, as well as air traffic control and manufacturing plant control. Because of their generality (they include as special cases continuous and discrete systems), deriving rigorous controller synthesis procedures is difficult. The most effective hybrid control algorithms are based on full state feedback. However, in the majority of cases, only partial information about the internal state of the hybrid plant can be measured. Observability and detectability are concepts of fundamental importance that establish the conditions for reconstruction of the state of a system and have been thoroughly investigated in the continuous and discrete domain but not as systematically for hybrid systems.

Hybrid systems' observability involves both the discrete structure and the continuous dynamics of the system. A hybrid system is said to be *observable* when it is possible to reconstruct the *discrete as well as the continuous state* of the system from the observed output information.

This paper reviews and places in context how the continuous and the discrete dynamics, as well as their interactions, intervene in the observability property of a quite general class of hybrid systems: linear hybrid systems called $H-$systems. Our specific objective is to show how the hybrid characteristics of the system come into play and give rise to particular aspects and properties that do not simply generalize the ones that are well-known for traditional dynamical systems. This paper intends to provide a tutorial approach to hybrid systems observability in its various forms to students in control and its application as well as to practitioners in the field.

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1

Introduction

Safety-critical embedded control systems, such as the ones encountered in transportation systems (e.g., airplanes, cars, and trains) or industrial plants, have become increasingly important as autonomy is taking centre stage. When designing these control systems, it is essential to take all effects into consideration, including the interaction between the plant to be controlled and the embedded controller. This calls for methods that can deal with heterogeneous components exhibiting a variety of different behaviors. For example, discrete controllers can be represented mathematically as discrete event systems, while plants are mostly represented by continuous time systems. The properties of these heterogeneous systems, called *hybrid systems*, have to be proven under all foreseeable scenarios and this need calls for formal approaches to design. Indeed, theoretical properties of hybrid systems have been the subject of intense research in the last decades.

Because of the generality of hybrid systems (see e.g. Lin and Antsaklis [2014] where different models and analysis methods are illustrated in detail), deriving rigorous controller synthesis procedures is often difficult. In many cases, we must resort to either heuristics or approximations because the generality of hybrid models implies a high level of complexity. Even when the structure of the hybrid problem is such that a controller can be synthesized, strong assumptions on the availability of information about the system have to be used. For example, in the automotive domain, a hybrid formalism was proposed to solve power-train control problems and to derive control laws based on full state feedback, requiring that the entire state of the system under control be known at all times (see e.g. Balluchi et al. [2000]). However, in most cases, only partial information about the internal state of the hybrid plant is available. Hence, to adopt hybrid controllers, the design of hybrid state observers that can reconstruct the state from partial information is of fundamental importance.

Indeed, reconstructing the internal behavior of a dynamical system on the basis of the available measurements is a central problem in control theory in general, not only for hybrid systems. Starting from the seminal paper Kalman [1959], state observability has been investigated both in the continuous domain since the sixties (Luenberger [1971] for the linear case and Griffith and Kumar [1971] for the nonlinear case), and in the discrete state domain since the eighties (see e.g. Ozveren and Willsky [1990] and Ramadge [1986]). However, the observability question is far from being fully answered (see e.g., the recent papers on nonlinear observability and observers design Khalil and Praly [2013] and Sassano and Astolfi [2014]). Even for linear observer design, there are still open questions (see e.g. Blumthaler and Trumpf [2014], Trumpf et al. [2014]).

For a discrete state system, observability corresponds to the reconstruction of the current discrete state. A related property is diagnosability, which corresponds to the possibility of determining the past occurrence of some particular states, for example faulty states. Recent advances on diagnosis methods for discrete event systems can be found in the excellent survey Zaytoon and Lafortune [2013]). The paper De Santis and Di Benedetto [2015] offers a general framework where a number of observability and diagnosability properties can be framed as special cases, for example, "critical observability" that arises when dealing with safety critical applications, e.g. Air Traffic Management Di Benedetto et al. [2005b], De Santis et al. [2006a]. In these applications, the critical set of discrete states represents dangerous situations that must be detected to avoid unsafe or even catastrophic behavior of the system.

Hybrid systems' observability involves both the discrete structure and the continuous dynamics of the system. We say that a hybrid system is *observable* when it is possible to reconstruct the discrete as well as the continuous state of the system from the observed output information.

The reconstruction of the discrete state of a hybrid system corresponds to understanding which specific continuous dynamical system (corresponding to a state in the discrete abstraction) is evolving. This can be done either by using only the discrete output information, and in this case hybrid discrete state observability simply coincides with discrete observability, or by using only the continuous output information. In this latter case, the important property is the possibility of inferring based on the continuous output information which continuous system is indeed active (this property is referred to as *distinguishability* of a pair of dynamical systems). However, we can exploit the hybrid nature of the system and merge the two to yield weaker conditions for the identification of the discrete state. If the current discrete state can be identified - using discrete and/or continuous information - the system is said to be *current location observable*. The possibility of estimating the continuous state of the hybrid system is closely related to the identification of the discrete state: as it will be seen in this paper, the observability property is equivalent to the current location observability property.

In recent years, many researchers have considered the observability problem for hybrid systems (see e.g. the special issue De Santis and Di Benedetto [2009] (Eds.) on observability and observer-based control of hybrid systems and the references therein, Balluchi et al. [2002], Bemporad et al. [2000], Collins and van Schuppen [2007], Babaali and Pappas [2005], De Santis et al. [2003], Vidal et al. [2003], Balluchi et al. [2013], Tanwani et al. [2013] among others). The formal definition and analysis of observability properties depend on the model, on the available output information, and on the objective for which state reconstruction is needed, e.g. for control purposes, for detection of critical situations, and for diagnosis of past system evolutions. It is therefore hard, in general, to understand the precise relationships that exist between different observability notions, especially when dealing with hybrid systems and when the results are established with different formalisms.

In this paper, we present a tutorial view of this topic, for a general class of hybrid systems, with the specific objectives of

- demonstrating the roles that the continuous and the discrete dynamics, as well as their interactions, play in the observability property;
- showing how the hybrid characteristics of the system come into play and give rise to particular aspects and properties that do not simply generalize the ones that are well-known for traditional dynamical systems.

Given our intent, we chose not to delve into the many nuances of the research on observability that are valid for particular versions of hybrid systems (e.g., controlled and uncontrolled switching systems and impulsive systems, under special assumptions), but to focus on a general class of hybrid systems for which strong results can be obtained: linear hybrid systems called $LH-$systems. For this class of systems, we first present definitions of observability and *detectability*, a weaker and more general form of observability (see De Santis et al. [2009]). Then, in order to clarify the function of the hybrid nature of the system, we proceed step by step, by first analyzing the discrete structure and the continuous dynamics separately. Then we address the problems (and opportunities) posed by the interaction between the two parts of the system. We also address the observer design problem following the methodology presented in Balluchi et al. [2013] where the identification of the current discrete state and the estimation of the continuous state are intertwined. Further we show how the observability conditions ensure the existence of such an observer. An application in the automotive domain proves how the theoretical conditions illustrated in the paper can indeed be used to construct an observer.

Because of the multiplicity of different notions of observability existing in the literature, in our exposition we need rigor and precision in the definitions and derivations in order to avoid confusion. As a consequence, notations are at times complex and not always intuitive. However, when necessary, we sacrifice mathematical precision to provide intuition and a working knowledge of the topics. Hence the way we address the audience of the paper is a compromise between mathematical rigor and informal descriptions.

We do not cover more specific research topics that are concerned with, for example, the use of the observer in output feedback control, distributed observability, diagnosis by abstraction. However, we inserted in every chapter a review section that presents the most relevant literature on the subject matter of the chapter. This review is by no means exhaustive and it is also intended to offer potential avenues of further analysis of the material presented.

Our paper is organized as follows.

In Chapter 2, we define the general hybrid system model, called $H-$system, and describe some of its properties.

In Chapter 3, we focus on the discrete structure of the hybrid system, which is a Finite State Machine (FSM) associated with the original system. We introduce the notions of strongly connected components, persistent states and traps, which are used in the sequel to characterize observability and detectability. We also illustrate some transformations of the FSM, which do not alter the information that is relevant for checking the detectability of the hybrid system.

In Chapter 4, always with reference to the discrete structure of the $H-$system, we define and characterize two observability properties, called current location observability and critical observability. These two notions will be extended in the following chapters to a general hybrid system.

In Chapter 5, observability and detectability for hybrid systems are defined. We use simple examples to illustrate how these properties are not only related to the corresponding properties of the same concepts for linear systems, but depend also, for example, on the topology of the discrete system, on the resets, the minimum and maximum dwell times in each discrete state.

In Chapter 6, we suppose that no output information is available from the discrete part of the system and investigate the possibility of determining the current discrete state of a hybrid system by using only the continuous output information. The notion of distinguishability of two dynamical systems plays the main role in the solution to this problem. We also analyze how and when it is possible to determine the times at which a discrete transition takes place (called switching times), without necessarily identifying which discrete mode is active.

In Chapter 7, we define the class of current location observable hybrid systems, that is systems for which the current discrete state can be identified after a finite number of steps, either independently from the continuous evolution, or by using also the continuous evolution. Current location observability is characterized in terms of set membership and some computationally efficient algorithms for the determination of the sets of interest are proposed.

In Chapter 8, we first define the unobservable sub-system associated to a hybrid system. Then, we show that detectability is equivalent to the observability of an appropriate hybrid system associated with the original one and the asymptotic stability of its unobservable part. Then, we provide a characterization of detectability by using a Kalman-like approach.

In the last Chapter 9, we address the observer design problem. We show how, under the observability conditions illustrated in the previous chapters, it is possible to design hybrid observers for current location observable hybrid systems. The hybrid observer consists in the construction of two sub-systems: a *location observer* that identifies the current discrete state of the hybrid plant, and a *continuous observer* that produces an estimate of the evolution of the continuous state of the hybrid plant. The application to an automotive test case is described.

For the reader's convenience, the notations are summarized in the Appendix.

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