Precomputation-Based Rendering

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Precomputation-Based Rendering

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Abstract

High quality image synthesis is a long-standing goal in computer graphics. Complex lighting, reflection, shadow and global illumination effects can be rendered with modern image synthesis algorithms, but those methods are focused on offline computation of a single image. They are far from interactive, and the image must be recomputed from scratch when any aspect of the scene changes. On the other hand, real-time rendering often fixes the object geometry and other attributes, such as relighting a static image for lighting design. In these cases, the final image or rendering is a linear combination of basis images or radiance distributions due to individual lights. We can therefore *precompute* offline solutions to each individual light or lighting basis function, combining them efficiently for real-time image synthesis. *Precomputation-based relighting and radiance transfer* has a long history with a spurt of renewed interest, including adoption in commercial video games, due to recent mathematical developments and hardware advances. In this survey, we describe the mathematical foundations, history, current research and future directions for precomputation-based rendering.

Contents

1

Introduction

High quality image synthesis is one of the oldest goals of computer graphics. A standard to aspire for is often referred to as *photorealism* — rendering images indistinguishable from real photographs. Achieving this goal requires considering a variety of complicated shading effects in the real world, such as complex natural illumination from a skylit scene, soft shadows from the leaves of a tree in sunlight, glossy reflections from a velvet cushion, and caustics from a wineglass. Three decades of research in offline global illumination algorithms has enabled substantial progress towards these goals, and a variety of complex lighting, reflection, shadow and global illumination effects can be rendered. The evidence is for all to see in the form of completely computer-generated movies, or the increasing use of computer graphics in the movie industry to seamlessly combine live action and synthetic elements.

There is another class of applications however, that requires real-time performance. Examples include video games, lighting design for architects and animators, interactive simulation and training, visualization of artifacts for archaeology and e-commerce, and virtual worlds. Historically, there has been a large chasm between interactivity and realism. While offline computer graphics rendering achieved more and more realistic effects, real-time imagery focused on increasing raw performance for geometry and textures. Interactive applications usually did not include complex natural lighting, realistic materials or accurate shading — indeed, cast shadows and global illumination were often completely missing in real-time rendering.

Over the past decade, serious efforts began to be made to bridge this chasm between photorealism and real-time. Two critical developments in computing power played a key role. First, graphics hardware became increasingly fast and flexible. Over a number of years, a new brand of graphics hardware and graphics processing units or GPUs evolved [16, 84], along with associated programming languages [17, 90, 105]. This enabled complex shading models and physically realistic computations to be performed at each vertex or pixel. The focus thus shifted from raw performance to high quality real-time image synthesis; an early paper in this direction is by Heidrich and Seidel [56], and a survey on this topic is by Kautz [67]. A second important development was the retargetting of traditional global illumination and ray tracing methods to modern CPU and GPU hardware. More efficient algorithms adapted to modern hardware, coupled with several iterations of Moore's law, enabled the first methods for real-time ray tracing [54, 106, 114, 131, 135, 136, 149]. A good recent survey of the work on the topic, including animated scenes, is the STAR report by Wald et al. [137].

However, many complex shading effects still proved quite difficult to address at real-time rates. A key challenge is the complexity of illumination. Real scenes are lit, not by a single point light source, but by multiple lights including area sources (and even virtual lights for instant global illumination [73]). Moreover, over the last decade, there has been substantial interest in high-dynamic range representations of full incident illumination, known as an *environment map* [15, 28, 93], where each of the million or more pixels can be viewed as a light source. Furthermore, area and environment lighting create a new type of look, involving more diffuse shading and softer shadows, that is often a desirable mood. However, rendering with multiple lights involves adding up or integrating their contributions. Even if we had an interactive method for a single light source, it will become very slow once we consider the hundreds to thousands of lights needed for realistic incident illumination.

This survey is about a class of techniques that use precomputation to address the challenge of complex illumination. They are collectively referred to as *precomputation-based relighting* or *precomputed radiance transfer* (*PRT*). The key idea is that the final radiance distribution in the scene is linear in the individual light sources — we can first simply render or *precompute* the results offline for each light source or lighting basis function, and then rapidly sum up the results in a second real-time phase. Of course, doing this efficiently, especially when there are thousands of basis lights as in an environment map, is non-trivial and an important intellectual challenge.

It is important to note the key distinction between offline rendering and PRT. Traditional global illumination takes as input the geometry, view, lighting and object material properties, and can produce very realistic images, albeit slowly. However, the algorithm must be re-run completely when any of these attributes change. In real-time rendering applications, we must be able to update the image interactively when, for example, illumination changes, but can often assume that other properties like object geometry remain fixed. Indeed, a precomputation-based approach does require fixing certain scene attributes. The earliest techniques allowed only the lighting to be dynamic, with viewpoint, geometry and materials all fixed. These are essentially approaches for *image relighting*, and the initial seminal work in this area by Nimeroff et al. [99], Dobashi et al. [34], and Dorsey et al. [35] was motivated by applications like a time-sequence of a scene lit by natural illumination, and the interactive design of operatic stage lighting.

Starting in 2002, with the publication of a seminal paper by Sloan et al. [121], the term "*precomputed radiance transfer* (*PRT*)" became common in computer graphics and a topic of renewed interest. One key innovation was the ability to address complex broad-area environment lighting, based on spherical harmonic representations inspired by a theoretical result the previous year on reflection as convolution [11, 108, 110]. A second important idea was to represent the radiance distribution on geometric meshes, enabling the idea to progress from 2D image relighting to rendering on 3D graphics hardware.

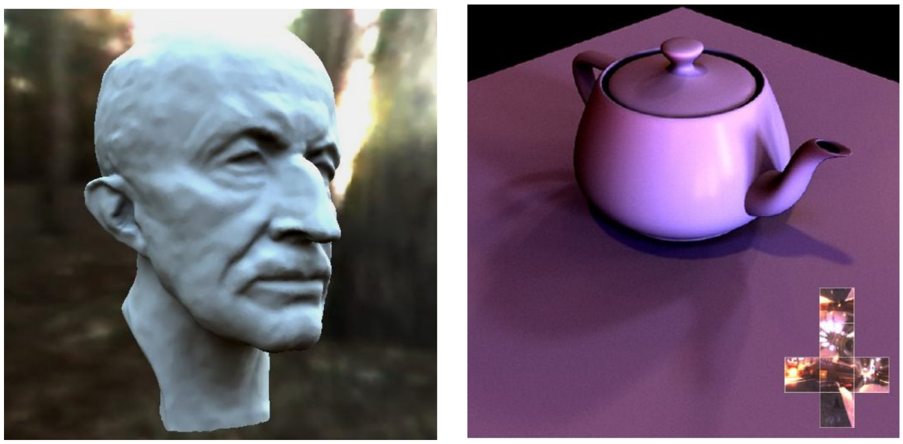

Fig. 1.1 Some examples of the types of renderings produced by Precomputed Radiance Transfer systems. The lighting can be varied dynamically with a variety of complex reflection and shadowing effects rendered in real-time. The image on the left is from the seminal initial work of Sloan et al. [121], while the image on the right is for relighting from a detailed illumination cubemap while preserving shadow and other features at all frequencies [96]. To enable precomputation, the geometry of the scene is assumed fixed in these examples.

Example images from early PRT algorithms [96, 121] are shown in Figure 1.1.

Since its inception, precomputed radiance transfer has led to a large number of papers as well as commercial adoption, with a number of video game companies (e.g., Microsoft and Bungie) incorporating variants, and a version included in Microsoft's DirectX 9 Utility Library. It has also led to a variety of new theoretical insights on basis representations and decompositions and analyses of light transport, that are broadly applicable to other domains as well. In this survey, we present a unified mathematical view of precomputation-based rendering, while discussing its motivation, history, and current and future research directions. Advanced readers may also be interested in a deeper technical discussion of a framework for precomputed and captured light transport by Lehtinen [79].

The remainder of this survey is organized as follows. Section 2 discusses background on the rendering equation and early seminal work in precomputation-based image relighting. Section 3 describes early and recent work on environment maps, a spherical distribution of the lighting in the scene. We also discuss key theoretical results developed in 2001, that showed that an effective representation of light reflection could be achieved using spherical harmonics. Section 4 introduces the 2002 paper by Sloan et al. [121] that has spurred much of the recent work in PRT. Section 5 describes different types of basis representations and compressions that have since been applied to precomputed light transport. Section 6 discusses the relaxation of a number of restrictions including varying viewpoint, materials and dynamic geometry. Section 7 describes some of the recent work on variants of PRT, that address issues like global illumination, lighting design, and volumetric scattering. Following this, Section 8 discusses future research directions. We conclude the survey in Section 9.

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