Full text available at: http://dx.doi.org/10.1561/080000010

Dealing with Endogeneity in Regression Models with Dynamic Coefficients

# Dealing with Endogeneity in Regression Models with Dynamic Coefficients

# **Chang-Jin Kim**

University of Washington USA and Korea University Korea changjin@u.washington.edu



## the essence of knowledge

Boston – Delft

# Foundations and Trends<sup>®</sup> in Econometrics

Published, sold and distributed by: now Publishers Inc. PO Box 1024 Hanover, MA 02339 USA Tel. +1-781-985-4510 www.nowpublishers.com sales@nowpublishers.com

Outside North America: now Publishers Inc. PO Box 179 2600 AD Delft The Netherlands Tel. +31-6-51115274

The preferred citation for this publication is C.-J. Kim, Dealing with Endogeneity in Regression Models with Dynamic Coefficients, Foundations and Trends<sup>(R)</sup> in Econometrics, vol 3, no 3, pp 165–266, 2008

ISBN: 978-1-60198-312-1 © 2010 C.-J. Kim

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, mechanical, photocopying, recording or otherwise, without prior written permission of the publishers.

Photocopying. In the USA: This journal is registered at the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923. Authorization to photocopy items for internal or personal use, or the internal or personal use of specific clients, is granted by now Publishers Inc for users registered with the Copyright Clearance Center (CCC). The 'services' for users can be found on the internet at: www.copyright.com

For those organizations that have been granted a photocopy license, a separate system of payment has been arranged. Authorization does not extend to other kinds of copying, such as that for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale. In the rest of the world: Permission to photocopy must be obtained from the copyright owner. Please apply to now Publishers Inc., PO Box 1024, Hanover, MA 02339, USA; Tel. +1-781-871-0245; www.nowpublishers.com; sales@nowpublishers.com

now Publishers Inc. has an exclusive license to publish this material worldwide. Permission to use this content must be obtained from the copyright license holder. Please apply to now Publishers, PO Box 179, 2600 AD Delft, The Netherlands, www.nowpublishers.com; e-mail: sales@nowpublishers.com

# Foundations and Trends<sup>®</sup> in Econometrics Volume 3 Issue 3, 2008

### **Editorial Board**

#### Editor-in-Chief: William H. Greene

Department of Economics New York University 44 West Fourth Street, 7–78 New York, NY 10012 USA wgreene@stern.nyu.edu

#### Editors

Manuel Arellano, CEMFI Spain Wiji Arulampalam, University of Warwick Orley Ashenfelter, Princeton University Jushan Bai, NYU Badi Baltagi, Syracuse University Anil Bera, University of Illinois Tim Bollerslev, Duke University David Brownstone, UC Irvine Xiaohong Chen, NYU Steven Durlauf, University of Wisconsin Amos Golan, American University Bill Griffiths, University of Melbourne James Heckman, University of Chicago Jan Kiviet, University of Amsterdam Gary Koop, Leicester University Michael Lechner, University of St. Gallen Lung-Fei Lee, Ohio State University Larry Marsh, Notre Dame University James MacKinnon, Queens University Bruce McCullough, Drexel University Jeff Simonoff, NYU Joseph Terza, University of Florida Ken Train, UC Berkeley Pravin Travedi, Indiana University Adonis Yatchew, University of Toronto

## **Editorial Scope**

Foundations and Trends<sup>®</sup> in Econometrics will publish survey and tutorial articles in the following topics:

- Identification
- Model Choice and Specification Analysis
- Non-linear Regression Models
- Simultaneous Equation Models
- Estimation Frameworks
- Biased Estimation
- Computational Problems
- Microeconometrics
- Treatment Modeling
- Discrete Choice Modeling
- Models for Count Data
- Duration Models
- Limited Dependent Variables
- Panel Data
- Dynamic Specification
- Inference and Causality
- Continuous Time Stochastic Models

- Modeling Non-linear Time Series
- Unit Roots
- Cointegration
- Latent Variable Models
- Qualitative Response Models
- Hypothesis Testing
- Interactions-based Models
- Duration Models
- Financial Econometrics
- Measurement Error in Survey Data
- Productivity Measurement and Analysis
- Semiparametric and Nonparametric Estimation
- Bootstrap Methods
- Nonstationary Time Series
- Robust Estimation

#### Information for Librarians

Foundations and Trends<sup>®</sup> in Econometrics, 2008, Volume 3, 4 issues. ISSN paper version 1551-3076. ISSN online version 1551-3084. Also available as a combined paper and online subscription.

Foundations and Trends<sup>®</sup> in Econometrics Vol. 3, No. 3 (2008) 165–266 © 2010 C.-J. Kim DOI: 10.1561/0800000010



## Dealing with Endogeneity in Regression Models with Dynamic Coefficients

### Chang-Jin Kim\*

University of Washington, USA, and Korea University, Korea, changjin@u.washington.edu

#### Abstract

The purpose of this monograph is to present a unified econometric framework for dealing with the issues of endogeneity in Markovswitching models and time-varying parameter models, as developed by Kim (2004, 2006, 2009), Kim and Nelson (2006), Kim et al. (2008), and Kim and Kim (2009). While Cogley and Sargent (2002), Primiceri (2005), Sims and Zha (2006), and Sims et al. (2008) consider estimation of simultaneous equations models with stochastic coefficients as a system, we deal with the LIML (limited information maximum likelihood) estimation of a single equation of interest out of a simultaneous equations model. Our main focus is on the two-step estimation procedures based on the control function approach, and we show how the problem of generated regressors can be addressed in second-step regressions.

<sup>&</sup>lt;sup>\*</sup> The author would like to thank Richard Startz and Yunmi Kim for their helpful comments and suggestions. The author acknowledges financial support from the National Research Foundation of Korea (KRF-2008-342-B00006) and the Bryan C. Cressey Professorship at the University of Washington.

# Contents

1 ]	Introduction	1
]	The Control Function Approach to Dealing with Endogeneity for Models with Constant Coefficients: Basic Framework	5
	Coefficients. Dasic Framework	0
2.1	The Case of i.i.d. Disturbances	5
2.2	The Case of $GARCH(1,1)$ Disturbances	9
2.3	The Case of Serially Correlated Regressors	
	and Disturbances	11
2.4	A Note on the Errors-in-Variables Problem in Rational	
	Expectations Models and Moving-Average Disturbances	15
2.5	Testing for Endogeneity	17
3	Markov-Switching Models with	
]	Endogenous Regressors	19
3.1	When the Relationship Between Endogenous	
	Variables and Instrumental Variables is	
	Time-Invariant: Basic Idea	20
3.2	When the Relationship Between Endogenous Variables	
	and Instrumental Variables is Markov-Switching:	
	A Common Latent State Variable	27

3.3	When the Relationship Between Endogenous Variables and Instrumental Variables is Markov-Switching: Two	
	Potentially Correlated Latent State Variables	31
3.4	When the Relationship Between Endogenous Variables	01
0.1	and Instrumental Variables is Markov-Switching: Further	
	Generalization	39
3.5	Application #1: Is the Backward-Looking Component	00
0.0	Important in a New Keynesian Phillips Curve?	45
3.6	Application #2: The Nature of the Structural Break	10
0.0	in the Excess Sensitivity of Consumption to Income	49
	In the Excess Sensitivity of Consumption to income	49
4	Markov-Switching Models with Endogenous	
	Switching: When the State Variable and	
	Regression Disturbance are Correlated	53
1	Regression Disturbance are Correlated	00
4.1	Model Specification	54
4.2	Derivation of the Hamilton Filter and the Log-Likelihood	
	Function	55
4.3	An Example in Finance: A Volatility Feedback	
	Model of Stock Returns	58
5 1	Time-Varying Parameter Models with Endogenous	
	Regressors	<b>65</b>
۲ 1	When the Deletion Determine the Findemanney Variables	
5.1	When the Relation Between the Endogenous Variables	07
5.0	and Instrumental Variables is Time-Invariant	67
5.2	When the Relation Between the Endogenous Variables	70
50	and Instrumental Variables is Time-Varying	73
5.3	When the Relation Between the Endogenous Variables	
	and Instrumental Variables is Time-Varying: An	
	Alternative Two-Step Approach and a Practical Solution	
	When $K$ is Large	79
5.4	Generalization and an Application: Estimation	
	of a Forward-Looking Monetary Policy Rule	
	with Time-Varying Responses to Inflation	
	and Output Gap	85

6	Concluding Remarks	97

99



Consider the following regression model with dynamic coefficients:

$$y_t = x'_t \beta_t + e_t, \ e_t \sim N(0, \sigma_{e,t}^2),$$
 (1.1)

where  $y_t$  is  $1 \times 1$  and  $x_t$  is a  $K \times 1$  vector of regressors. The  $K \times 1$  vector of regression coefficients  $\beta_t$  is stochastic and time-dependent. Depending on the assumptions on the stochastic nature of  $\beta_t$ , we have either a Markov-switching model or a time-varying parameter model. For a time-varying parameter model, we conventionally assume that  $\beta_t$  is subject to a continuous shock. For example, we assume that:

$$\beta_t = \beta_{t-1} + v_t, \quad v_t \sim i.i.d.N(0,Q),$$
(1.2)

where the  $K \times K$  matrix Q is the variance–covariance matrix of  $v_t$ . For a Markov-switching model,  $\beta_t$  is subject to a discrete shock. For example, we assume that  $\beta_t$  is dependent upon a first-order, J-state Markov-switching process  $S_t$  in the following way:

$$\beta_t = \beta_1 S_{1,t}^{\dagger} + \beta_2 S_{2,t}^{\dagger} + \dots + \beta_J S_{J,t}^{\dagger}, \qquad (1.3)$$

$$S_{j,t}^{\dagger} = \begin{cases} 1, & \text{if } S_t = j; \quad j = 1, 2, \dots J \\ 0, & \text{otherwise,} \end{cases}$$
(1.4)

#### 2 Introduction

where the transitional dynamics of  $S_t$  are defined as:

$$Pr[S_t = j | S_{t-1} = i] = p_{ji}, \quad \sum_{j=1}^J p_{ji} = 1.$$
 (1.5)

Time-varying parameter models, which date to Cooley and Prescott (1973, 1976), Rosenberg (1973), Sarris (1973), and Markov regimeswitching models, originally introduced by Goldfeld and Quandt (1973) and further extended by Hamilton (1989), and have been widely used in modeling instability in economic relations.<sup>1</sup> With a growing body of recent empirical evidence on widespread instability in macroeconomic relations (Diebold, 1998; Stock and Watson, 1998; Perron and Qu, 2007), the importance of these time series models with dynamic coefficients has been recognized by more macroeconomists and financial economists than ever before.

However, almost all applications of these models so far have been limited to the cases of exogenous regressors or exogenous coefficients, with the following assumptions:

Assumption #1: 
$$E(e_t|x_t) = 0$$
 (1.6)

Assumption #2: 
$$E(e_t|\beta_t) = 0.$$
 (1.7)

When either one of the above assumptions is violated, inferences about the model based on the conventional Kalman (1960) filter or the conventional Hamilton (1989) filter are invalid. In particular, in the case of endogenous regressors where Assumption #1 is violated, one is tempted to employ the conventional two-step procedure. That is, defining  $z_t$  to be a vector of instrumental variables, one may regress  $x_t$  on  $z_t$  to get  $\hat{x}_t$ , the fitted value of  $x_t$ , in the first step and then estimate Equation (1.1) by replacing  $x_t$  with  $\hat{x}_t$  in the second step. However, this conventional two-step procedure is problematic when the regression coefficients are stochastic. For example, even in the case in which  $e_t$ in Equation (1.1) is independent and identically distributed (i.i.d.), the disturbance term in the second-step regression is heteroscedastic. Ignoring this heteroscedasticity would result in inefficiency in the two-step

<sup>&</sup>lt;sup>1</sup> For a comprehensive review of these models, readers are referred to Kim and Nelson (1999).

estimation of the model. A more serious issue is that, unlike the case of constant coefficients, it is not easy to solve the problem of generated regressors in calculating the standard errors of the coefficient estimators.

The purpose of this monograph is to present a unified econometric framework for dealing with the issues of endogeneity in Markovswitching models and time-varying parameter models, as developed by Kim (2004, 2006, 2009), Kim and Nelson (2006), Kim et al. (2008), and Kim and Kim (2009). Note that, while Cogley and Sargent (2002), Primiceri (2005), Sims and Zha (2006), and Sims et al. (2008) consider estimation of simultaneous equations models with stochastic coefficients as a system, we focus on the LIML (limited information maximum likelihood) estimation of a single equation of interest out of a simultaneous equations model.

The control function approach, which is an econometric method used to correct for biases that arise as a consequence of selection or endogeneity, will be the main tool in dealing with the problem of endogeneity throughout this article. While the approach has been extensively applied to the sample-selection models and disequilibrium models in the microeconometrics literature, its application in the time-series econometrics literature is relatively new. The basic idea behind the control function is to model the dependence of the disturbance term on the endogenous variables in a way that allows us to construct a function such that, conditional on the function, the endogeneity problem in the regression equation of interest disappears. For example, in the case of a linear regression with constant coefficients, the two-step estimation procedure based on the control function approach proceeds as follows. In the first step, the residuals of the reduced-form equations for the endogenous regressors are estimated. Then, in the second step, the primary equation of interest is estimated with these residuals included as additional regressors.

The outline of this monograph is as follows. In Section 2, we review the basic issues associated with the control function approach, which is the main tool for dealing with endogeneity in this monograph. We investigate these issues within the framework of constant regression coefficients. In Section 3, we consider estimation of Markov-switching

3

#### 4 Introduction

models with endogenous regressors, by dropping Assumption #1 in Equation (1.6) but maintaining Assumption #2 in Equation (1.7). Section 4 deals with estimation of a Markov-switching model in which Assumption #1 is maintained but Assumption #2 is dropped. In this model, while the regressors are exogenous or predetermined, the Markov-switching coefficients are correlated with regression disturbances. The issues of endogeneity within the time-varying parameter models are discussed in Section 5. In Sections 3–5, we will see how the basic Hamilton (1989) filter and the basic Kalman (1960) filter can be modified to deal with different types of endogeneity. Furthermore, we will see how the problem of generated regressors in the two-step procedure can be addressed, in light of Pagan (1984) and the results in Section 2. Finally, Section 6 provides concluding remarks.

- Adam, K. and M. Padula (2003), 'Inflation dynamics and subjective expectations in the United States'. ECB Working Paper No. 222.
- Belsley, D. (1973), 'On the determination of systematic parameter variation in the linear regression model'. Annals of Economic and Social Measurement 2, 487–494.
- Boivin, J. (2006), 'Has U.S. monetary policy changed? Evidence from drifting coefficients and real-time data'. *Journal of Money, Credit* and Banking 38(5), 1149–1174.
- Campbell, J. Y. and L. Hentschel (1992), 'No news is good news: An asymmetric model of changing volatility in stock returns'. *Journal of Financial Economics* **31**, 281–318.
- Campbell, J. Y. and N. G. Mankiw (1989), 'Consumption, income, and interest rates: Reinterpreting the time series evidence'. In: O. J. Blanchard and S. Fischer (eds.): National Bureau of Economic Research Macroeconomics Annual 1989. Cambridge, MA: MIT Press, pp. 185– 216.
- Campbell, J. Y. and N. G. Mankiw (1991), 'The response of consumption to income: A cross-country investigation'. *European Economic Review* 35, 723–67.

- Carroll, C. (2003), 'Macroeconomic expectations of households and professional forecasters'. Quarterly Journal of Economics 118(1), 269–298.
- Chib, S. (1998), 'Estimation and comparison of multiple change-point models'. Journal of Econometrics 86, 221–241.
- Chow, G. C. (1984), 'Random and changing coefficient models'. In:Z. Griliches and M. D. Intriligator (eds.): *Handbook of Econometrics*. North Holland: Amsterdam, Chapter 21.
- Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005), 'Nominal rigidities and the dynamic effects of a shock to monetary policy'. *Journal of Political Economy* **113**(1), 1–45.
- Clarida, R., J. Gali, and M. Gertler (2000), 'Monetary policy rules and macroeconomic stability: Evidence and some theory'. *Quarterly Journal of Economics* pp. 147–180.
- Cogley, T. and T. J. Sargent (2002), 'Evolving post-world war II U.S. inflation dynamics'. In: *NBER Macroeconomics Annual 16*. Cambridge, MA: MIT Press, pp. 331–373.
- Cogley, T. and T. J. Sargent (2005), 'Drifts and volatilities: Monetary policies and outcomes in the post world war II U.S.'. *Review of Economic Dynamics* 8, 262–302.
- Cogley, T. and A. M. Sbordone (2008), 'Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips curve'. *American Economic Review* 98(5), 2101–2126.
- Cooley, T. F. and E. C. Prescott (1973), 'Varying parameter regression: A theory and some applications'. Annals of Economic and Social Measurement 2, 463–473.
- Cooley, T. F. and E. C. Prescott (1976), 'Tests of an adaptive regression model'. Estimation in the Presence of Stochastic Parameter Variation, Econometrica 44, 167–184.
- Diebold, F. X. (1998), 'The past, present, and future of macroeconomics'. Journal of Economic Perspectives 12, 175–19.
- Diebold, F. X., J.-H. Lee, and G. C. Weinbach (1994), 'Regime switching with time-varying transition probabilities'. In: C. Hargreaves (ed.): Nonstationary Time Series Analysis and Cointegration. Oxford: Oxford University Press.

#### Full text available at: http://dx.doi.org/10.1561/080000010

- Engle, R. F. and M. W. Watson (1987), 'The Kalman filter model: Applications to forecasting and rational expectations'. In: T. Bewley (ed.): Advances in Econometrics: Fifth World Congress of Econometric Society. Cambridge, U.K.: Cambridge University Press.
- Erceg, C. J. and A. T. Levin (2003), 'Imperfect credibility and inflation persistence'. Journal of Monetary Economics 50, 915–944.
- Filardo, A. J. (1994), 'Business-cycle phases and their transitional dynamics'. Journal of Business and Economic Statistics 12, 299–308.
- Fuhrer, J. C. (1997), 'Inflation/output variance trade-offs and optimal monetary policy'. Journal of Money, Credit, and Banking 29, 214–234.
- Fuhrer, J. C. and G. R. Moore (1995), 'Inflation persistence'. Quarterly Journal of Economics 110, 127–159.
- Gali, J. and M. Gertler (1999), 'Inflation dynamics: A structural econometric analysis'. Journal of Monetary Economics 44(2), 195–222.
- Gali, J., M. Gertler, and J. D. Lopez-Salido (2001), 'European inflation dynamics'. *European Economic Review* 44(7), 1237–1270.
- Goldfeld, S. M. and R. E. Quandt (1973), 'A Markov model for switching regressions'. Journal of Econometrics 1, 3–16.
- Hamilton, J. D. (1989), 'A new approach to the economic analysis of nonstationary time series and the business cycle'. *Econometrica* 57(2), 357–384.
- Harvey, A. C. (1981), *Time Series Models*. Oxford: Philip Allan and Humanities Press.
- Harvey, A. C. (1991), Forecasting, Structural Time Series Models, and the Kalman Filter. Cambridge, U.K.: Cambridge University Press.
- Harvey, A. C., E. Ruiz, and E. Sentana (1992), 'Unobserved component time series models with ARCH disturbances'. *Journal of Econometrics* 52, 129–157.
- Heckman, J. (1979), 'Sample selection bias as a specification error'. *Econometrica* 47(1), 153–162.
- Kalman, R. E. (1960), 'A new approach to linear filtering and prediction problems'. Transactions of the ASME Journal of Basic Engineering D82, 35–45.
- Kim, C.-J. (1994), 'Dynamic linear models with Markov-switching'. Journal of Econometrics 60, 1–22.

- Kim, C.-J. (2004), 'Markov-switching models with endogenous explanatory variables'. Journal of Econometrics 122, 127–136.
- Kim, C.-J. (2006), 'Time-varying parameter models with endogenous regressors'. *Economic Letters* **91**, 21–26.
- Kim, C.-J. (2009), 'Markov-switching models with endogenous explanatory variables II: A two-step MLE procedure'. *Journal of Econometrics* 148, 46–55.
- Kim, C.-J. and S. Chaudhuri (2009), 'Markov-switching models with endogenous explanatory variables: Generalization'. Working Paper, University of Washington and University of North Carolina.
- Kim, C.-J. and Y. Kim (2008), 'Is the backward-looking component important in a New Keynesian Phillips curve?'. Studies in Nonlinear Dynamics and Econometrics 12(3), Article 5.
- Kim, C.-J., J. C. Morley, and C. R. Nelson (2004), 'Is there a positive relationship between stock market volatility and the equity premium?'. *Journal of Money, Credit and Banking* 36, 339–360.
- Kim, C.-J. and C. R. Nelson (1999), State-Space Models with Regime-Switching: Classical and Gibbs-Sampling Approaches with Applications. MIT Press.
- Kim, C.-J. and C. R. Nelson (2006), 'Estimation of a forward-looking monetary policy rule: A time-varying parameter model using ex-post data'. *Journal of Monetary Economics* 53, 1949–1966.
- Kim, C.-J., J. Piger, and R. Startz (2008), 'Estimation of Markov regime-switching regression models with endogenous switching'. *Journal of Econometrics* 143(2), 263–273.
- Kim, Y. and C.-J. Kim (2009), 'Dealing with endogeneity in a timevarying-parameter model: Joint estimation and two-step estimation procedures'. Working Paper, University of Washington and University of Manitoba.
- Kozicki, S. and P. A. Tinsley (2001), 'Term structure views of monetary policy under alternative models of agent expectations'. *Journal of Economic Dynamics and Control* 23, 149–184.
- Kozicki, S. and P. A. Tinsley (2002), 'Alternative sources of the lag dynamics of inflation'. Federal Reserve Bank of Kansas City, RWP 02-12.

#### Full text available at: http://dx.doi.org/10.1561/080000010

- Lucas, R. E. (1976), 'Econometric policy evaluation: A critique'. Carnegie-Rochester Conference Series on Public Policy 1, 19–46.
- Maddala, G. and F. Nelson (1974), 'Maximum likelihood methods for models of markets in disequilibrium'. *Econometrica* 42(6), 1013– 1030.
- Maddala, G. S. (1983), Limited-Dependent and Qualitative Variables in Econometrics. Econometric Society Monographs No. 3, Cambridge University Press.
- Maddala, G. S. and F. Nelson (1975), 'Switching regression models with exogenous and endogenous switching'. Proceedings of the American Statistical Association pp. 423–426.
- Monokroussos, G. (2009), 'Dynamic limited dependent variable modeling and U.S. Monetary policy'. Forthcoming, in *Journal of Money*, *Credit*, and *Banking*.
- Orphanides, A. (2004), 'Monetary policy rules, macroeconomic stability and inflation: A view from the trenches'. *Journal of Money, Credit* and Banking **35**(6).
- Pagan, A. (1984), 'Econometric issues in the analysis of regressions with generated regressors'. *International Economic Review* 25(1), 221–247.
- Perron, P. and J. Qu (2007), 'Estimating and testing multiple structural changes in multivariate regressions'. *Econometrica* 75(2007), 459–502.
- Primiceri, G. E. (2005), 'Time varying structural vector autoregressions and monetary policy'. *Review of Economic Studies* 72, 821–852.
- Quandt, R. E. (1958), 'The estimation of the parameters of a linear regression system obeying two separate regimes'. Journal of the American Statistical Association 53, 873–880.
- Roberts, J. M. (1995), 'New Keynesian economics and the Phillips curve'. Journal of Money, Credit and Banking 27(4), 975–984.
- Roberts, J. M. (1997), 'Is inflation sticky?'. Journal of Monetary Economics 39, 176–196.
- Roberts, J. M. (1998), 'Inflation expectations and the transmission of monetary policy'. Federal Reserve Board FEDS working paper 1998-43.

- Rosenberg, B. (1973), 'The analysis of a cross-section of time series by stochastically convergent parameter regression'. Annals of Economic and Social Measurement 2, 399–428.
- Sarris, A. H. (1973), 'A bayesian approach to estimation of time-varying regression coefficients'. Annals of Economic and Social Measurement 2, pp. 501–524.
- Sbordone, A. (2001), 'Price and unit labor costs: A new test of price stickiness'. Journal of Monetary Economics 49, 265–292.
- Sbordone, A. (2002), 'An optimizing model of U.S. wage and price dynamics'. Proceedings, Federal Reserve Bank of San Francisco, Mar.
- Sims, C. A., D. F. Waggoner, and T. Zha (2008), 'Methods for inference in large multiple-equation Markov-switching models'. *Journal* of Econometrics 146, 255–274.
- Sims, C. A. and T. Zha (2006), 'Were there regime switches in US Monetary policy?'. American Economic Review 96(1), 54–81.
- Stock, J. H. and M. W. Watson (1998), 'Evidence on structural instability in macroeconomic time series relations'. *Journal of Business* and Economic Statistics 14, 11–30.
- Turner, C. M., R. Startz, and C. R. Nelson (1989), 'A Markov model of heteroscedasticity, risk, and learning in the stock market'. *Journal* of Financial Economics 25, 3–22.