Port-Hamiltonian Systems Theory: An Introductory Overview

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Dedicated to the memory of Jan C. Willems,
inspiring teacher and friend.

Contents

Abstract

An up-to-date survey of the theory of port-Hamiltonian systems is given, emphasizing novel developments and relationships with other formalisms. Port-Hamiltonian systems theory yields a systematic framework for network modeling of multi-physics systems. Examples from different areas show the range of applicability. While the emphasis is on modeling and analysis, the last part provides a brief introduction to control of port-Hamiltonian systems.

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1

Introduction

1.1 Origins of port-Hamiltonian systems theory

The theory of port-Hamiltonian systems brings together different traditions in physical systems modeling and analysis.

Firstly, from a modeling perspective it originates in the theory of *port-based modeling* as pioneered by Henry Paynter in the late 1950s Paynter (1960); Breedveld (1984, 2009). Port-based modeling is aimed at providing a unified framework for the modeling of systems belonging to different physical domains (mechanical, electrical, hydraulic, thermal, etc.). This is achieved by recognizing *energy* as the 'lingua franca' between physical domains, and by identifying ideal system components capturing the main physical characteristics (energy-storage, energy-dissipation, energy-routing, etc.). Historically port-based modeling comes along with an insightful graphical notation emphasizing the structure of the physical system as a collection of ideal components linked by edges capturing the energy-flows between them. In analogy with chemical species these edges are called *bonds*, and the resulting graph is called a *bond graph*. Motivated by, among others, electrical circuit theory the energy flow along the bonds is represented by pairs of variables, whose product equals power. Typical examples of such pairs of variables (in different physical domains) are voltages and currents, velocities and forces, flows and pressures, etc.. A port-Hamiltonian formulation of bond graph models can be found in Golo et al. (2003). Port-based modeling can be seen to be a further abstraction of the theory of *across* and *through* variables (cf. MacFarlane (1970)) in the network modeling of physical systems[1].

A second origin of port-Hamiltonian systems theory is *geometric mechanics*; see e.g. Arnol'd (1978); Abraham & Marsden (1994); Marsden & Ratiu (1999); Bloch (2003); Bullo & Lewis (2004). In this branch of mathematical physics the Hamiltonian formulation of classical mechanics is formalized in a geometric way. The basic paradigm of geometric mechanics is to represent Hamiltonian dynamics in a coordinate-free manner using a state space (commonly the phase space of the system) endowed with a symplectic or Poisson structure, together with a Hamiltonian function representing energy. This geometric approach has led to an elegant and powerful theory for the analysis of the complicated dynamical behavior of Hamiltonian systems, displaying their intrinsic features, such as symmetries and conserved quantities, in a transparant way. Also infinite-dimensional Hamiltonian systems have been successfully cast into this framework Olver (1993).

Finally, a third pillar underlying the framework of port-Hamiltonian systems is *systems and control theory*, emphasizing dynamical systems as being open to interaction with the environment (e.g. via inputs and outputs), and as being susceptible to control interaction. The description and analysis of physical subclasses of control systems has roots in electrical network synthesis theory. Its geometric formulation was especially pioneered in Brockett (1977); see e.g. van der Schaft (1984, 1982a,b); Crouch (1981, 1984); Crouch & van der Schaft (1987); Nijmeijer & van der Schaft (1990); Maschke & van der Schaft (1992); Bloch (2003); Bullo & Lewis (2004) for some of the main developments, especially with regard to the anal-

[1]'Abstraction' since the theory of across and through variables emphasizes the *balance laws* in the system; an aspect which is usually not emphasized in port-based modeling. In Chapter 12 and in Chapter 14 we will see how port-Hamiltonian systems can be also defined starting with the basic balance laws of the system.

ysis and control of nonlinear mechanical systems (e.g. with nonholonomic kinematic constraints).

A main difference of port-Hamiltonian systems theory with geometric mechanics lies in the fact that for port-Hamiltonian systems the underlying geometric structure is not necessarily the symplectic structure of the phase space, but in fact is determined by the *interconnection structure* of the system. In this sense port-Hamiltonian systems theory intrinsically merges *geometry* with *network theory*. The appropriate geometric object appears to be the notion of a *Dirac structure*, which was explored before in Weinstein (1983); Courant (1990); Dorfman (1993) as a geometric object generalizing at the same time symplectic and Poisson structures[2]. The usefulness of Dirac structures for a geometric theory of port-based modeling and analysis was first recognized in van der Schaft & Maschke (1995); Bloch & Crouch (1999); Dalsmo & van der Schaft (1999). Among others it has led to a theory of Hamiltonian *differential-algebraic equations*. Extensions to the distributed-parameter case were first explored in van der Schaft & Maschke (2002). A key property of Dirac structures is the fact that compositions of Dirac structures are again Dirac structures. This has the crucial consequence that the power-conserving interconnection of port-Hamiltonian systems (through their external ports) is again a port-Hamiltonian system; a fundamental property for network modeling and control.

Another main extension of port-Hamiltonian systems theory with respect to geometric mechanics is the inclusion of *energy-dissipating elements*, which are largely absent in classical Hamiltonian systems. This greatly broadens the range of applicability of port-Hamiltonian systems compared to that of Hamiltonian systems in analytical dynamics. In fact, the framework of port-based modeling and port-Hamiltonian systems emerges as a general theory for the modeling of complex physical systems as encountered in many areas of engineering[3]. Fur-

[2]The terminology 'Dirac structure' seems to be largely inspired by the 'Dirac bracket' introduced by Paul Dirac in order to cope with Hamiltonian systems subject to constraints due to degeneracy of the underlying Lagrangian function Dirac (1950, 1958). This was motivated in its turn by quantization theory.

[3]It should be added here that our emphasis in physical system modeling is on

thermore, because of its emphasis on *energy* and *power* as the lingua franca between different physical domains, port-Hamiltonian systems theory is ideally suited for a systematic mathematical treatment of *multi-physics systems*, i.e., systems containing subsystems from different physical domains (mechanical, electro-magnetic, hydraulic, chemical, etc.).

Apart from offering a systematic and insightful framework for modeling and analysis of multi-physics systems, port-Hamiltonian systems theory provides a natural starting point for control. Especially in the nonlinear case it is widely recognized that physical properties of the system (such as balance and conservation laws and energy considerations) should be exploited and/or respected in the design of control laws which are robust and physically interpretable. Port-Hamiltonian systems theory offers a range of concepts and tools for doing this, including the shaping of energy-storage and energy-dissipation, as well as the interpretation of controller systems as virtual system components. In this sense, port-Hamiltonian theory is a natural instance of a 'cyber-physical' systems theory: it admits the extension of physical system models with virtual ('cyber') system components, which may or may not mirror physical dynamics. From a broader perspective port-Hamiltonian systems theory is also related to multi-physics[4] network modeling approaches aimed at numerical *simulation*, such as 20-sim (based on bond graphs) and Modelica/Dymola.

1.2 Summary of contents

In these lecture notes we want to highlight a number of directions in port-Hamiltonian systems theory. Previous textbooks covering material on port-Hamiltonian systems are van der Schaft (2000) (Chapter 4), and Duindam et al. (2009). Especially Duindam et al. (2009) goes into more detail about a number of topics, and presents a wealth of

'modeling for control'. Since the addition of control will anyway modify the dynamical properties of the system the emphasis is on relatively simple models reflecting the main dynamical characteristics of the system.

[4]For specific physical domains (e.g., mechanical, electrical, chemical, hydraulic, ..) there are many network modeling and simulation software packages available.

material on various application domains. The current lecture notes present an up-to-date account of the basic theory, emphasizing novel developments.

Chapter 2 provides the basic definition of port-Hamiltonian systems and elaborates on the concept of a Dirac structure. Chapter 3 deals with Dirac structures on manifolds, and the resulting definition of port-Hamiltonian systems on manifolds. A brief discussion concerning integrability of Dirac structures is given, and the relation with the theory of kinematic constraints is provided. Chapter 4 details the special, but important, subclass of input-state-output port-Hamiltonian systems arising from the assumption of absence of algebraic constraints and the linearity of energy-dissipation relations. The resulting class of port-Hamiltonian systems is often taken as the starting point for the development of control theory for port-Hamiltonian systems.

With the general definition of port-Hamiltonian systems given in a geometric, coordinate-free, way, it is for many purposes important to represent the resulting dynamics in suitable coordinates, and in a form that is convenient for the system at hand. Chapter 5 shows how this amounts to finding a suitable representation of the Dirac structure, and how one can move from one representation to another. In Chapter 6 it is discussed how the power-conserving interconnection of port-Hamiltonian systems again defines a port-Hamiltonian system. This fundamental property of port-Hamiltonian system is based on the result that the composition of Dirac structures is another Dirac structure. Chapter 7 investigates the close connection of port-Hamiltonian systems with the concept of passivity, which is a key property for analysis and control. In Chapter 8 other structural properties of port-Hamiltonian systems are studied, in particular the existence of conserved quantities (Casimirs) and algebraic constraints.

Chapter 9 takes a step in a new direction by replacing the composition of the Dirac structure and the resistive structure by a general maximal monotone relation, leading to the novel class of incrementally port-Hamiltonian systems. In Chapter 10 the relation of port-Hamiltonian systems with the older class of input-output Hamiltonian systems is explored, and the key property of preservation of stability of input-output Hamiltonian systems under positive feedback (in contrast with negative feedback for port-Hamiltonian and passive systems) is discussed. Finally Chapter 11 makes the connection of port-Hamiltonian systems to another class of systems, namely the pseudo-gradient systems extending the Brayton-Moser equations of electrical circuits.

Chapter 12 deals with port-Hamiltonian systems on graphs, starting from the basic observation that the incidence structure of the graph defines a Poisson structure on the space of flow and effort variables associated to the vertices and edges of the graph. This is illustrated on a number of examples. In Chapter 13 the framework is extended to switching port-Hamiltonian systems, including a formulation of a jump rule generalizing the classical charge and flux conservation principle from electrical circuits with switches. Chapter 14 deals with the port-Hamiltonian formulation of distributed-parameter systems, based on the formulation of the Stokes-Dirac structure expressing the basic balance laws. Finally, Chapter 15 gives an introduction to the control theory of port-Hamiltonian systems, exploiting their basic properties such as passivity and existence of conserved quantities.

What is not in these lecture notes

The overview of port-Hamiltonian systems theory presented in this article is far from being complete: a number of topics are not treated at all, or only superficially. Notable omissions are the theory of scattering of port-Hamiltonian systems Stramigioli et al. (2002); van der Schaft (2009), treatment of symmetries and conservation laws of port-Hamiltonian systems van der Schaft (1998); Blankenstein & van der Schaft (2001), controllability and observability for input-output Hamiltonian systems and port-Hamiltonian systems van der Schaft (1984, 1982a,b); Maschke & van der Schaft (1992), realization theory of input-output Hamiltonian systems and port-Hamiltonian systems Crouch & van der Schaft (1987), port-Hamiltonian formulation of thermodynamical systems Eberard et al. (2007), model reduction of port-Hamiltonian systems Polyuga & van der Schaft (2011), well-posedness and stability of distributed-parameter port-Hamiltonian systems Villegas (2007); Jacob & Zwart (2012), and structure-preserving discretization of distributed-parameter port-Hamiltonian systems Golo et al. (2004); Seslija et al. (2012). Furthermore, Chapter 15 on control of port-Hamiltonian systems only highlights a number of the developments in this area; for further information we refer to the extensive literature including Ortega et al. (2001a,b); Duindam et al. (2009); Ortega et al. (2008).

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