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Entropy, Double Entry Accounting and Quantum Entanglement

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Contents

Entropy, Double Entry Accounting and Quantum Entanglement

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ABSTRACT

This monograph analyzes accounting using information theory developed by Claude Shannon and others. A three-way framing equivalence is derived (i) when states are observable; and (ii) when states are not observable and only a signal is observable where the signal reports the state with error. The equivalence establishes equality of accounting numbers, firm rate of return, and the amount of information available to the firm where Shannon's entropy is the information metric. The major assumptions used in deriving the state observable equivalences are constant relative risk aversion preferences, arbitrage free prices, and geometric mean accounting valuation. State unobservability is modeled using the quantum axioms, and, hence, quantum probabilities; the state is unobservable in the same way quantum objects are unobservable. The state observable equivalence is seen to be a special case of the state unobservable equivalence.

Quantum probabilities allow analyzing the effects of entanglement, a phenomenon not occurring when classical probabilities are used. Entanglement is seen to be a powerful

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economic force, and caused by instantaneous communication of information. We speculate double entry accounting can be a mechanism for creating entanglement effects as (i) double entry accounting conveys information relevant to the expected return maximization and entropy reduction; and (ii) it does so instantaneously as the same number is simultaneously available in two places (due to double entry).

Keywords: double entry; entropy; entanglement; instantaneous information transfer; uncertainty; Shannon theory; quantum information processing; quantum computation

1

Introduction

The double entry accounting system is over five centuries old. In that time, commerce and technology have changed in dramatic and unforeseen ways. The double entry system, however, continues to survive and even thrive. The reasons for its durability and longevity remain something of a mystery. In this monograph we offer some speculations about the reasons for double entry accounting's apparent usefulness. To arrive at a place where speculations can be made requires some work. The work is done in three steps.

Step one derives a framing equivalence which establishes an equality between expected rate of return (denominated in dollars) and entropy (denominated in probabilities).

$$E[r|X] = r_f + H(p) - H(p|X) \tag{1.1}$$

$E[r|X]$ is the expected rate of return given an information source X; r_f is the risk-free rate of return. $H(p) = -\sum_i p_i \ln(p_i)$ is the entropy of a probability distribution specified by state probabilities p_i and measures the amount of uncertainty (Shannon, 1948). $H(p|X)$ is the entropy of conditional probabilities derived from X.

The framing equivalence changes the frame of analysis from economic decision making with dollars to an information theory context with probabilities. This allows interrogating economic decisions in an information frame using entropy and related information concepts. Some other academic disciplines have made intellectual progress by changing the frame to information. For example, the science of thermodynamics originally confronted questions involving heat and energy. Changing the frame to information allowed resolving long-standing paradoxes in thermodynamics, notably the puzzle of Maxwell's demon.[1]

We propose to follow a similar line of inquiry in accounting: the attempt is to illuminate accounting questions by working in the information frame; deriving the framing equivalence (1.1) is the first step. Questions about the economic rate of return, $E\left[r|X\right]$, can be reframed as questions about information. An increase (decrease) in $E\left[r|X\right]$ due to information source X equals a decrease (increase) in the entropy expression; the latter is the metric for the amount of information in source X in our analysis.

$$\text{Information in } X = H\left(p\right) - H\left(p|X\right)$$

In addition, using geometric mean accounting (that is, the book value of the assets at the end of the period is valued at the geometric mean of returns), we establish that $E\left[r|X\right]$ is the logarithm of one plus the accounting rate of return, and, hence, the three equivalence relations.

$$\ln\left(1 + \frac{Income}{Assets}\right) = E[r|X] = r_f + H(p) - H\left(p|X\right)$$

Three assumptions are required to derive these equivalence relations: constant relative risk aversion preferences, arbitrage free prices, and geometric mean accounting valuation.

The second step is to invoke the assumption that the states of the world are not directly observable; all that can be observed is a measurement about the state where the measurement includes some random errors.[2] The existence of unobservable states makes the analysis of information source X more interesting. In particular, state unobservability

[1] Good references on this topic include Seife (2007) and Sen (2021).

[2] The analysis begins with the specification of a state-act-outcome matrix wherein outcomes are jointly determined by acts (controllable) and states of the world (not controllable). State unobservability is the only difference between the first and the second steps. As shown later, state unobservability causes a decrease in the value of information.

decreases the benefit associated with the information source (quantified by $E\left[r|X\right] - E\left[r\right]$). We denote the decreases in the benefit of X as the "observability gap." We propose to investigate the determinants of the gap and the possible ways to reduce the gap.

The signal is modeled using the axioms of quantum information as in Nielsen and Chuang (2004). Notably, the state of the world is unobservable in (exactly) the same way as a quantum state is unobservable. A revised framing equivalence is derived under the conditions of state unobservability,

$$\ln\left(1 + \frac{Income}{Assets}\right) = E[r|X] = r_f + H(\lambda) - H\left(\lambda|X\right), \tag{1.2}$$

where λ_j is the (quantum) probability of a particular measurement j determined by the quantum axioms. Quantum axioms imply that the quantum probabilities are defined on a vector space; this is in contrast to the probabilities (of classical states) that are defined on a set.[3] When a probability is defined over a vector, it is a more general characterization and can be used to describe richer (and more interesting) settings. For us, the enrichment is entanglement. Here we are not presuming that economic phenomena are like quantum phenomena in a small scale. Instead, we use quantum axioms to account for state unobservability and to explore a richer probability analysis.

We should emphasize how we view the role of the "qubits;" what they are and what they do in our setting. In the quantum world qubits, quantum bits, are used to describe the states of the world—qubits convey information about the states; based on quantum axioms, qubits are vectors. We view qubits as equivalent to the decision-making units in the firm: individuals, say, or divisions, or processes; wherever firm decisions are made. What qubits do is to acquire and process information. Acquisition of information reduces entropy and, by the framing equivalences (1.2), entropy is all that matters to explain the expected rate of return. If entropy goes down, the expected rate of return goes up by exactly the same amount.

[3]The contrast between the classical probabilities and the quantum probabilities is discussed and illustrated in Bradley (2019a,b).

In step three we run thought experiments. Taking advantage of the framing equivalences (1.1) and (1.2), it is much more convenient to run the thought experiments in an information frame. The purpose of the thought experiments is to determine how to achieve more efficient information processing by the qubits: more efficient in the sense that entropy is reduced; and, hence, the expected rate of return is increased (the observability gap is reduced). The answer to more efficient information processing is entanglement. Entanglement reduces entropy.

The quantum axioms, while supplying a definition of entanglement, don't explain very well what entanglement does. That's what the Bell Theorem does; it explains what entanglement does. Entanglement is a powerful way to communicate information. One possible manifestation of entanglement is when one qubit acquires information, that information is instantly available to another (entangled) qubit.

It is at this point that we (finally) feel justified in engaging in some speculation about the role and efficiency of double entry accounting in an information processing setting. Because of the framing equivalences, accounting numbers reflect the same phenomena as captured in the expected rate of return as well as the entropy reduction. Double entry supplies a mechanism for (virtually) simultaneous communication of information and, hence, can serve as a way to exploit the benefits of entanglement among information processors (decision-makers). In particular, writing down the same number twice ensures it arrives simultaneously to two different decision-makers. As all transactions are captured in the accounting system, much information can be dispersed throughout the firm in this fashion.

Admittedly, the connections we make between double entry accounting and entanglement are speculative. Nonetheless, given the remarkable explanatory power of entanglement with respect to information in the physical world, we view the speculations as potentially fruitful.

Shannon's entropy was introduced in a significant way to the accounting literature by Lev's monograph (Lev, 1969) for the American Accounting Association. Lev (1969) uses entropy to analyze aggregation issue in financial statements, evaluate the accuracy of budgets and assess losses associated with financial failure. Lev and Theil (1978) use the principle of maximum entropy as a criterion for the selection of depreciation schemes when imperfect information is available. More recent studies in the rational inattention literature view accounting as communication channel to investors and employ entropy to measure the channel capacity (Sims, 1998, 2003). Jiang and Yang (2017) apply entropy to measure the informativeness of accounting disclosure in a signaling setting and Bertomeu *et al.* (2020) apply entropy to measure investors' attention allocation of managers' voluntary disclosure. Our study establishes the equivalence between accounting numbers and information and proposes the double entry accounting as a mechanism for instantaneous information transfer.

The remainder of the monograph is organized as follows. In Section 2 we establish the three-way classical equivalence (step one). In Section 3 we establish the three-way quantum equivalence (step two). The classical equivalence is a special case of the quantum equivalence. In Section 4, we define observability gap and run thought experiments to show that entanglement can reduce the observability gap (step three). We interpret entanglement in the quantum world using the Bell Theorem and conjecture that double entry accounting is a mechanism for entanglement in the business world. We conclude in Section 5. All proofs are provided in Appendix A. Appendix B discusses the Bell Theorem and computes a Bell inequality.

References

Aczel, A. D. (2003). *Entanglement*. First Plume Printing.

Arrow, K. and G. Debreu (1954). "Existence of an equilibrium for a competitive economy". *Econometrica*. 22: 265–290.

Becker, A. (2019). *What is Real? The Unfinished Quest for the Meaning of Quantum Physics*. New York: Basic Books, Hachette Book Group, Inc.

Bertomeu, J., K. Hu, and Y. Liu (2020). "Disclosure and investor inattention". Working paper. DOI: 10.2139/ssrn.3673225.

Bradley, T.-D. (2019a). "A First Look at Quantum Probability, Part 1". In: *Math3ma*.

Bradley, T.-D. (2019b). "A First Look at Quantum Probability, Part 2". In: *Math3ma*.

Christensen, J. A. and J. Demski (2003). *Accounting Theory: An Information Content Perspective*. McGraw-Hill Higher Education.

Cover, T. M. and J. A. Thomas (1991). *Elements of Information Theory*. John Wiley & Sons, Inc.

Debreu, G. (1959). *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*. New Haven: Yale University Press.

Demski, J. S. (2008). *Managerial Uses of Accounting Information*. 2nd ed. New York: Springer.

Einstein, A., B. Podolsky, and N. Rosen (1935). "Can quantum-mechanical Description of physical reality be considered complete?" *Physical Review*. 47: 777–780.

Fellingham, J. C. (2017). *Accounting: An Information Science*. Ohio State University. URL: https://u.osu.edu/fellingham.1/homepage/.

Fellingham, J. C. and H. Lin (2020). "Is accounting an information science?" *Accounting, Economics, and Law: A Convivium, De Gruyter*. 10(1): 1–17.

Fellingham, J. C., H. Lin, and D. Schroeder (2019). "Entropy and accounting". Working paper.

Grossman, S. J. and O. D. Hart (1983). "An analysis of the principal-agency problem". *Econometrica*. 51(1): 7–45.

Hakansson, N. (1971). "Multi-period mean-variance analysis: Towards a general theory of portfolio choice". *Journal of Finance*. 26(4): 857–884.

Holmstrom, B. (1979). "Moral hazard and observability". *Bell Journal of Economics*. 10: 324–340.

Jaynes, E. T. (1957). "Information theory and statistical mechanics". *Physical Review*. 106(4): 620–630.

Jaynes, E. T. (2003). *Probability Theory: The Logic of Science*. Cambridge University Press.

Jean, W. H. (1980). "The geometric mean and stochastic dominance". *The Journal of Finance*. 35(1): 151–158.

Jiang, X. and M. Yang (2017). "Properties of optimal accounting rules in a signaling game". *Journal of Accounting and Economics*. 63(2–3): 499–512.

Kelly, J. (1956). "A new interpretation of information rate". *The Bell System Technical Journal*. 35: 917–926.

Lambert, R. A. (2001). "Contracting theory and accounting". *Journal of Accounting and Economics*. 32(1–3): 3–87.

Latane, H. A. (1959). "Criteria for choice among risky ventures". *Journal of Political Economy*. 67(2): 144–155.

Lev, B. (1969). *Accounting and Information Theory*. Studies in Accounting Research #2; Sarasota, Florida: The American Accounting Association.

Lev, B. and H. Theil (1978). “A maximum entropy approach to the choice of asset depreciation”. *Journal of Accounting Research*. 16(2): 286–293.

Lloyd, S. (2007). *Programming the Universe: A Quantum Computer Scientist Takes on the Cosmos*. New York: Vintage Books, a Division of Random House, Inc.

Markowitz, H. (1952). “Portfolio selection”. *Journal of Finance*. 7: 77–91.

Nielsen, M. A. and I. L. Chuang (2004). *Quantum Computation and Quantum Information*. Cambridge University Press.

Ross, S. A. (2005). *Neoclassical Finance*. Princeton and Oxford: Princeton University Press.

Ross, S. A. (2015). “The recovery theorem”. *The Journal of Finance*. LXX(2): 615–648.

Samuelson, P. (1971). “The ‘fallacy’ of maximizing the geometric mean in long sequences of investing or gambling”. *Proceedings of the National Academy of Science*. 68: 2493–2496.

Savage, L. J. (1954). *The Foundations of Statistics*. New York: John Wiley.

ScienceDaily (2020). “Quantum entanglement offers unprecedented precision for GPS, imaging and beyond”. URL: https://www.sciencedaily.com/releases/2020/04/200420104856.htm.

Seife, C. (2007). *Decoding the Universe: How the New Science of Information is Explaining Everything in the Cosmos, from Our Brains to Black Holes*. Penguin (Non-Classics).

Sen, P. (2021). *Einstein’s Fridge: How the Difference Between Hot and Cold Explains the Universe*. Scribner Book Company.

Shannon, C. E. (1948). “A mathematical theory of communication”. *The Bell System Technical Journal*. 27(3): 379–423.

Sims, C. A. (1998). “Stickiness”. *Carnegie-Rochester Conference Series on Public Policy*. North-Holland. 49: 317–356.

Sims, C. A. (2003). “Implications of rational inattention”. *Journal of Monetary Economics*. 50(3): 665–690.

Thorp, E. O. (2006). "The kelly criterion in blackjack sports betting and the stock market". In: *Chapter 9 in Handbook of Asset and Liability Management*. Ed. by S. A. Zenios and W. Ziemba. Vol. 1. Elsevier B. V.