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Computational Symmetry in Computer Vision and Computer Graphics

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Abstract

In the arts and sciences, as well as in our daily lives, symmetry has made a profound and lasting impact. Likewise, a computational treatment of symmetry and group theory (the ultimate mathematical formalization of symmetry) has the potential to play an important role in computational sciences. Though the term *computational symmetry* was formally defined a decade ago by the first author, referring to algorithmic treatment of symmetries, seeking symmetry from digital data has been attempted for over four decades. *Computational symmetry* on real world data turns out to be challenging enough that, after decades of effort, a fully automated symmetry–savvy system remains elusive for real world applications. The recent resurging interests in computational symmetry for computer vision and computer graphics applications have shown promising results. Recognizing the fundamental relevance and

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potential power that computational symmetry affords, we offer this survey to the computer vision and computer graphics communities. This survey provides a succinct summary of the relevant mathematical theory, a historic perspective of some important symmetry-related ideas, a partial yet timely report on the state of the arts symmetry detection algorithms along with its first quantitative benchmark, a diverse set of real world applications, suggestions for future directions and a comprehensive reference list.

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Contents

Full text available at: http://dx.doi.org/10.1561/0600000008

Full text available at: http://dx.doi.org/10.1561/0600000008

1

Introduction

Symmetry is a pervasive phenomenon presenting itself in all forms and scales in natural and man-made environments, from galaxies to biological structures (Figure 1.1), as well as in the arts (Figure 1.2 from the classic book by Jones [106]). Much of our understanding of the world is based on the perception and recognition of repeated patterns that are generalized by the mathematical concept of symmetries [47, 277]. Humans and animals have an innate ability to perceive and take advantage of symmetry in everyday life, but harnessing this powerful insight for machine intelligence remains an elusive goal for computer science.

In the basic sciences, the understanding of symmetry played a profound role in several important discoveries, including: the theory of relativity (the discovery of the isometries of Minkowski spacetime under the Poincaré group, the full symmetry group) [195]; the double helix structure of DNA (with two-fold rotation symmetry) [275]; the discovery of quasi-crystals (the first observation of an unusual fivefold symmetry indicated by diffraction pictures of samples from an alloy of aluminium and manganese) [247] and their mathematical counterpart Penrose tiles [209]. It is not a coincidence that all of these symmetry-related discoveries led, directly or indirectly, to Nobel prizes — an indication of the fundamental relevance of symmetry in science.

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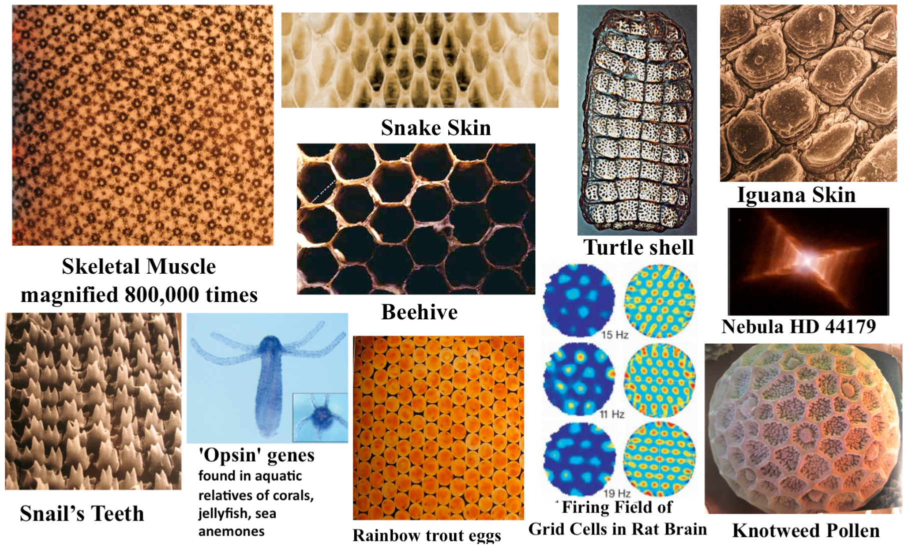

Fig. 1.1 The ubiquitous appearance of real world symmetries in nature: from Nebula to the firing field of grid cells in rats' brains. The photos from four corners, and bottom-middle, are courtesy of [182].

Given the evidence of the powerful role of symmetry in the history of the natural sciences, we hypothesize that *computational symmetry*, defined by the first author as *using computers to model, analyze, synthesize and manipulate symmetries in digital forms, imagery or otherwise* [152], will likewise play a crucial role in the advancement of our understanding in artificial/machine intelligence.

In human perception, symmetry is considered a pre-attentive feature that enhances object recognition [40, 144, 268]. Although humans, primates, dolphins, birds, and insects have an innate ability to recognize and use real world symmetries that have been quantitatively documented [74, 226, 268], the symmetry cue is hardly used in today's object recognition, categorization or scene understanding systems due, largely, to a lack of computational models and available robust algorithms.

In computational science, the development of symmetry detection algorithms has had a long history. The earliest attempt at an algorithmic treatment of bilateral reflection symmetry detection predates computer vision itself [18]. In spite of years of effort, we are still short of

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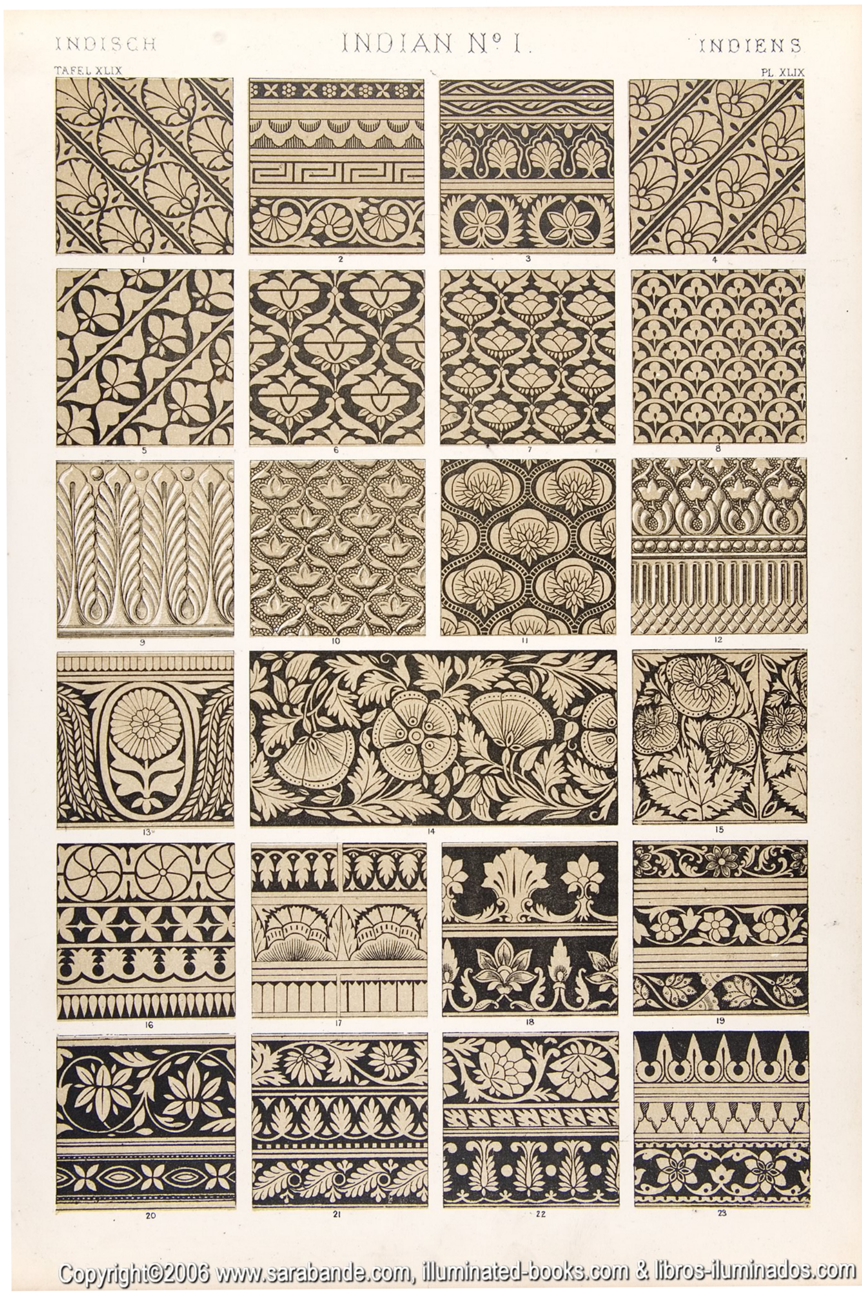

Fig. 1.2 Symmetries in art.

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a robust, widely applicable "symmetry detector" for real images. Furthermore, though an initial effort has been made [33, 203], we have yet to see a large-scale, systematic, quantitative evaluation and a publicly available test-image database to gauge the progress in this important, widely applicable research direction.

Without attempting to cover all published related-work in this survey, we primarily focus on the theory and techniques that make explicit use of symmetry groups. Due to a lack of coverage in the literature on *discrete* finite and infinite symmetry groups and their relevance to and impact on computer vision and computer graphics problems, we pay special attention to these types of algorithms without excluding affine, perspective, and non-Euclidan geometries. We hope to provide a clear conceptual roadmap of group theory (Section 2), and its multi-facet applications in computer vision and computer graphics. In particular, we hope to achieve these goals:

(1) to de-mystify group theory, discrete and finitely generated (infinite) groups in particular, using concrete examples from 2D and 3D Euclidean, projective and hyperbolic geometries;
(2) to illustrate the ubiquitous and persistent appearances of symmetry structures, particularly those associated with discrete symmetry groups, in real world data;
(3) to appreciate the substantial computational challenges as well as promises in current and future computational symmetry research.

1.1 What is symmetry?

From the spirit of the Felix Klein's Erlangen program [79]: geometry is the study of a space that is invariant under a given transformation group, to the Gestalt principles of perception [7]: among others, *Laws of Symmetry*, symmetries and group theory play an important role in describing the geometry and the apperance of an object. Informally, we may think of symmetry as expressing the notion that a figure or object is made from multiple copies of the same smaller unit that are interchangeable somehow. Mathematically, we formalize this notion by examining the effect of transformations on the object in a certain space

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such that its sub-parts coincide (map to each other). The following quote from Weyl [277] captures the essence of symmetry eloquently:

> "Starting from the somewhat vague notion of symmetry = harmony of proportions, ... rise to the general idea ... of invariance of a configuration of elements under a group of automorphic transformations."

More formally, in a metric space M, a *symmetry* $g \in G$ of a set $S \subseteq M$ is an isometry (a distance preserving transformation) that maps S to itself (an automorphism), $g(S) = S$. The transformation g keeps S invariant as a whole while permuting its parts. Symmetries G of S form a mathematical *group* $\{G, *\}$, closed under transformation composition $*$, called the *symmetry group* of S [44].

Group theory provides a level of abstraction that leads to simplicity and completeness in practical algorithm design and execution. In two-dimensional (2D) Euclidean space, for example, there are four distinct *atomic* transformations as *primitive* symmetries [41, 44, 277]: translation, rotation, reflection and glide-reflection[1] (Figure 1.3). One somewhat surprising mathematical discovery of a century ago is the answer to the first part of Hilbert's 18th problem: there is only a finite number of symmetry groups for all periodic patterns in R^n of any n. These groups are referred to as *crystallographic groups* [44, 84, 277]. While the number of distinct 1D (frieze) and 2D (wallpaper) crystallographic groups is finite and relatively small, there is an infinite number of potential instantiations; some samples can be observed in Figures 1.1, 1.2 and 1.3.

1.2 Why is symmetry relevant to computational science?

A computational model for symmetry is especially pertinent to computer vision and computer graphics, or machine intelligence in general,

[1] Glide-reflection is defined as a symmetry composed of a translation along and a reflection about the same axis.

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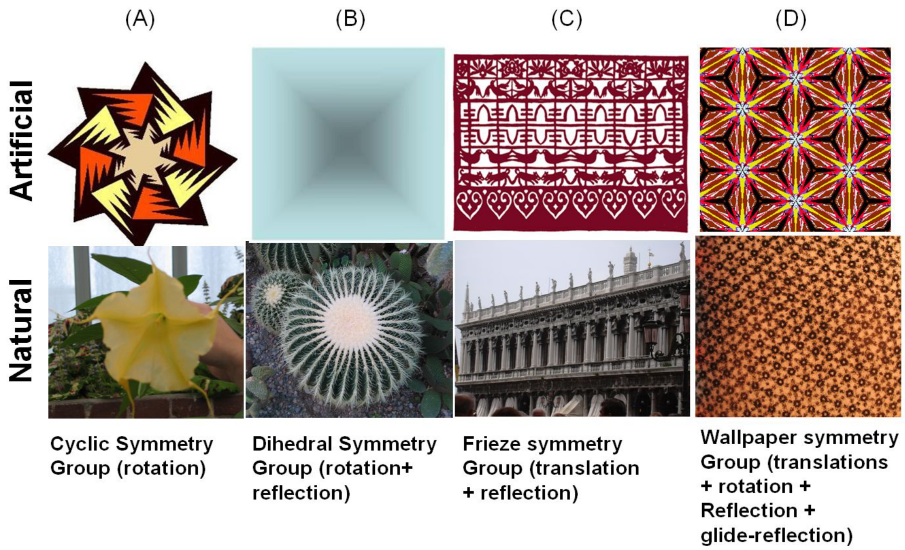

Fig. 1.3 Sample images of symmetry categorized by their respective ideal symmetry groups: column (A) cyclic group containing rotation symmetries only, (B) dihedral group (reflection and rotation), (C) frieze group (translation plus reflection), and (D) wallpaper group (translation, rotation, reflection and glide-reflection). The top row contains synthetic patterns while the bottom row shows photos of real world scenes (bottom-right is an image of a transverse slice of skeletal muscle magnified with a transmitter microscope 800,000 times).

because of its

- *ubiquitousness*: both the physical and the digital worlds are filled with various forms of symmetry, near-symmetry and distorted symmetry patterns (Figures 1.1, 1.2 and 1.3). The applicability of such a computational model can only be limited by one's imagination;
- *essentiality*: intelligent beings perceive and interact with the chaotic real world in the most efficient and effective manner by capturing its essential structures and sub-structures — the generators of symmetry, near-symmetry, distorted symmetry and/or repeated patterns;
- *compactness*: the recognition of symmetries is the first step towards minimizing redundancy, often leading to drastic reductions in computation; and

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- *aestho-physiology*: from a butterfly to an elephant, from a tea cup to a building, symmetry or deviation from it, has been a time-honored principle for design (by nature or by human) that can guide machine perception, detection, recognition and synthesis of the real world.

Since the earliest bilateral reflection symmetry detection algorithm [18], attempts in computational treatment of symmetry and regularity have been made continuously. Figure 1.4 shows the statistics of published papers[2] in several major computer vision/graphics conferences/journals during the period of 1974–2009 (36 years). An increasing level of interests can be observed in both computer vision and computer graphics.

1.3 Why is computational symmetry challenging?

Humans are experts in symmetry detection and appreciation [144, 268]. Our ability to recognize and tolerate departures from prefect symmetries reflects a level of sophistication in human perception. From an engineering point of view, however, it remains unclear how to capture and simulate this perceptual capability of humans and animals for machine/artifitial intelligence. From a theoretical point of view, even though group theory itself (especially Euclidean group and its subgroups) is a mature field, little formal theory exists connecting the elegant group theory to the noisy, incomplete and often inconsistent real world. From an educational point of view, group theory is usually introduced in classrooms as an abstract theory instead of a theoretical basis for algorithmic treatment of real world problems, such like those in computer vision and computer graphics.

Regardless of how powerful computers have become, one fundamental limitation of computers is their finite representation power. One simple floating point round-up error destroys any perfect symmetry in the data. In addition, and perhaps more importantly, the non-coherent

[2] A list of the papers counted for can be found here: http://www.cse.psu.edu/∼yanxi/symmetryCitation.htm.

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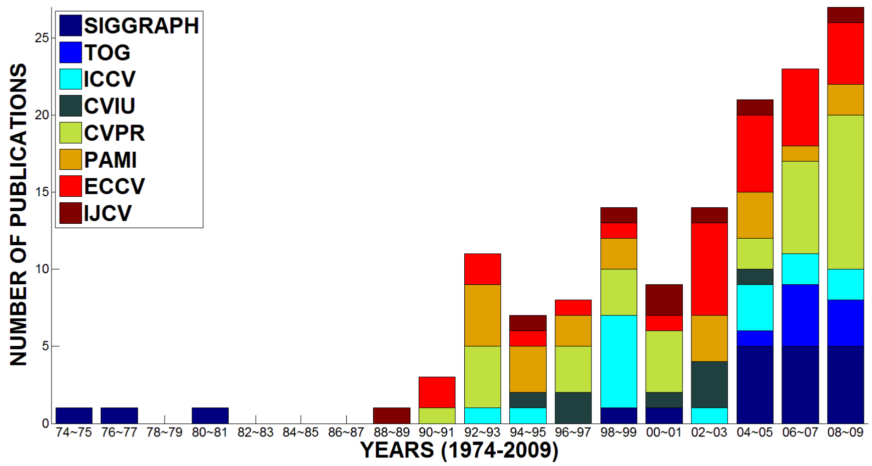
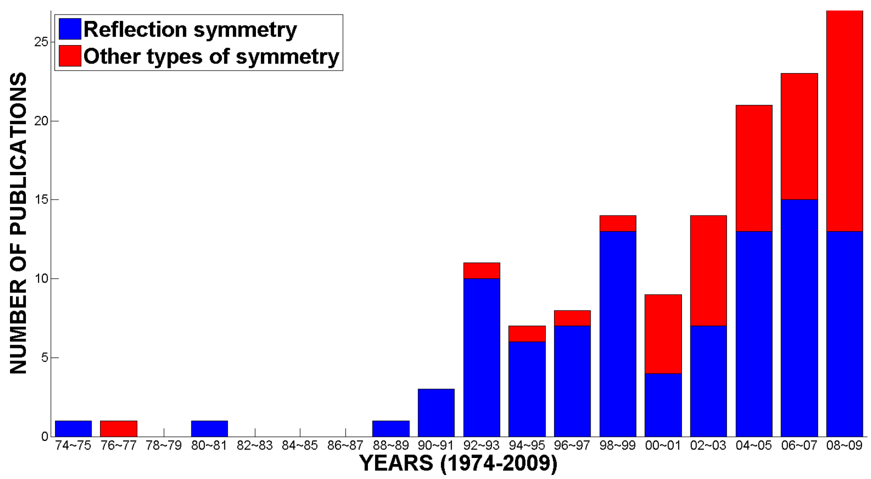


(A) Publications on symmetry detection and applications in major computer vision and computer graphics conferences/journals.



(B) Dividing papers into on reflection symmetry alone versus other types of symmetries (rotation, translation and glide-reflection).

Fig. 1.4 From the publication statistics, it is obvious that research on reflection symmetry has been dominating the field in the past, with a growing awareness of the whole symmetry spectrum. A similar reflection-symmetry-dominating trend has also been observed in the psychology literature for human perception of symmetries [268].

(discrete versus continuous, and finite versus infinite) topological nature of symmetry groups poses serious problems for their representation and computation on computers under a uniform framework [151].

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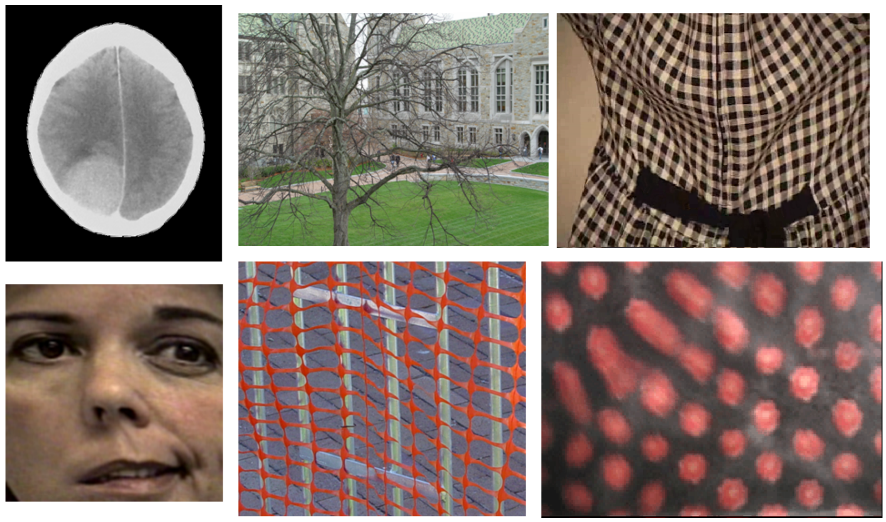

Fig. 1.5 Sample images of distorted, disguised and layered real world symmetries. Top and bottom-middle photos are courtesy of David R. Martin.

In summary, computational symmetry is challenged by at least two acute discrepancies:

- the clean formal concepts of group theory versus imperfect, noisy, ambiguous, distorted and often hidden symmetry patterns in digitized real world data (Figure 1.5).
- the complete, concise and uniform mathematical theory of symmetry, group theory, versus limitations of representational power of computers (hardware) and a lack of computational models for real world symmetry (software);

Without explicitly and effectively addressing each of these challenges, it is impossible for computational symmetry to release its potential power and play a substantial role in computer vision and computer graphics research even though the importance and relevance of symmetry in these fields become increasingly obvious (Figure 1.4).

1.4 Historical Perspective

To gain some historic perspective and insight, interested readers can find several influential symmetry-related papers, for example, the

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wonderful exposition on the role of symmetry in "Biological Shape and Visual Science" by Blum in 1973 [20]; in 1977, the "Description and Recognition of Curved Objects" reported by Nevatia and Binford, where bilateral symmetry of the object about different axes is examined [197][3]; the method of detecting angle/side regularities of closed curves and plateaus in one-dimensional patterns by Davis et al. [22, 48]; the introduction of the term *skewed symmetry* by Takeo Kanade in 1981 [109]; the exposition on "Smoothed Local Symmetries and Their Implementation" by Brady and Asada (1984) [24]; the theory of *recognition-by-components* (RBC) proposed by Biederman in 1985 [14][4]; "Perceptual Grouping and the Natural Representation of Natural Form" using superquadrics as restricted *generalized cylinders* (GC) by Pentland in 1986 [210]; "Perceptual Organization and Visual Recognition" by Lowe [175], where the non-coincidental appearance of symmetry in the real world was noted; and the "Symmetry-seeking Models for 3D Object Reconstruction" (1987) illustrated by Terzopoulos, Witkin and Kass [256].

1.5 Organization of this Survey

In order to make this survey concise, self-contained and easily searchable, we organize this survey into eight relatively independent sections. They are:

(1) **Introduction**, where we motivate the necessity of this survey;
(2) **Symmetry and symmetry groups**, where we provide a set of standard definitions and proofs with intuitive explanations of these concepts. The theoretical basis for symmetry is

[3] The authors stated: because of the simple descriptors used for pieces, the symmetry calculation is correspondingly crude. However, we feel that with improved descriptions, symmetry can be very useful." (p. 90) [197].

[4] The fundamental assumption of the proposed theory of RBC "is that a modest set of components [N probably $5 \leq 36$] can be derived from contrasts of five readily detectable properties of edges in a two-dimensional image: curvature, collinearity, symmetry, parallelism, and cotermination. The detection of these properties is generally invariant over viewing position and image quality and consequently, allows robust object perception when the image is projected from a novel viewpoint or degraded."

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established under Euclidean, affine and perspectively-skewed and non-Euclidean geometries;

- Symmetry in Euclidean geometry
- Symmetry-based invariants in perspective transformation (affine as a special case)
- Symmetry in non-Euclidean geometry

(3) **Symmetry and Symmetry group detection**, where we describe and demonstrate the methods and the output of several representative state of the art symmetry detection algorithms followed by the results from the first quantified benchmarking in this area;
(4) **Near regular textures**, where we show their mathematical roots to crystallographic groups, and their increasingly wide applicability in both computer vision and computer graphics research;
(5) **Continuous and quantified symmetry**, where we demonstrate a variety of real world applications using quantified symmetry (or deviations from it) as a continuous measure;
(6) **Symmetry in graphics**, where a range of visually appealing symmetry-based graphics applications is illustrated;
(7) **Summary**, where we summarize this effort, provide pointers to existing resources and lay out tangible future directions in this research area; and
(8) **References**, a comprehensive reference list in computational symmetry completes this survey.

Given limitations in space and time, some important aspects of symmetry related topics are not covered in this survey, they include:

- *Medial axis* and its wide range of applications: for a recent survey on this topic, refer to [249]; a classic shock graph reference [250], and some recent developments [140, 253].
- Human and animal symmetry perceptions: these are well studied and well documented. Interested readers can start with [268, 144]; some excellent exploration of symmetry and culture research can be found in these books [236, 274].

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- Alternative representations of symmetry groups: in this survey, we follow the classic group theory for symmetry. But we recommend the book by Conway et al. [41] for great intuition and a modern representation of symmetry groups.

Full text available at: http://dx.doi.org/10.1561/0600000008

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