(Hyper)-Graphs Inference through Convex Relaxations and Move Making Algorithms: Contributions and Applications in Artificial Vision

Nikos Komodakis
University Paris Est
Ecole des Ponts ParisTech, France

M. Pawan Kumar
Department of Engineering Science
University of Oxford, UK

Nikos Paragios
CentraleSupelec, INRIA
Universite Paris-Saclay, France

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Nikos Komodakis[1]
University Paris Est
Ecole des Ponts ParisTech, France
nikos.komodakis@enpc.fr

M. Pawan Kumar[1]
Department of Engineering Science
University of Oxford, UK
pawan@robots.ox.ac.uk

Nikos Paragios
CentraleSupelec, INRIA
Universite Paris-Saclay, France
nikos.paragios@ecp.fr

[1]The first two authors contributed equally.

Contents

Abstract

Computational visual perception seeks to reproduce human vision through the combination of visual sensors, artificial intelligence and computing. To this end, computer vision tasks are often reformulated as mathematical inference problems where the objective is to determine the set of parameters corresponding to the lowest potential of a task-specific objective function. Graphical models have been the most popular formulation in the field over the past two decades where the problem is viewed as a discrete assignment labeling one. Modularity, scalability and portability are the main strengths of these methods which once combined with efficient inference algorithms they could lead to state of the art results. In this tutorial we focus on the inference component of the problem and in particular we discuss in a systematic manner the most commonly used optimization principles in the context of graphical models. Our study concerns inference over low rank models (interactions between variables are constrained to pairs) as well as higher order ones (arbitrary set of variables determine hyper-cliques on which constraints are introduced) and seeks a concise, self-contained presentation of prior art as well as the presentation of the current state of the art methods in the field.

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1

Introduction

Graphical models (Conditional Random Fields (CRFs) or Markov Random Fields (MRFs)) have been introduced in the field of computer vision almost four decades ago. CRFs were introduced in Fischler and Elschlager [1973] while MRFs were introduced in Geman and Geman [1984] to address the problem of image restoration. The central idea is to express perception through an inference problem over graph. The process is defined using a set of nodes, a set of labels, and a neighborhood system. The graph nodes often correspond to the parameters to be determined, the labels to a quantized/discrete version of the search space and the connectivity of the graph to the constraints/interactions between variables. Graph-based methods are endowed with numerous advantages as it concerns inference when compared to their alternative that refers to continuous formulations. These methods are in general gradient-free and therefore can easily accommodate changes of the model (graph structure), changes of the objective function (perception task), or changes of the discretization space (precision).

Early approaches to address graph-based optimization in the field of computer vision were primarily based either on annealing like approaches or on local minimum update principles. Simulated annealing was an alternative direction that provides in theory good guarantees as it concerns the optimality properties of the obtained solution. The central idea is to perform a search with a decreasing radius/temperature where at a given iteration the current state is updated to a new state with a tolerance (as it concerns the objective function) that is related to the temperature. Such meta-heuristic methods could lead to a good approximation of the optimal solution if temperature/radius are appropriately defined that in general is not that trivial. Iterated conditional modes or highest confidence first were among the first attempts exploiting local minimum iterative principles. Their underlying principle was to solve the problem progressively through a repetitive local update of the optimal solution towards a new local optimum. These methods were computationally efficient and deterministic in the expense of quite inefficient in terms of capturing the global optimum of the solution and the complete absences of guarantee as it concerns the optimality properties of the obtained solution.

Despite the elegance, modularity and scalability of MRFs/CRFs, their adoption was quite limited (over eighties and nineties) from the image processing/computer vision community and beyond due lack of efficient optimization methods to address their inference. The introduction of efficient inference algorithms inspired from the networks community, like for example the max flow/min cut principle at late nineties that is a special case of the duality theorem for linear programs as well their efficient implementations towards taking advantage of image like graphs Boykov et al. [1998] or message passing methods Pearl [1998] that are based on the calculation of the marginal for a given node given the states of the other nodes have re-introduced graphical models in the field of computer vision. During the past two decades we have witnessed a tremendous progress both on their use to address visual perception tasks Wang et al. [2013], Kappes et al. [2015], Paragios and Komodakis [2014], Szeliski et al. [2008], Blake et al. [2011], Komodakis and Tziritas [2007a] as well as it concerns their inference. This tutorial aims to provide an overview of the state of the art methods in the field for inference as well as the most recent advances in that direction using move making algorithms and convex relations. The reminder of this paper is organized as follows; Section 2 presents briefly the context and a short review of the the most representative inference methods. Section 3 is dedicated to move making algorithms, while section 4 presents efficient linear programming-inspired principles for graph inference. The last section introduces dual decomposition, a generic, modular and scalable framework to perform (hyper) graph inference.

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