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The Path to Path-Traced Movies

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Contents

Abstract

Path tracing is one of several techniques to render photorealistic images by simulating the physics of light propagation within a scene. The roots of path tracing are outside of computer graphics, in the Monte Carlo simulations developed for neutron transport. A great strength of path tracing is that it is conceptually, mathematically, and often-times algorithmically simple and elegant, yet it is very general. Until recently, however, brute-force path tracing techniques were simply too noisy and slow to be practical for movie production rendering. They therefore received little usage outside of academia, except perhaps to generate an occasional reference image to validate the correctness of other (faster but less general) rendering algorithms. The last ten years have seen a dramatic shift in this balance, and path tracing techniques are now widely used. This shift was partially fueled by steadily increasing computational power and memory, but also by significant improvements in sampling, rendering, and denoising techniques. In this survey, we provide an overview of path tracing and highlight important milestones in its development that have led to it becoming the preferred movie rendering technique today.

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1

Introduction

Rendering computer-generated (CG) movies is tough. There are 130,000 high-resolution frames in a 90-minute movie, with each frame requiring computation of typically two million pixel colors (many more for IMAX movies). This is several hundred billion pixel colors that will be scrutinized by the movie director and by audiences worldwide. The images are often computed with motion blur and depth of field, to mimic these characteristic effects (limitations) of real cameras. The images must be free of visual noise—one particularly pesky type of noise is occasional bright pixels known as "fireflies". There can be no spatial or temporal aliasing (affectionately known as "jaggies" and "crawlies") in the images. The color of each pixel depends not only on what object is visible in that pixel (including its orientation, material, texture, illumination, etc.), but also—through shadows and reflected light—on objects in other parts of the scene. The surface color calculations have to be programmable, with the computations specified in standalone programs called "shaders". Typical scenes contain huge amounts of geometry and texture data, often straining the available memory even on high-end computers. There are often dozens of textures specifying the material properties of each surface, and a scene can contain

Figure 1.1: Frames from the movies *Toy Story* (1995) and *Finding Dory* (2016). (© 1995, 2016 Pixar/Disney.)

millions of surfaces. With all the data that goes into rendering each frame, production-strength renderers are sometimes jokingly referred to as data management systems with images as a by-product. Rendering times are crucial as well, both for quick test images during the development of the movie, and for the final-quality frames that will appear in movie theaters.

Figure 1.1(a) shows a frame from the first computer-generated animated feature film: Pixar's *Toy Story* from 1995. This movie was rendered with the RenderMan renderer using the Reyes scan-line algorithm [25] with shadow maps and reflection maps [4]. For many years, the Reyes algorithm was the work-horse of most CG and visual effects work at major studios.

The last 15 years has seen hybrid renderers combining the Reyes algorithm for directly visible objects with ray-traced shadows and reflections. Soft indirect illumination has been computed with a variety of methods, including distribution ray tracing and point-based global illumination.

At the same time, other renderers, such as Arnold, pushed for a complete switch to path tracing. Compared to Reyes-based hybrid renderers, path tracing is a simpler and more brute-force approach. It has its roots in a statistical sampling method called Monte Carlo, which was first used for particle simulation in nuclear engineering. Path tracing can render shadows and reflections in a conceptually simple recursive manner, but on the other hand it is more noisy and less memory efficient than some Reyes hybrids. Path tracing is not necessarily the fastest method to render final movie-quality images with indirect illumination, depth of field, and motion blur: for example, point-based global illumination has no noise, and distribution ray tracing with irradiance gradients and radiosity caching is better able to exploit sample domain coherency. However, path tracing's natural ability to handle complex light transport effects, along with its potential to simplify the production pipeline, reduce iteration time during layout and lighting, and improve overall workflow, are enticing advantages.

Figure 1.1(b) shows a frame from the recent movie *Finding Dory*. Here all the direct visibility, shadows, reflections, refractions, indirect diffuse illumination, and subsurface scattering are computed with RenderMan's implementation of path tracing.

Even though the algorithmic developments for the switch to path tracing have been under way for quite some time, there is a fairly sudden wave of studios switching their pipelines over. One might even talk about a path tracing "revolution" [61]. This article is our attempt to retrace the steps the industry has taken on its journey to path-traced movies. We will identify major hurdles that stood in the way of that transition, describe the technical milestones that pushed the field forward over the last couple of decades, and discuss the combination of circumstances that came together to propel the CG and VFX movie industry into a path-traced world. Since the journey is not yet complete, we will also discuss on-going challenges and open questions that practitioners and researchers will need to address in the years to come.

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