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Computational Imaging Through Atmospheric Turbulence

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Computational Imaging Through Atmospheric Turbulence

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ABSTRACT

Since the seminal work of Andrey Kolmogorov in the early 1940's, imaging through atmospheric turbulence has grown from a pure scientific pursuit to an important subject across a multitude of civilian, space-mission, and national security applications. Fueled by the recent advancement of deep learning, the field is further experiencing a new wave of momentum of applying these learning-based techniques to the problem. However, because of the complexity of the physics of atmospheric turbulence, significant gaps remain to be filled before the power of deep learning can be fully unleashed. In particular, the goal of building the most accurate turbulence model to mimic nature is gradually shifted to designing a compromised model that can maximize the image reconstruction performance. This leads to a new field which this book is trying to explain, Computational Imaging Through Atmospheric Turbulence.

The goal of this book is to present the basic concepts of turbulence physics while framing it under the theme of computational imaging. Emphasis is put on elaborating the principles of how waves propagate through atmospheric turbulence and propagation-free approaches to reproduce the

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effect without needing wave propagation equations. This allows for a much faster simulation while preserving the physics of turbulence, hence creating the possibility of integrating turbulence physics into the design of image reconstruction algorithms. The book is written for readers with an image processing background who are seeking to understand the physics of turbulence. Connections with deep learning are emphasized throughout the book.

Preface

The physics of imaging through atmospheric turbulence has been around for more than 80 years, and over this time it has generated a rich collection of texts and papers. We came to the field with an image processing background without having even read an optics textbook. Upon reading some of the imaging through turbulence literature, we very quickly realized the depth and breadth of the subject, but more strikingly the lack of an easy-to-read reference for people with a background like us.

The goal of this book is to provide a short introduction to the subject from the perspective of an image processing person. By image processing, we are thinking of scientists and engineers working on inverse problems in imaging systems with the goal of recovering signals from corrupted measurements. To this end, we are targeting readers who would like to know the physics of atmospheric turbulence so that they can improve their algorithms. Because of the specific perspective we take here and the targeted audience group, we shall not take a very rigorous physics-based approach. Unless the reader is already familiar with wave optics, the learning barrier will be so high that an average person would not be able to master the concepts quickly. Democratizing the ideas and educating the image processing community is an important mission of this book.

As we write this book, we aim in delivering the "big pictures" of the subject. Whenever needed, we streamline background materials including probability, optics, and optimization. Some sacrifices in the material are made to balance precision and clarity. Therefore, we do not regard this book as any substitution of the great optics books of our time. Whenever possible, we will connect the technical details back to our theme of computational imaging.

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West Lafayette, IN, United States
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References

[1] R. Ahmad, C. A. Bouman, G. T. Buzzard, S. Chan, S. Liu, E. T. Reehorst, and P. Schniter, "Plug-and-play methods for magnetic resonance imaging: Using denoisers for image recovery," *IEEE Signal Processing Magazine*, vol. 37, no. 1, pp. 105–116, 2020. DOI: 10.1109/MSP.2019.2949470.

[2] K. Akiyama, A. Alberdi, W. Alef, K. Asada, R. Azulay, A.-K. Baczko, D. Ball, M. Baloković, J. Barrett, D. Bintley, *et al.*, "First m87 event horizon telescope results. iv. imaging the central supermassive black hole," *The Astrophysical Journal Letters*, vol. 875, no. 1, p. L4, 2019.

[3] N. Anantrasirichai, "Atmospheric turbulence removal with complex-valued convolutional neural network," 2022. URL: https://arxiv.org/abs/2204.06989.

[4] N. Anantrasirichai, A. Achim, and D. Bull, "Atmospheric turbulence mitigation for sequences with moving objects using recursive image fusion," in *IEEE International Conference on Image Processing*, pp. 2895–2899, Oct. 2018. DOI: 10.1109/ICIP.2018.8451755.

[5] N. Anantrasirichai, A. Achim, N. G. Kingsbury, and D. R. Bull, "Atmospheric turbulence mitigation using complex wavelet-based fusion," *IEEE Transactions on Image Processing*, vol. 22, no. 6, pp. 2398–2408, Jun. 2013. DOI: 10.1109/TIP.2013.2249078.

[6] N. Antipa, G. Kuo, R. Heckel, B. Mildenhall, E. Bostan, R. Ng, and L. Waller, "Diffusercam: Lensless single-exposure 3d imaging," *Optica*, vol. 5, no. 1, pp. 1–9, Jan. 2018. DOI: 10.1364/OPTICA.5.000001.

[7] E. Anzuola and S. Gladysz, "Modeling dynamic atmospheric turbulence using temporal spectra and Karhunen–Loève decomposition," *Optical Engineering*, vol. 56, no. 7, p. 071 508, 2017. DOI: 10.1117/1.OE.56.7.079803.

[8] M. Aubailly, M. A. Vorontsov, G. W. Carhart, and M. T. Valley, "Automated video enhancement from a stream of atmospherically-distorted images: The lucky-region fusion approach," in *Atmospheric Optics: Models, Measurements, and Target-in-the-Loop Propagation III*, Proc. SPIE 7463, Aug. 2009. DOI: 10.1117/12.828332.

[9] S. Basu, J. E. McCrae, and S. T. Fiorino, "Estimation of the path averaged atmospheric refractive index structure constant from time lapse imagery," in *Laser Radar Technology and Applications XX; and Atmospheric Propagation XII*, p. 94650T, Proc. SPIE 9465, May 2015. DOI: 10.1117/12.2177330.

[10] M. Born and E. Wolf, *Principles of Optics; Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, Seventh ed. Cambridge University Press, 1999. DOI: 10.1017/CBO9781139644181.

[11] J. P. Bos and M. C. Roggemann, "Technique for simulating anisoplanatic image formation over long horizontal paths," *Optical Engineering*, vol. 51, no. 10, p. 101 704, 2012. DOI: 10.1117/1.OE.51.10.101704.

[12] J. P. Bos, M. C. Roggemann, and V. S. R. Gudimetla, "Anisotropic non-kolmogorov turbulence phase screens with variable orientation," *Applied Optics*, vol. 54, no. 8, pp. 2039–2045, Mar. 2015. DOI: 10.1364/AO.54.002039.

[13] "Bridging the gap between computational photography and visual recognition: 5th UG2+ prize challenge," Track 3. URL: http://cvpr2022.ug2challenge.org/dataset22_t3.html.

[14] A. Buades, B. Coll, and J. Morel, “A non-local algorithm for image denoising,” in *IEEE Conference on Computer Vision and Pattern Recognition*, vol. 2, pp. 60–65, 2005.

[15] W. P. Burckel and R. N. Gray, “Turbulence phase screens based on polar-logarithmic spectral sampling,” *Applied Optics*, vol. 52, no. 19, pp. 4672–4680, Jul. 2013. DOI: 10.1364/AO.52.004672.

[16] J.-F. Cai, H. Ji, C. Liu, and Z. Shen, “Blind motion deblurring from a single image using sparse approximation,” in *IEEE Conference on Computer Vision and Pattern Recognition*, pp. 104–111, 2009.

[17] T. Çaliskan and N. Arica, “Atmospheric turbulence mitigation using optical flow,” in *International Conference on Pattern Recognition*, pp. 883–888, 2014. DOI: 10.1109/ICPR.2014.162.

[18] W. H. Chak, C. P. Lau, and L. M. Lui, “Subsampled turbulence removal network,” *Mathematics, Computation and Geometry of Data*, vol. 1, pp. 1–33, 2021. DOI: 10.4310/MCGD.2021.v1.n1.a1.

[19] S. H. Chan, “Performance analysis of plug-and-play admm: A graph signal processing perspective,” *IEEE Transactions on Computational Imaging*, vol. 5, no. 2, pp. 274–286, 2019. DOI: 10.1109/TCI.2019.2892123.

[20] S. H. Chan, “Tilt-then-blur or blur-then-tilt? Clarifying the atmospheric turbulence model,” *IEEE Signal Processing Letters*, vol. 29, pp. 1833–1837, 2022. DOI: 10.1109/LSP.2022.3200551.

[21] S. H. Chan, X. Wang, and O. A. Elgendy, “Plug-and-play ADMM for image restoration: Fixed-point convergence and applications,” *IEEE Transactions on Computational Imaging*, vol. 3, no. 1, pp. 84–98, 2017. DOI: 10.1109/TCI.2016.2629286.

[22] G. A. Chanan, “Calculation of wave-front tilt correlations associated with atmospheric turbulence,” *Journal of the Optical Society of America A*, vol. 9, no. 2, pp. 298–301, Feb. 1992. DOI: 10.1364/JOSAA.9.000298.

[23] M. Charnotskii, J. Gozani, V. Tatarskii, and V. Zavorotny, “Wave propagation theories in random media based on the path-integral approach,” *Progress in Optics*, vol. 32, pp. 203–266, 1993. DOI: 10.1016/S0079-6638(08)70164-1.

[24] G. Chen, Z. Gao, Q. Wang, and Q. Luo, "U-net like deep autoencoders for deblurring atmospheric turbulence," *Journal of Electronic Imaging*, vol. 28, no. 5, p. 053 024, 2019. DOI: 10.1117/1.JEI.28.5.053024.

[25] N. Chimitt and S. H. Chan, "Simulating anisoplanatic turbulence by sampling intermodal and spatially correlated Zernike coefficients," *Optical Engineering*, vol. 59, no. 8, p. 083 101, 2020. DOI: 10.1117/1.OE.59.8.083101.

[26] N. Chimitt and S. H. Chan, "Anisoplanatic optical turbulence simulation for near-continuous C_n^2 profiles without wave propagation," 2023. URL: https://arxiv.org/abs/2305.09036.

[27] N. Chimitt, X. Zhang, Y. Chi, and S. H. Chan, "Scattering and gathering for spatially varying blurs," 2023. URL: https://arxiv.org/abs/2303.05687.

[28] N. Chimitt, X. Zhang, Z. Mao, and S. H. Chan, "Real-time dense field phase-to-space simulation of imaging through atmospheric turbulence," *IEEE Transactions on Computational Imaging*, 2022. DOI: 10.1109/TCI.2022.3226293.

[29] P. H. Christensen and W. Jarosz, "The path to path-traced movies," *Foundations and Trends® in Computer Graphics and Vision*, vol. 10, no. 2, pp. 103–175, 2016. DOI: 10.1561/0600000073.

[30] W. A. Coles, J. P. Filice, R. G. Frehlich, and M. Yadlowsky, "Simulation of wave propagation in three-dimensional random media," *Applied Optics*, vol. 34, no. 12, pp. 2089–2101, Apr. 1995. DOI: 10.1364/AO.34.002089.

[31] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, "Image denoising by sparse 3D transform-domain collaborative filtering," *IEEE Transactions on Image Processing*, vol. 16, no. 8, pp. 2080–2095, Aug. 2007. DOI: 10.1109/TIP.2007.901238.

[32] R. Dashen, "Path integrals for waves in random media," *Journal of Mathematical Physics*, vol. 20, no. 5, pp. 894–920, May 1979. DOI: 10.1063/1.524138.

[33] M. Delbracio and G. Sapiro, "Removing camera shake via weighted fourier burst accumulation," *IEEE Transactions on Image Processing*, vol. 24, no. 11, pp. 3293–3307, 2015. DOI: 10.1109/TIP.2015.2442914.

[34] D. R. Droege, R. C. Hardie, B. S. Allen, A. J. Dapore, and J. C. Blevins, "A real-time atmospheric turbulence mitigation and super-resolution solution for infrared imaging systems," in *Infrared Imaging Systems: Design, Analysis, Modeling, and Testing XXIII*, Proc. SPIE 8355, 2012. DOI: 10.1117/12.920323.

[35] M. A. Ealey and J. A. Wellman, "Deformable mirrors: Design fundamentals, key performance specifications, and parametric trades," in *Active and Adaptive Optical Components*, Proc. SPIE 1543, 1992. DOI: 10.1117/12.51167.

[36] B. Y. Feng, M. Xie, and C. A. Metzler, "TurbuGAN: An adversarial learning approach to spatially-varying multiframe blind deconvolution with applications to imaging through turbulence," 2022. URL: https://arxiv.org/pdf/2203.06764.pdf.

[37] R. Feng, J. Gu, Y. Qiao, and C. Dong, "Suppressing model overfitting for image super-resolution networks," 2019. URL: https://arxiv.org/abs/1906.04809.

[38] R. Fergus, B. Singh, A. Hertzmann, S. Roweis, and W. Freeman, "Removing camera shake from a single photograph," *ACM Transactions on Graphics*, vol. 25, pp. 787–794, Jul. 2006. DOI: 10.1145/1141911.1141956.

[39] R. P. Feynman, *The Feynman Lectures on Physics*. Reading, Mass.: Addison-Wesley Pub. Co., 1963–1965., 1965 1963.

[40] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals*, ser. International series in pure and applied physics. McGraw-Hill, 1965.

[41] D. H. Frakes, J. W. Monaco, and M. J. T. Smith, "Suppression of atmospheric turbulence in video using an adaptive control grid interpolation approach," in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 1881–1884, 2001.

[42] R. Frehlich, "Simulation of laser propagation in a turbulent atmosphere," *Applied Optics*, vol. 39, no. 3, pp. 393–397, Jan. 2000. DOI: 10.1364/AO.39.000393.

[43] D. Fried, "Anisoplanatism in adaptive optics," *Journal of the Optical Society of America*, vol. 72, no. 1, pp. 52–61, 1982. DOI: 10.1364/JOSA.72.000052.

[44] D. L. Fried, "Statistics of a geometric representation of wavefront distortion," *Journal of the Optical Society of America*, vol. 55, no. 11, pp. 1427–1435, Nov. 1965. DOI: 10.1364/JOSA.55.001427.

[45] D. L. Fried, "Optical resolution through a randomly inhomogeneous medium for very long and very short exposures," *Journal of the Optical Society of America*, vol. 56, no. 10, pp. 1372–1379, 1966. DOI: 10.1364/JOSA.56.001372.

[46] D. L. Fried, "Probability of getting a lucky short-exposure image through turbulence," *Journal of the Optical Society of America*, vol. 68, no. 12, pp. 1651–1658, Dec. 1978. DOI: 10.1364/JOSA.68.001651.

[47] D. L. Fried, "Differential angle of arrival: Theory, evaluation, and measurement feasibility," *Radio Science*, vol. 10, no. 1, pp. 71–76, 1975. DOI: 10.1029/RS010i001p00071.

[48] D. L. Fried, "Varieties Of Isoplanatism," in *Imaging Through the Atmosphere*, Proc. SPIE 0075, 1976.

[49] D. L. Fried and T. Clark, "Extruding kolmogorov-type phase screen ribbons," *Journal of the Optical Society of America A*, vol. 25, no. 2, pp. 463–468, Feb. 2008. DOI: 10.1364/JOSAA.25.000463.

[50] D. Fried, "Optical heterodyne detection of an atmospherically distorted signal wave front," *Proceedings of the IEEE*, vol. 55, no. 1, pp. 57–77, 1967. DOI: 10.1109/PROC.1967.5377.

[51] U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov*. Cambridge University Press, 1995. DOI: 10.1017/CBO9781139170666.

[52] G. Gilles and S. Osher, "Wavelet burst accumulation for turbulence mitigation," *Journal of Electronic Imaging*, vol. 25, no. 3, p. 033 003, 2016. DOI: 10.1117/1.JEI.25.3.033003.

[53] J. Gilles and N. B. Ferrante, "Open turbulent image set (OTIS)," *Pattern Recognition Letters*, vol. 86, pp. 38–41, 2017. DOI: 10.1016/j.patrec.2016.12.020.

[54] J. W. Goodman, *Introduction to Fourier Optics*, Third ed. Roberts and Company, 2005.

[55] J. W. Goodman, *Speckle Phenomena in Optics: Theory and Applications*. Roberts & Company, 2007.

[56] J. W. Goodman, *Statistical Optics*, Second ed. John Wiley and Sons Inc., 2015.

[57] R. Gray, "Zernikecalc: A MATLAB function to work with zernike polynomials over circular and non-circular pupils," URL: https://www.mathworks.com/matlabcentral/fileexchange/33330-zernikecalc.

[58] D. P. Greenwood, "Bandwidth specification for adaptive optics systems," *Journal of the Optical Society of America*, vol. 67, no. 3, pp. 390–393, Mar. 1977. DOI: 10.1364/JOSA.67.000390.

[59] D. N. Groff, K. J. Miller, and T. W. D. Bosq, "Towards development of improved metrics for quantifying turbulence imposed degradation of long-range video," in *Infrared Imaging Systems: Design, Analysis, Modeling, and Testing XXXII*, p. 117400P, Proc. SPIE 11740, 2021. DOI: 10.1117/12.2587971.

[60] K. K. Halder, M. Tahtali, and S. G. Anavatti, "Geometric correction of atmospheric turbulence-degraded video containing moving objects," *Optics Express*, vol. 23, no. 4, pp. 5091–5101, Feb. 2015. DOI: 10.1364/OE.23.005091.

[61] R. C. Hardie, J. D. Power, D. A. LeMaster, D. R. Droege, S. Gladysz, and S. Bose-Pillai, "Simulation of anisoplanatic imaging through optical turbulence using numerical wave propagation with new validation analysis," *Optical Engineering*, vol. 56, no. 7, p. 071 502, 2017. DOI: 10.1117/1.OE.56.7.071502.

[62] R. C. Hardie, M. A. Rucci, S. R. Bose-Pillai, R. V. Hook, and B. K. Karch, "Modeling and simulation of multispectral imaging through anisoplanatic atmospheric optical turbulence," *Optical Engineering*, vol. 61, no. 9, p. 093 102, 2022. DOI: 10.1117/1.OE.61.9.093102.

[63] R. C. Hardie, M. A. Rucci, A. J. Dapore, and B. K. Karch, "Block matching and wiener filtering approach to optical turbulence mitigation and its application to simulated and real imagery with quantitative error analysis," *Optical Engineering*, vol. 56, no. 7, p. 071 503, 2017. DOI: 10.1117/1.OE.56.7.071503.

[64] R. He, Z. Wang, Y. Fan, and D. Feng, "Atmospheric turbulence mitigation based on turbulence extraction," in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 1442–1446, Mar. 2016. DOI: 10.1109/ICASSP.2016.7471915.

[65] E. Hecht, *Optics*, Fifth ed. Pearson Education, Inc., 2015.

[66] G. Heidbreder, "Image degradation with random wavefront tilt compensation," *IEEE Transactions on Antennas and Propagation*, vol. 15, no. 1, pp. 90–98, 1967. DOI: 10.1109/TAP.1967.1138846.

[67] B. J. Herman and L. A. Strugala, "Method for inclusion of low-frequency contributions in numerical representation of atmospheric turbulence," in *Propagation of High-Energy Laser Beams Through the Earth's Atmosphere*, Proc. SPIE 1221, 1990. DOI: 10.1117/12.18342.

[68] M. Hirsch, S. Sra, B. Schölkopf, and S. Harmeling, "Efficient filter flow for space-variant multiframe blind deconvolution," in *IEEE Conference on Computer Vision and Pattern Recognition*, pp. 607–614, 2010. DOI: 10.1109/CVPR.2010.5540158.

[69] C. S. Huebner, "Turbulence mitigation of short exposure image data using motion detection and background segmentation," in *Infrared Imaging Systems: Design, Analysis, Modeling, and Testing XXIII*, Proc. SPIE 8355, May 2012. DOI: 10.1117/12.918255.

[70] R. E. Hufnagel, "Propagation through atmospheric turbulence," *The Infrared Handbook,* Chapter 6, 1978.

[71] R. E. Hufnagel and N. R. Stanley, "Modulation transfer function associated with image transmission through turbulent media," *Journal of the Optical Society of America*, vol. 54, no. 1, pp. 52–61, Jan. 1964. DOI: 10.1364/JOSA.54.000052.

[72] T.-W. Hui, X. Tang, and C. C. Loy, "LiteFlowNet: A Lightweight Convolutional Neural Network for Optical Flow Estimation," in *IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 8981–8989, 2018.

[73] B. R. Hunt, A. L. Iler, C. A. Bailey, and M. A. Rucci, "Synthesis of atmospheric turbulence point spread functions by sparse and redundant representations," *Optical Engineering*, vol. 57, no. 2, p. 024 101, 2018. DOI: 10.1117/1.OE.57.2.024101.

[74] E. Ilg, N. Mayer, T. Saikia, M. Keuper, A. Dosovitskiy, and T. Brox, "Flownet 2.0: Evolution of optical flow estimation with deep networks," in *IEEE Conference on Computer Vision and Pattern Recognition*, pp. 2462–2470, 2017. DOI: 10.1109/CVPR.2017.179.

[75] A. Ishimaru, "Theory and application of wave propagation and scattering in random media," *Proceedings of the IEEE*, vol. 65, no. 7, pp. 1030–1061, 1977. DOI: 10.1109/PROC.1977.10612.

[76] A. Ishimaru, *Wave Propagation and Scattering in Random Media*, vol. 2. New York: Academic Press, 1978.

[77] J. D. Jackson, *Classical Electrodynamics*, Third ed. New York, NY: Wiley, 1999.

[78] A. J. E. M. Janssen, "Extended nijboer–zernike approach for the computation of optical point-spread functions," *Journal of the Optical Society of America A*, vol. 19, no. 5, pp. 849–857, May 2002. DOI: 10.1364/JOSAA.19.000849.

[79] D. Jin, Y. Chen, Y. Lu, J. Chen, P. Wang, Z. Liu, S. Guo, and X. Bai, "Neutralizing the impact of atmospheric turbulence on complex scene imaging via deep learning," *Nature Machine Intelligence*, vol. 3, pp. 876–884, 2021. DOI: 10.1038/s42256-021-00392-1.

[80] N. Joshi and M. F. Cohen, "Seeing Mt. Rainier: Lucky imaging for multi-image denoising, sharpening, and haze removal," in *IEEE International Conference on Computational Photography*, pp. 1–8, 2010. DOI: 10.1109/ICCPHOT.2010.5585096.

[81] M. Khorasaninejad, W. T. Chen, R. C. Devlin, J. Oh, A. Y. Zhu, and F. Capasso, "Metalenses at visible wavelengths: Diffraction-limited focusing and subwavelength resolution imaging," *Science*, vol. 352, no. 6290, pp. 1190–1194, 2016. DOI: 10.1126/science.aaf6644.

[82] A. N. Kolmogorov, "Dissipation of energy in the locally isotropic turbulence," *Doklady Akademii Nauk SSSR*, vol. 32, pp. 16–18, 1941.

[83] A. N. Kolmogorov, "The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers," *Doklady Akademii Nauk SSSR*, vol. 30, pp. 301–305, 1941.

[84] A. N. Kolmogorov, *Foundations of the Theory of Probability*, ser. AMS Chelsea Publishing Series. Chelsea Publishing Company, 1956.

[85] O. Korotkova, *Random Light Beams*. CRC Press, 2017.

[86] D. Krishnan, T. Tay, and R. Fergus, "Blind deconvolution using a normalized sparsity measure," in *IEEE Conference on Computer Vision and Pattern Recognition*, IEEE, pp. 233–240, 2011. DOI: 10.1109/CVPR.2011.5995521.

[87] S. L. Lachinova, M. A. Vorontsov, V. V. Dudorov, V. V. Kolosov, and M. T. Valley, "Anisoplanatic imaging through atmospheric turbulence: Brightness function approach," in *Atmospheric Optics: Models, Measurements, and Target-in-the-Loop Propagation*, Proc. SPIE 6708, 2007. DOI: 10.1117/12.738796.

[88] S. L. Lachinova, M. A. Vorontsov, G. A. Filimonov, D. A. LeMaster, and M. E. Trippel, "Comparative analysis of numerical simulation techniques for incoherent imaging of extended objects through atmospheric turbulence," *Optical Engineering*, vol. 56, no. 7, 2017. DOI: 10.1117/1.OE.56.7.071509.

[89] R. G. Lane, A. Glindemann, and J. C. Dainty, "Simulation of a Kolmogorov phase screen," *Waves in Random Media*, vol. 2, no. 3, pp. 209–224, 1992. DOI: 10.1088/0959-7174/2/3/003.

[90] C. P. Lau, Y. H. Lai, and L. M. Lui, "Restoration of atmospheric turbulence-distorted images via RPCA and quasiconformal maps," *Inverse Problems*, Mar. 2019. DOI: 10.1088/1361-6420/ab0e4b.

[91] C. P. Lau and L. M. Lui, "Subsampled turbulence removal network," *Mathematics, Computation and Geometry of Data*, vol. 1, no. 1, pp. 1–33, 2021. DOI: 10.4310/MCGD.2021.v1.n1.a1.

[92] C. P. Lau, H. Souri, and R. Chellappa, "Atfacegan: Single face semantic aware image restoration and recognition from atmospheric turbulence," *IEEE Transactions on Biometrics, Behavior, and Identity Science*, vol. 3, no. 2, pp. 240–251, Feb. 2021. DOI: 10.1109/TBIOM.2021.3058316.

[93] C. P. Lau, Y. H. Lai, and L. M. Lui, "Restoration of atmospheric turbulence-distorted images via RPCA and quasiconformal maps," *Inverse Problems*, Mar. 2019.

[94] K. R. Leonard, J. Howe, and D. E. Oxford, "Simulation of atmospheric turbulence effects and mitigation algorithms on stand-off automatic facial recognition," in *Optics and Photonics for Counterterrorism, Crime Fighting, and Defence VIII*, Proc. SPIE 8546, Oct. 2012. DOI: 10.1117/12.979480.

[95] A. Levin, "Blind motion deblurring using image statistics," *Advances in Neural Information Processing Systems*, vol. 19, 2006. DOI: 10.7551/mitpress/7503.003.0110.

[96] A. Levin, R. Fergus, F. Durand, and W. T. Freeman, "Image and depth from a conventional camera with a coded aperture," *ACM Transactions on Graphics*, vol. 26, no. 3, 70–es, Jul. 2007. DOI: 10.1145/1276377.1276464.

[97] A. Levin, Y. Weiss, F. Durand, and W. T. Freeman, "Understanding blind deconvolution algorithms," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 33, no. 12, pp. 2354–2367, 2011. DOI: 10.1109/TPAMI.2011.148.

[98] N. Li, S. Thapa, C. Whyte, A. W. Reed, S. Jayasuriya, and J. Ye, "Unsupervised non-rigid image distortion removal via grid deformation," in *IEEE/CVF International Conference on Computer Vision*, pp. 2522–2532, Oct. 2021.

[99] C. Liu, "Beyond pixels: Exploring new representations and applications for motion analysis," Ph.D. dissertation, Massachusetts Institute of Technology, Jan. 2009, 2009.

[100] Y. Lou, S. Ha Kang, S. Soatto, and A. Bertozzi, "Video stabilization of atmospheric turbulence distortion," *Inverse Problems and Imaging*, vol. 7, no. 3, pp. 839–861, Aug. 2013. DOI: 10.3934/ipi.2013.7.839.

[101] C. Macaskill and T. E. Ewart, "Computer simulation of two-dimensional random wave propagation," *IMA Journal of Applied Mathematics*, vol. 33, no. 1, pp. 1–15, Jul. 1984. DOI: 10.1093/imamat/33.1.1.

[102] Y. Mao and J. Gilles, "Non rigid geometric distortions correction - application to atmospheric turbulence stabilization," *Inverse Problems and Imaging*, vol. 3, pp. 531–546, 2012. DOI: 10.3934/ipi.2012.6.531.

[103] Z. Mao, N. Chimitt, and S. H. Chan, "Image reconstruction of static and dynamic scenes through anisoplanatic turbulence," *IEEE Transactions on Computational Imaging*, vol. 6, pp. 1415–1428, Oct. 2020. DOI: 10.1109/TCI.2020.3029401.

[104] Z. Mao, N. Chimitt, and S. H. Chan, "Accelerating atmospheric turbulence simulation via learned phase-to-space transform," in *IEEE/CVF International Conference on Computer Vision*, pp. 14 759–14 768, Oct. 2021. DOI: 10.1109/ICCV48922.2021.01449.

[105] Z. Mao, A. Jaiswal, Z. Wang, and S. H. Chan, "Single frame atmospheric turbulence mitigation: A benchmark study and a new physics-inspired transformer model," in *European Conference on Computer Vision*, 2022.

[106] J. M. Martin and S. M. Flatté, "Intensity images and statistics from numerical simulation of wave propagation in 3-d random media," *Applied Optics*, vol. 27, no. 11, pp. 2111–2126, Jun. 1988. DOI: 10.1364/AO.27.002111.

[107] B. L. McGlamery, "Restoration of turbulence-degraded images," *Journal of the Optical Society of America*, vol. 57, no. 3, pp. 293–297, Mar. 1967. DOI: 10.1364/JOSA.57.000293.

[108] T. Michaeli and M. Irani, "Blind deblurring using internal patch recurrence," in *European Conference on Computer Vision*, Springer, pp. 783–798, 2014. DOI: 10.1007/978-3-319-10578-9_51.

[109] K. J. Miller and T. D. Bosq, "A machine learning approach to improving quality of atmospheric turbulence simulation," in *Infrared Imaging Systems: Design, Analysis, Modeling, and Testing XXXII*, p. 117400N, Proc. SPIE 11740, 2021. DOI: 10.1117/12.2587749.

[110] K. J. Miller, B. Preece, T. W. D. Bosq, and K. R. Leonard, "A data-constrained algorithm for the emulation of long-range turbulence-degraded video," in *Infrared Imaging Systems: Design, Analysis, Modeling, and Testing XXX*, G. C. Holst and K. A. Krapels, Eds., International Society for Optics and Photonics, vol. 11001, 110010J, SPIE, 2019. DOI: 10.1117/12.2519069.

[111] N. G. Nair, K. Mei, and V. M. Patel, "AT-DDPM: Restoring faces degraded by atmospheric turbulence using denoising diffusion probabilistic models," in *IEEE/CVF Winter Conference on Applications of Computer Vision*, pp. 3434–3443, 2023.

[112] R. Nieuwenhuizen, J. Dijk, and K. Schutte, "Dynamic turbulence mitigation for long-range imaging in the presence of large moving objects," *EURASIP Journal on Image and Video Processing*, vol. 2, no. 2, 2019. DOI: 10.1186/s13640-018-0380-9.

[113] R. Nieuwenhuizen and K. Schutte, "Deep learning for software-based turbulence mitigation in long-range imaging," in *Artificial Intelligence and Machine Learning in Defense Applications*, Proc. SPIE 11169, 2019. DOI: 10.1117/12.2532603.

[114] R. J. Noll, "Zernike polynomials and atmospheric turbulence," *Journal of the Optical Society of America*, vol. 66, no. 3, pp. 207–211, Mar. 1976. DOI: 10.1364/JOSA.66.000207.

[115] K. Novak and A. T. Watnik, "Imaging through deconvolution with a spatially variant point spread function," in *Computational Imaging VI*, p. 1 173 105, Proc. SPIE 11731, 2021. DOI: 10.1117/12.2585632.

[116] A. M. Obukhov, "On the distribution of energy in the sprectrum of turbulent flow," *Doklady Akademii Nauk SSSR*, vol. 32, pp. 22–24, 1941.

[117] A. M. Obukhov, "Spectral energy distribution in a turbulent flow," *Izv. Akad. Nauk S.S.S.R, Ser. Georgr. Geofiz.*, vol. 5, pp. 453–466, 1941.

[118] A. M. Obukhov, "Effect of weak inhomogeneities in the atmosphere on sound and light propagation," *Izv. Akad. Nauk S.S.S.R, Ser. Geofiz.*, vol. 2, pp. 155–165, 1953.

[119] A. V. Oppenheim, A. S. Willsky, and S. H. Nawab, *Signals & Systems*. USA: Prentice-Hall, Inc., 1996.

[120] O. Oreifej, X. Li, and M. Shah, "Simultaneous video stabilization and moving object detection in turbulence," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 35, no. 2, pp. 450–462, Feb. 2013. DOI: 10.1109/TPAMI.2012.97.

[121] R. R. Parenti and R. J. Sasiela, "Laser-guide-star systems for astronomical applications," *Journal of the Optical Society of America A*, vol. 11, no. 1, pp. 288–309, Jan. 1994. DOI: 10.1364/JOSAA.11.000288.

[122] C. J. Pellizzari, M. T. Banet, M. F. Spencer, and C. A. Bouman, "Demonstration of single-shot digital holography using a bayesian framework," *Journal of the Optical Society of America A*, vol. 35, no. 1, pp. 103–107, Jan. 2018. DOI: 10.1364/JOSAA.35.000103.

[123] C. J. Pellizzari, M. F. Spencer, and C. A. Bouman, "Imaging through distributed-volume aberrations using single-shot digital holography," *Journal of the Optical Society of America A*, vol. 36, no. 2, A20–A33, Feb. 2019. DOI: 10.1364/JOSAA.36.000A20.

[124] C. J. Pellizzari, R. Trahan, H. Zhou, S. Williams, S. E. Williams, B. Nemati, M. Shao, and C. A. Bouman, "Optically coherent image formation and denoising using a plug and play inversion framework," *Applied Optics*, vol. 56, no. 16, pp. 4735–4744, Jun. 2017. DOI: 10.1364/AO.56.004735.

[125] J. R. Peterson, J. G. Jernigan, S. M. Kahn, A. P. Rasmussen, E. Peng, Z. Ahmad, J. Bankert, C. Chang, C. Claver, D. K. Gilmore, E. Grace, M. Hannel, M. Hodge, S. Lorenz, A. Lupu, A. Meert, S. Nagarajan, N. Todd, A. Winans, and M. Young, "Simulation of astronomical images from optical survey telescopes using a comprehensive photon monte carlo approach," *The Astrophysical Journal Supplement Series*, vol. 218, no. 1, p. 14, May 2015. DOI: 10.1088/0067-0049/218/1/14.

[126] J. D. Phillips, M. E. Goda, and J. Schmidt, "Atmospheric turbulence simulation using liquid crystal spatial light modulators," in *Advanced Wavefront Control: Methods, Devices, and Applications III*, International Society for Optics and Photonics, vol. 5894, p. 589 406, SPIE, 2005. DOI: 10.1117/12.620407.

[127] G. Potvin, J. L. Forand, and D. Dion, "A simple physical model for simulating turbulent imaging," in *Infrared Imaging Systems: Design, Analysis, Modeling, and Testing XXII*, G. C. Holst and K. A. Krapels, Eds., International Society for Optics and Photonics, vol. 8014, 80140Y, SPIE, 2011. DOI: 10.1117/12.884989.

[128] R. Ramamoorthi, "Precomputation-based rendering," *Foundations and Trends® in Computer Graphics and Vision*, vol. 3, no. 4, pp. 281–369, 2009. DOI: 10.1561/0600000021.

[129] A. Ranjan and M. J. Black, "Optical flow estimation using a spatial pyramid network," in *IEEE Conference on Computer Vision and Pattern Recognition*, pp. 4161–4170, 2017. DOI: 10.1109/CVPR.2017.291.

[130] R. Raskar, A. Agrawal, and J. Tumblin, "Coded exposure photography: Motion deblurring using fluttered shutter," *ACM Transactions on Graphics*, vol. 25, no. 3, pp. 795–804, Jul. 2006. DOI: 10.1145/1141911.1141957.

[131] E. Repasi and R. Weiss, "Analysis of image distortions by atmospheric turbulence and computer simulation of turbulence effects," in *Infrared Imaging Systems: Design, Analysis, Modeling, and Testing XIX*, Proc. SPIE 6941, 2008. DOI: 10.1117/12.775600.

[132] E. Repasi and R. Weiss, "Computer simulation of image degradations by atmospheric turbulence for horizontal views," in *Infrared Imaging Systems: Design, Analysis, Modeling, and Testing XXII*, Proc. SPIE 8014, 2011. DOI: 10.1117/12.883805.

[133] E. Repasi and R. Weiss, "Computer simulation of image degradations by atmospheric turbulence for horizontal views," in *Infrared Imaging Systems: Design, Analysis, Modeling, and Testing XXII*, International Society for Optics and Photonics, vol. 8014, 80140U, 2011. DOI: 10.1117/12.883805.

[134] F. Roddier, *Adaptive Optics in Astronomy.* Cambridge University Press, 1999. DOI: 10.1017/CBO9780511525179.

[135] N. A. Roddier, "Atmospheric wavefront simulation using Zernike polynomials," *Optical Engineering*, vol. 29, no. 10, pp. 1174–1180, 1990. DOI: 10.1117/12.55712.

[136] B. Rodenburg, M. Mirhosseini, M. Malik, O. S. Magaña-Loaiza, M. Yanakas, L. Maher, N. K. Steinhoff, G. A. Tyler, and R. W. Boyd, "Simulating thick atmospheric turbulence in the lab with application to orbital angular momentum communication," *New Journal of Physics*, vol. 16, no. 3, p. 033 020, Mar. 2014. DOI: 10.1088/1367-2630/16/3/033020.

[137] M. C. Roggemann and B. M. Welsh, *Imaging Through Atmospheric Turbulence*. Taylor & Francis, 1996.

[138] M. C. Roggemann, C. A. Stoudt, and B. M. Welsh, "Image-spectrum signal-to-noise-ratio improvements by statistical frame selection for adaptive-optics imaging through atmospheric turbulence," *Optical Engineering*, vol. 33, no. 10, pp. 3254–3264, 1994. DOI: 10.1117/12.181250.

[139] M. C. Roggemann, B. M. Welsh, D. Montera, and T. A. Rhoadarmer, "Method for simulating atmospheric turbulence phase effects for multiple time slices and anisoplanatic conditions," *Applied Optics*, vol. 34, no. 20, pp. 4037–4051, Jul. 1995. DOI: 10.1364/AO.34.004037.

[140] Y. Romano, M. Elad, and P. Milanfar, "The little engine that could: Regularization by denoising (RED)," *SIAM Journal on Imaging Sciences*, vol. 10, no. 4, pp. 1804–1844, 2017. DOI: 10.1137/16M1102884.

[141] M. A. Rucci, R. C. Hardie, and R. K. Martin, "Simulation of anisoplanatic lucky look imaging and statistics through optical turbulence using numerical wave propagation," *Applied Optics*, vol. 60, no. 25, G19–G29, Sep. 2021. DOI: 10.1364/AO.427716.

[142] A. S. S. Gepshtein and B. Fishbain, "Restoration of atmospheric turbulent video containing real motion using rank filtering and elastic image registration," in *European Signal Processing Conference*, pp. 477–480, Sep. 2004.

[143] S. M. Safdarnejad, X. Liu, L. Udpa, B. Andrus, J. Wood, and D. Craven, "Sports videos in the wild (svw): A video dataset for sports analysis," in *2015 11th IEEE International Conference and Workshops on Automatic Face and Gesture Recognition*, IEEE, vol. 1, pp. 1–7, 2015. DOI: 10.1109/FG.2015.7163105.

[144] J. D. Schmidt, *Numerical Simulation of Optical Wave Propagation: With Examples in MATLAB*. SPIE Press, 2010, pp. 1–197. DOI: 10.1117/3.866274.

[145] M. Shimizu, S. Yoshimura, M. Tanaka, and M. Okutomi, "Super-resolution from image sequence under influence of hot-air optical turbulence," in *IEEE Conference on Computer Vision and Pattern Recognition*, pp. 1–8, 2008.

[146] M. Spivack and B. Uscinski, "The split-step solution in random wave propagation," *Journal of Computational and Applied Mathematics*, vol. 27, no. 3, pp. 349–361, 1989. DOI: 10.1016/0377-0427(89)90022-8.

[147] N. Takato and I. Yamaguchi, "Spatial correlation of Zernike phase-expansion coefficients for atmospheric turbulence with finite outer scale," *Journal of the Optical Society of America A*, vol. 12, no. 5, pp. 958–963, May 1995. DOI: 10.1364/JOSAA.12.000958.

[148] V. I. Tatarskii, *Wave Propagation in a Turbulent Medium*. New York: Dover Publications, 1961. DOI: 10.1063/1.3057286.

[149] V. Tatarskii, A. Ishimaru, and V. Zavorotny, *Wave Propagation in Random Media (Scintillation): Proceedings of the Conference Held 3-7 August, 1992 at the University of Washington, USA*, ser. SPIE press monographs. Taylor & Francis, 1993.

[150] V. Tatarskii and V. Zavorotnyi, "Strong fluctuations in light propagation in a randomly inhomogeneous medium," *Progress in Optics*, vol. 18, pp. 204–256, 1980. DOI: 10.1016/S0079-6638(08)70214-2.

[151] C. Tian, Y. Xu, and W. Zuo, "Image denoising using deep cnn with batch renormalization," *Neural Networks*, vol. 121, pp. 461–473, 2020. DOI: 10.1016/j.neunet.2019.08.022.

[152] Y. Tian and S. G. Narasimhan, "Seeing through water: Image restoration using model-based tracking," in *IEEE International Conference on Computer Vision*, pp. 2303–2310, 2009. DOI: 10.1109/ICCV.2009.5459440.

[153] Y. Tian, S. G. Narasimhan, and A. J. Vannevel, "Depth from optical turbulence," in *IEEE Conference on Computer Vision and Pattern Recognition*, pp. 246–253, 2012. DOI: 10.1109/CVPR.2012.6247682.

[154] D. H. Tofsted, D. Quintis, S. G. O'Brien, J. Yarbrough, M. D. Bustillos, and G. M. T. Vaucher, Tech. Rep.

[155] D. Tofsted, S. O'Brien, J. Yarbrough, D. Quintis, and M. Bustillos, "Characterization of atmospheric turbulence during the nato rtg-40 land field trials," in *Atmospheric Propagation IV*, Proc. SPIE 6551, 2007. DOI: 10.1117/12.720696.

[156] R. Tyson, *Principles of Adaptive Optics*. CRC Press, 2010. DOI: 10.1201/EBK1439808580.

[157] R. K. Tyson, "Adaptive optics and ground-to-space laser communications," *Applied Optics*, vol. 35, no. 19, pp. 3640–3646, Jul. 1996. DOI: 10.1364/AO.35.003640.

[158] G. C. Valley, "Isoplanatic degradation of tilt correction and short-term imaging systems," *Applied Optics*, vol. 19, no. 4, pp. 574–577, Feb. 1980. DOI: 10.1364/AO.19.000574.

[159] S. van Haver, "The extended Nijboer-Zernike diffraction theory and its applications," Ph.D. dissertation, Delft University of Technology, 2010, 2010.

[160] A. Veeraraghavan, R. Raskar, A. Agrawal, A. Mohan, and J. Tumblin, "Dappled photography: Mask enhanced cameras for heterodyned light fields and coded aperture refocusing," *ACM Transactions on Graphics*, vol. 26, no. 3, 69–es, 2007. DOI: 10.1145/1276377.1276463.

[161] M. T. Velluet, C. Bell, J. F. Daigle, J. Dijk, S. Gladysz, A. Kanaev, A. Lambert, D. Lemaster, G. Potvin, and M. Vorontsov, "Data collection and preliminary results on turbulence characterisation and mitigation techniques," in *Electro-Optical and Infrared Systems: Technology and Applications XVI*, Proc. SPIE 11159, 2019. DOI: 10.1117/12.2533821.

[162] S. V. Venkatakrishnan, C. A. Bouman, and B. Wohlberg, "Plug-and-play priors for model based reconstruction," in *IEEE Global Conference on Signal and Information Processing*, IEEE, pp. 945–948, 2013. DOI: 10.1109/GlobalSIP.2013.6737048.

[163] D. Vint, G. D. Caterina, J. Soraghan, R. Lamb, and D. Humphreys, "Analysis of deep learning architectures for turbulence mitigation in long-range imagery," in *Artificial Intelligence and Machine Learning in Defense Applications II*, Proc. SPIE 11543, 2020. DOI: 10.1117/12.2573927.

[164] D. G. Voelz, *Computational Fourier Optics: A MATLAB Tutorial*. SPIE Press, 2011. DOI: 10.1117/3.858456.

[165] D. G. Voelz, E. Wijerathna, A. Muschinski, and X. Xiao, "Computer simulations of optical turbulence in the weak- and strong-scattering regime: Angle-of-arrival fluctuations obtained from ray optics and wave optics," *Optical Engineering*, vol. 57, no. 10, p. 104 102, 2018. DOI: 10.1117/1.OE.57.10.104102.

[166] M. A. Vorontsov and G. W. Carhart, "Anisoplanatic imaging through turbulent media: Image recovery by local information fusion from a set of short-exposure images," *Journal of the Optical Society of America A*, vol. 18, no. 6, pp. 1312–1324, Jun. 2001. DOI: 10.1364/JOSAA.18.001312.

[167] M. A. Vorontsov and V. Kolosov, "Target-in-the-loop beam control: Basic considerations for analysis and wave-front sensing," *Journal of the Optical Society of America A*, vol. 22, no. 1, pp. 126–141, Jan. 2005. DOI: 10.1364/JOSAA.22.000126.

[168] F. Wang, I. Toselli, and O. Korotkova, "Two spatial light modulator system for laboratory simulation of random beam propagation in random media," *Applied Optics*, vol. 55, no. 5, pp. 1112–1117, Feb. 2016. DOI: 10.1364/AO.55.001112.

[169] J. Y. Wang and J. K. Markey, "Modal compensation of atmospheric turbulence phase distortion," *Journal of the Optical Society of America*, vol. 68, no. 1, pp. 78–87, Jan. 1978. DOI: 10.1364/JOSA.68.000078.

[170] A. T. Watnik and D. F. Gardner, "Wavefront sensing in deep turbulence," *Optics and Photonics News*, vol. 29, no. 10, pp. 38–45, Oct. 2018. DOI: 10.1364/OPN.29.10.000038.

[171] B. M. Welsh, "Fourier-series-based atmospheric phase screen generator for simulating anisoplanatic geometries and temporal evolution," in *Propagation and Imaging through the Atmosphere*, Proc. SPIE 3125, 1997. DOI: 10.1117/12.279029.

[172] M. R. Whiteley, M. C. Roggemann, and B. M. Welsh, "Temporal properties of the zernike expansion coefficients of turbulence-induced phase aberrations for aperture and source motion," *Journal of the Optical Society of America A*, vol. 15, no. 4, pp. 993–1005, Apr. 1998. DOI: 10.1364/JOSAA.15.000993.

[173] Y. Xie, W. Zhang, D. Tao, W. Hu, Y. Qu, and H. Wang, "Removing turbulence effect via hybrid total variation and deformation-guided kernel regression," *IEEE Transactions on Image Processing*, vol. 25, no. 10, pp. 4943–4958, Oct. 2016. DOI: 10.1109/TIP.2016.2598638.

[174] B. Xue, Y. Liu, L. Cui, X. Bai, X. Cao, and F. Zhou, "Video stabilization in atmosphere turbulent conditions based on the Laplacian-Riesz pyramid," *Optics Express*, vol. 24, no. 24, pp. 28 092–28 103, Nov. 2016. DOI: 10.1364/OE.24.028092.

[175] R. Yasarla and V. M. Patel, "CNN-Based restoration of a single face image degraded by atmospheric turbulence," *IEEE Transactions on Biometrics, Behavior, and Identity Science*, vol. 4, no. 2, pp. 222–233, 2022. DOI: 10.1109/TBIOM.2022.3169697.

[176] S. W. Zamir, A. Arora, S. Khan, M. Hayat, F. S. Khan, and M.-H. Yang, "Restormer: Efficient transformer for high-resolution image restoration," in *IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 5728–5739, 2022. DOI: 10.1109/CVPR52688.2022.00564.

[177] V. U. Zavorotnyi, V. I. Kliatskin, and V. I. Tatarskii, "Strong fluctuations of electromagnetic-wave intensity in randomly inhomogeneous media," *Zhurnal Eksperimentalnoi i Teoreticheskoi Fiziki*, vol. 73, pp. 481–497, Aug. 1977.

[178] K. Zhang, W. Zuo, Y. Chen, D. Meng, and L. Zhang, "Beyond a gaussian denoiser: Residual learning of deep CNN for image denoising," *IEEE Transactions on Image Processing*, vol. 26, no. 7, pp. 3142–3155, 2017. DOI: 10.1109/TIP.2017.2662206.

[179] X. Zhang, Z. Mao, N. Chimitt, and S. H. Chan, "Imaging through the atmosphere using turbulence mitigation transformer," URL: https://arxiv.org/abs/2207.06465.

[180] Y. Zhang, K. Li, K. Li, L. Wang, B. Zhong, and Y. Fu, "Image super-resolution using very deep residual channel attention networks," in *European Conference on Computer Vision*, pp. 294–310, 2018. DOI: 10.1007/978-3-030-01234-2_18.

[181] Y. Zhang, Y. Tian, Y. Kong, B. Zhong, and Y. Fu, "Residual dense network for image restoration," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 43, no. 7, pp. 2480–2495, 2021. DOI: 10.1109/TPAMI.2020.2968521.

[182] B. Zhou, A. Lapedriza, A. Khosla, A. Oliva, and A. Torralba, "Places: A 10 million image database for scene recognition," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2017. DOI: 10.1109/TPAMI.2017.2723009.

[183] X. Zhu and P. Milanfar, "Removing atmospheric turbulence via space-invariant deconvolution," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 35, no. 1, pp. 157–170, Jan. 2013. DOI: 10.1109/TPAMI.2012.82.