
**Reliability Criteria in
Information Theory
and in Statistical
Hypothesis Testing**

Reliability Criteria in Information Theory and in Statistical Hypothesis Testing

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Foundations and Trends[®] in Communications and Information Theory

Published, sold and distributed by:

now Publishers Inc.
PO Box 1024
Hanover, MA 02339
USA
Tel. +1-781-985-4510
www.nowpublishers.com
sales@nowpublishers.com

Outside North America:

now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is E. A. Haroutunian, M. E. Haroutunian and A. N. Harutyunyan, Reliability Criteria in Information Theory and in Statistical Hypothesis Testing, Foundations and Trends[®] in Communications and Information Theory, vol 4, nos 2–3, pp 97–263, 2007

ISBN: 978-1-60198-046-5

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Foundations and Trends[®] in Communications and Information Theory, 2007, Volume 4, 6 issues. ISSN paper version 1567-2190. ISSN online version 1567-2328. Also available as a combined paper and online subscription.

Foundations and Trends[®] in
Communications and Information Theory
Vol. 4, Nos. 2–3 (2007) 97–263
© 2008 E. A. Haroutunian, M. E. Haroutunian
and A. N. Harutyunyan
DOI: 10.1561/01000000008



Reliability Criteria in Information Theory and in Statistical Hypothesis Testing

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To the memory of Roland Dobrushin the outstanding scientist and
wonderful teacher.

Abstract

This survey is devoted to one of the central problems of Information Theory — the problem of determination of interdependence between coding rate and error probability exponent for different information transmission systems. The overview deals with memoryless systems of finite alphabet setting. It presents material complementary to the contents of the series of the most remarkable in Information Theory books of Feinstein, Fano, Wolfowitz, Gallager, Csiszar and Körner, Kolesnik and Poltirev, Blahut, Cover and Thomas and of the papers by Dobrushin, Gelfand and Prelov.

We briefly formulate fundamental notions and results of Shannon theory on reliable transmission via coding and give a survey of results obtained in last two-three decades by the authors, their colleagues and some other researchers. The paper is written with the goal to make accessible to a broader circle of readers the theory of rate-reliability. We regard this concept useful to promote the noted problem solution in parallel with elaboration of the notion of reliability-reliability dependence relative to the statistical hypothesis testing and identification.

Preface

This monograph is devoted to one of the central problems of Information Theory — the problem of determination of interdependence between coding rate and error probability exponent for different information transmission systems. The overview deals with memoryless systems of finite alphabet setting. It presents material complementary to the contents of the series of the most remarkable in Information Theory books.

We briefly formulate fundamental notions and results of Shannon theory on reliable transmission via coding and give a survey of results obtained in last two–three decades by coauthors, their colleagues, and some other researchers. The review was written with the goal to make accessible to a broader circle of readers the concept of rate-reliability. We regard this concept useful to promote the noted problem solution in parallel with elaboration of the notion of reliability–reliability dependence relative to the statistical hypothesis testing and identification.

The authors are grateful to R. Ahlswede, V. Balakirsky, P. Harremöes, N. Cai for their useful inputs to the earlier versions of the manuscript.

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The comments and suggestions of S. Shamai (Shitz), G. Kramer, and the anonymous reviewers are highly appreciated.

The participation of our colleagues P. Hakobyan and S. Tonoyan in the manuscript revision was helpful.

A. Harutyunyan acknowledges the support by the Alexander von Humboldt Foundation to his research at the Institute for Experimental Mathematics, Essen University.

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1

Introduction

1.1 Information Theory and Problems of Shannon Theory

The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at an other point.

Claude Shannon, 1948

Information Theory as a scientific discipline originated from the landmark work “A mathematical theory of communication” [191] of an American genius engineer and mathematician Claude E. Shannon in 1948, and thereafter exists as a formalized science with more than a half century life. In the Guest Editorial [215] to “Commemorative issue 1948–1998” of IEEE Transactions on Information Theory Sergio Verdú certified: “With communication engineering in the epicenter of the bombshell, the sensational aftermath of Shannon’s paper soon reached Mathematics, Physics, Statistics, Computing, and Cryptology. Even Economics, Biology, Linguistics, and other fields in the natural and social sciences felt the ripples of Shannon’s new theory.” In his wise retrospective [82] on the founder’s life and scientific heritage Robert Gallager wrote: “Claude E. Shannon invented information theory and

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provided the concepts, insights, and mathematical formulations that now form the basis for modern communication technology. In a surprisingly large number of ways, he enabled the information age.” The exceptional role of Claude Shannon in development of modern science was noted earlier by Andrey Kolmogorov in Preface to Russian edition of Shannon’s “Works on Information Theory and Cybernetics” [197] and by Roland Dobrushin in Preface of Editor to the Russian translation of the book by Csiszár and Körner [51].

In [191] and another epochal work [194] Shannon mathematically addressed the basic problems in communications and gave their solutions, stating the three fundamental discoveries underlying the information theory concerning the transmission problem via noisy channel and its inherent concept — capacity, data compression with the central role of entropy in that, and source coding under fidelity criterion with specification of the possible performance limit in terms of the mutual information introduced by him.

Under the term “Shannon Theory” it is generally accepted now to mean the subfield of information theory which deals with the establishment of performance bounds for various parameters of transmission systems.

The relevant sections of this review treat noted fundamental results and go further in generalizations and solutions of those problems toward some classical and more complicated communication situations, focusing on the results and methodology developed mainly in the works of coauthors related to the role of the error probability exponent as a characteristic in the mathematical model of an information transmission system.

Taking into account the interconnection of the statistical, probabilistic, and information theoretical problems we hereby add results also on the error exponent (reliability function) investigation in statistical hypotheses testing models.

1.2 Concepts of Reliability Function and of Rate-Reliability Function

Important properties of each communication channel are characterized by the *reliability function* $E(R)$, which was introduced by

Shannon [195], as the optimal exponent of the exponential decrease

$$\exp\{-NE(R)\}$$

of the decoding error probability, when code length N increases, for given transmission rate R less than capacity C of the channel [191]. In an analogous sense one can characterize various communication systems. The reliability function $E(R)$ is also called the *error probability exponent*. Besides, by analogy with the concept of the rate-distortion function [26, 194], the function $E(R)$ may be called the *reliability-rate function*.

There is a large number of works devoted to studying of this function for various communication systems. Along with achievements in this part of Shannon theory a lot of problems have remained unsolved. Because of principal difficulty of finding the reliability function for the whole range of rates $0 < R < C$, this problem is completely solved only in rather particular cases. The situation is typical when obtained upper and lower bounds for the function $E(R)$ coincide only for rates R in some interval, say $R_{\text{crit}} < R < C$, where R_{crit} is the rate, for which the derivative of $E(R)$ by R equals -1 .

It is desirable to create a more harmonious general theory and more effective methods of usable bounds construction for new classes of more complicated information transmission systems. It seems, that the approach developed by the authors is fruitful for this purpose. It consists in studying the function $R(E) = C(E)$, inverse to $E(R)$ [98, 100, 102]. This is not a simple mechanical permutation of roles of independent and dependent variables, since the investigation of optimal rates of codes, ensuring when N increases the error probability exponential decrease with given exponent (reliability) E , can be more expedient than the study of the function $E(R)$.

At the same time, there is an analogy with the problem from coding theory about bounding of codes optimal volume depending on their correction ability. This allows to hope for profitable application of results and methods of one theory in the other. The definition of the function $C(E)$ is in natural conformity with Shannon's notions of the channel capacity C and of the zero-error capacity C_0 [152]. When E increases from zero to infinity the function $C(E)$ decreases from C to C_0 (it is

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so, if $C_0 > 0$, in the other case $C(E) = 0$ when E is great enough). So, by analogy with the definition of the capacity, this characteristic of the channel may be called *E-capacity*. From the other side the name *rate-reliability function* is also logical. One of the advantages of our approach is the convenience in study of the optimal rates of source codes ensuring given exponential decrease of probability of exceeding the given distortion level of messages restoration. This will be the *rate-reliability-distortion function* $R(E, \Delta, P)$ inverse to exponent function $E(R, \Delta, P)$ by Marton [171]. So the name shows which dependence of characteristics is in study. Later on, it is possible to consider also other arguments, for example, coding rates on the other inputs of channel or source, if their number is greater than one. This makes the theory more well-proportioned and comprehensible.

Concerning methods for the bounds construction, it is found that the Shannon's random coding method [191] of proving the existence of codes with definite properties, can be applied with the same success for studying of the rate-reliability function. For the converse coding theorem type upper bounds deduction (so called sphere packing bounds) E. Haroutunian proposed a simple combinatorial method [98, 102], which one can apply to various systems. This method is based on the proof of the strong converse coding theorem, as it was in the method put forth in [99] and used by other authors [35, 51], and [152] for deduction of the sphere packing bound for the reliability function. Moreover, derivation of the upper bound of $C(E)$ by passage to limit for $E \rightarrow \infty$ comes to be the upper bound for the zero-error capacity C_0 .

We note the following practically useful circumstance: the comparison of the analytical form of writing of the sphere packing bound for $C(E)$ with expression of the capacity C in some cases gives us the possibility to write down formally the bound for each system, for which the achievable rates region (capacity) is known. In rate-reliability-distortion theory an advantage of the approach is the technical ease of treatment of the coding rate as a function of distortion and error exponent which allows to convert readily the results from the rate-reliability-distortion area to the rate-distortion ones looking at the extremal values of the reliability, e.g., $E \rightarrow 0, E \rightarrow \infty$. That fact is especially important when one deals with multidimensional situation. Having solved the problem

of finding the rate-reliability-distortion region of a multiterminal system, the corresponding rate-distortion one can be deduced without an effort.

In literature we know an early attempt to consider the concept of E -capacity $R(E)$. In [51] (Section 2.5) Csiszár and Körner mention the concept of “generalized capacity” for DMC as “the capacity” corresponding to tolerated probability of error $\exp\{-NE\}$ (i.e., the largest R with $E(R) \geq E$). But they limited themselves with consideration (problem 15, Section 2.5) only of the case $E \leq E_{cr}(W)$, where $E_{cr}(W) = E(R_{cr}(W))$. In some of the earlier works the rate-reliability function was also considered (for e.g., Fu and Shen [77], Tuncel and Rose [206] and Chen [42]).

E. A. Haroutunian and M. E. Haroutunian [116] have been teaching the concept of E -capacity in Yerevan State University for many years.

1.3 Notations for Measures of Information and Some Identities

Here we introduce our notations for necessary characteristics of Shannon’s entropy and mutual information and Kullback–Leibler’s divergence.

In the review finite sets are considered, which are denoted by $\mathcal{U}, \mathcal{X}, \mathcal{Y}, \mathcal{S}, \dots$. The size of the set \mathcal{X} is denoted by $|\mathcal{X}|$. Random variables (RVs) with values in $\mathcal{U}, \mathcal{X}, \mathcal{Y}, \mathcal{S}, \dots$ are denoted by U, X, Y, S, \dots . Probability distributions (PDs) are denoted by $Q, P, V, W, PV, P \circ V, \dots$. Let PD of RV X be $P \triangleq \{P(x), x \in \mathcal{X}\}$, and V be conditional PD of RV Y for given value x

$$V \triangleq \{V(y|x), x \in \mathcal{X}, y \in \mathcal{Y}\},$$

joint PD of RV X and Y be

$$P \circ V \triangleq \{P \circ V(x, y) = P(x)V(y|x), x \in \mathcal{X}, y \in \mathcal{Y}\},$$

and PD of RV Y be

$$PV \triangleq \left\{ PV(y) = \sum_{x \in \mathcal{X}} P(x)V(y|x), y \in \mathcal{Y} \right\}.$$

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The set of messages to be transmitted are denoted by \mathcal{M} and its cardinality by M .

We use the following notations (here and in the sequel all log-s and exp-s are of base 2): for *entropy* of RV X with PD P :

$$H_P(X) \triangleq - \sum_{x \in \mathcal{X}} P(x) \log P(x),$$

for entropy of RV Y with PD PV :

$$H_{P,V}(Y) \triangleq - \sum_{y \in \mathcal{Y}} PV(y) \log PV(y),$$

for *joint entropy* of RV X and Y :

$$H_{P,V}(X, Y) \triangleq - \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} P \circ V(x, y) \log P \circ V(x, y),$$

for *conditional entropy* of RV Y relative to RV X :

$$H_{P,V}(Y|X) \triangleq - \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} PV(y) \log V(y|x),$$

for *mutual information* of RV X and Y :

$$I_{P,V}(X \wedge Y) = I_{P,V}(Y \wedge X) \triangleq \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} P(x)V(y|x) \log \frac{V(y|x)}{PV(y)},$$

for *conditional mutual information* of RV X and Y relative to RV U with PD $Q \triangleq \{Q(u), u \in \mathcal{U}\}$, $P \triangleq \{P(x|u), u \in \mathcal{U}, x \in \mathcal{X}\}$, $V \triangleq \{V(y|x, u), u \in \mathcal{U}, x \in \mathcal{X}, y \in \mathcal{Y}\}$,

$$I_{Q,P,V}(X \wedge Y|U) \triangleq \sum_{u \in \mathcal{U}, x \in \mathcal{X}, y \in \mathcal{Y}} Q(u)P(x|u)V(y|x, u) \log \frac{V(y|x, u)}{PV(y|u)},$$

for *informational divergence* of PD P and PD Q on \mathcal{X} :

$$D(P||Q) \triangleq \sum_{x \in \mathcal{X}} P(x) \log \frac{P(x)}{Q(x)},$$

and for *informational conditional divergence* of PD $P \circ V$ and PD $P \circ W$ on $\mathcal{X} \times \mathcal{Y}$, where $W \triangleq \{W(y|x), x \in \mathcal{X}, y \in \mathcal{Y}\}$:

$$D(P \circ V||P \circ W) = D(V||W|P) \triangleq \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} P(x)V(y|x) \log \frac{V(y|x)}{W(y|x)}.$$

The following identities are often useful

$$\begin{aligned}
D(P \circ V \| Q \circ W) &= D(P \| Q) + D(V \| W | P), \\
H_{P,V}(X, Y) &= H_P(X) + H_{P,V}(Y | X) \\
&= H_{P,V}(Y) + H_{P,V}(X | Y), \\
I_{P,V}(Y \wedge X) &= H_{P,V}(Y) - H_{P,V}(Y | X) \\
&= H_P(X) + H_{P,V}(Y) - H_{P,V}(X, Y), \\
I_{Q,P,V}(Y \wedge X | U) &= H_{Q,P,V}(Y | U) - H_{Q,P,V}(Y | X, U), \\
I_{Q,P,V}(X \wedge Y, U) &= I_{Q,P,V}(X \wedge Y) + I_{Q,P,V}(X \wedge U | Y) \\
&= I_{Q,P,V}(X \wedge U) + I_{Q,P,V}(X \wedge Y | U).
\end{aligned}$$

1.4 Basics of the Method of Types

Our proofs will be based on the method of types [49, 51], one of the important technical tools in Shannon Theory. It was one of Shannon's key notions, called "typical sequence," that served, developed, and applied in many works, particularly in the books of Wolfowitz [222], Csiszár and Körner [51], Cover and Thomas [48], and Yeung [224]. The idea of the method of types is to partition the set of all N -length sequences into classes according to their empirical distributions (types).

The *type* P of a sequence (or vector) $\mathbf{x} = (x_1, \dots, x_N) \in \mathcal{X}^N$ is a PD $P = \{P(x) = N(x|\mathbf{x})/N, x \in \mathcal{X}\}$, where $N(x|\mathbf{x})$ is the number of repetitions of symbol x among x_1, \dots, x_N . The *joint type* of \mathbf{x} and $\mathbf{y} \in \mathcal{Y}^N$ is the PD $\{N(x, y|\mathbf{x}, \mathbf{y})/N, x \in \mathcal{X}, y \in \mathcal{Y}\}$, where $N(x, y|\mathbf{x}, \mathbf{y})$ is the number of occurrences of symbols pair (x, y) in the pair of vectors (\mathbf{x}, \mathbf{y}) . In other words, joint type is the type of the sequence $(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$ from $(\mathcal{X} \times \mathcal{Y})^N$.

We say that the *conditional type* of \mathbf{y} for given \mathbf{x} is PD $V = \{V(y|x), x \in \mathcal{X}, y \in \mathcal{Y}\}$ if $N(x, y|\mathbf{x}, \mathbf{y}) = N(x|\mathbf{x})V(y|x)$ for all $x \in \mathcal{X}, y \in \mathcal{Y}$. The set of all PD on \mathcal{X} is denoted by $\mathcal{P}(\mathcal{X})$ and the subset of $\mathcal{P}(\mathcal{X})$ consisting of the possible types of sequences $\mathbf{x} \in \mathcal{X}^N$ is denoted by $\mathcal{P}_N(\mathcal{X})$. The set of vectors \mathbf{x} of type P is denoted by $\mathcal{T}_P^N(\mathcal{X})$ ($\mathcal{T}_P^N(\mathcal{X}) = \emptyset$ for PD $P \notin \mathcal{P}_N(\mathcal{X})$). The set of all sequences $\mathbf{y} \in \mathcal{Y}^N$ of conditional type V for given $\mathbf{x} \in \mathcal{T}_P^N(\mathcal{X})$ is denoted by $\mathcal{T}_{P,V}^N(Y|\mathbf{x})$ and

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is called *V-shell* of \mathbf{x} . The set of all possible *V-shells* for \mathbf{x} of type P is denoted by $\mathcal{V}_N(\mathcal{Y}, P)$.

In the following lemmas very useful properties of types are formulated, for proofs see [49, 51, 63].

Lemma 1.1. (Type counting)

$$|\mathcal{P}_N(\mathcal{X})| < (N + 1)^{|\mathcal{X}|}, \quad (1.1)$$

$$|\mathcal{V}_N(\mathcal{Y}, P)| < (N + 1)^{|\mathcal{X}||\mathcal{Y}|}. \quad (1.2)$$

Lemma 1.2. For any type $P \in \mathcal{P}_N(\mathcal{X})$

$$(N + 1)^{-|\mathcal{X}|} \exp\{NH_P(X)\} < |\mathcal{T}_P^N(X)| \leq \exp\{NH_P(X)\}, \quad (1.3)$$

and for any conditional type V and $\mathbf{x} \in \mathcal{T}_P^N(X)$

$$\begin{aligned} (N + 1)^{-|\mathcal{X}||\mathcal{Y}|} \exp\{NH_{P,V}(Y|X)\} \\ < |\mathcal{T}_{P,V}^N(Y|\mathbf{x})| \leq \exp\{NH_{P,V}(Y|X)\}. \end{aligned} \quad (1.4)$$

Lemma 1.3. If $\mathbf{x} \in \mathcal{T}_P^N(X)$, $\mathbf{y} \in \mathcal{T}_{P,V}^N(Y|\mathbf{x})$, then

$$Q^N(\mathbf{x}) = \exp\{-N(H_P(X) + D(P||Q))\}, \quad (1.5)$$

$$W^N(\mathbf{y}|\mathbf{x}) = \exp\{-N(H_{P,V}(Y|X) + D(V||W|P))\}. \quad (1.6)$$

Some authors frequently apply known facts of the theory of large deviations [48] for proofs of information-theoretical results. In tutorial [54] Csiszár and Shields deduce results on large deviations using the method of types. This method helps in a better perception of the subject because the process of inference in all cases is based on the examination of the types of vectors. That is why we prefer the usage of the method of types.

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