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# Optimal Resource Allocation in Coordinated Multi-Cell Systems

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## Abstract

The use of multiple antennas at base stations is a key component in the design of cellular communication systems that can meet high-capacity demands in the downlink. Under ideal conditions, the gain of employing multiple antennas is well-recognized: the data throughput increases linearly with the number of transmit antennas if the spatial dimension is utilized to serve many users in parallel. The practical performance of multi-cell systems is, however, limited by a variety of nonidealities, such as insufficient channel knowledge, high computational complexity, heterogeneous user conditions, limited backhaul capacity, transceiver impairments, and the constrained level of coordination between base stations.

This tutorial presents a general framework for modeling different multi-cell scenarios, including clustered joint transmission, coordinated beamforming, interference channels, cognitive radio, and spectrum sharing between operators. The framework enables joint analysis and insights that are both scenario independent and dependent.

The performance of multi-cell systems depends on the resource allocation; that is, how the time, power, frequency, and spatial resources are divided among users. A comprehensive characterization of resource allocation problem categories is provided, along with the signal processing algorithms that solve them. The inherent difficulties are revealed: (a) the overwhelming spatial degrees-of-freedom created by the multitude of transmit antennas; and (b) the fundamental trade-off between maximizing aggregate system throughput and maintaining user fairness. The tutorial provides a pragmatic foundation for resource allocation where the system utility metric can be selected to achieve practical feasibility. The structure of optimal resource allocation is also derived, in terms of beamforming parameterizations and optimal operating points.

This tutorial provides a solid ground and understanding for optimization of practical multi-cell systems, including the impact of the nonidealities mentioned above. The Matlab code is available online for some of the examples and algorithms in this tutorial.

# Contents

# 1

# Introduction

This section describes a general framework for modeling different types of multi-cell systems and measuring their performance — both in terms of system utility and individual user performance. The framework is based on the concept of dynamic cooperation clusters, which enables unified analysis of everything from interference channels and cognitive radio to cellular networks with global joint transmission. The concept of resource allocation is defined as allocating transmit power among users and spatial directions, while satisfying a set of power constraints that have physical, regulatory, and economic implications. A major complication in resource allocation is the inter-user interference that arises and limits the performance when multiple users are served in parallel. Resource allocation is particularly complex when multiple antennas are employed at each base station. However, the throughput, user satisfaction, and revenue of multi-cell systems can be greatly improved if we understand the nature of multi-cell resource allocation and how to exploit the spatial domain to obtain high spectral efficiencies.

Mathematically, resource allocation corresponds to the selection of a signal correlation matrix for each user. This enables computation of the corresponding signal-to-interference-and-noise ratio (SINR) of each user. For a given resource allocation, this section describes different ways of measuring the performance experienced by each user and the inherent conflict between maximizing the performance of different users. The performance region and channel gain regions are defined to illustrate this conflict. These regions provide a bridge between user performance and system utility. Resource allocation is then naturally formulated as a multi-objective optimization problem and the boundary of the performance region represents all efficient solutions.

This section formulates the general optimization problem, discusses the different solution strategies taken in later sections, and derives some basic properties of the optimal solution and the performance region. A detailed outline of this tutorial is given at the end of this section. Mathematical proofs are provided throughout the tutorial to facilitate a thorough understanding of multi-cell resource allocation.

## 1.1 Introduction to Multi-Antenna Communications

The purpose of communication is to transfer data between devices through a physical medium called the *channel*. This tutorial focuses on wireless communications, where the data is sent as electromagnetic radio waves propagating through the environment between the devices (e.g., air, building, trees, etc.). The wireless channel distorts the emitted signal, adds interference from other radio signals emitted in the same frequency band, and adds thermal background noise. As the radio frequency spectrum is a global resource used for many things (e.g., cellular/computer networks, radio/television broadcasting, satellite services, and military applications) it is very crowded and spectrum licenses are very expensive, at least in frequency bands suitable for long-range applications. Therefore, wireless communication systems should be designed to use their assigned frequency resources as efficiently as possible, for example, in terms of achieving high *spectral efficiency* (bits/s/Hz) for the system as a whole. This becomes particularly important as cellular networks are transitioning from low-rate voice/messaging services to high-rate low-latency data services. The overall efficiency and user satisfaction can be improved by dynamic allocation and management of the available resources, and service providers can even share spectrum to further improve their joint spectral efficiency.

The spectral efficiency of a single link (from one transmitter to one receiver) is fundamentally limited by the available transmit power [236], but the spectral efficiency can potentially be improved by allowing many devices to communicate in parallel and thereby contribute to the total spectral efficiency. This approach will however create inter-user interference that could degrade the performance if not properly controlled. As the power of electromagnetic radio waves attenuates with the propagation distance, the traditional way of handling interference is to only allow simultaneous use of the same resource (e.g., frequency band) by spatially well-separated devices. As the radio waves from a single transmit antenna follow a fixed radiation pattern, this calls for division of the landscape into cells and cell sectors. By applying fixed frequency reuse patterns such that adjacent sectors are not utilizing the same resources, interference can be greatly avoided. This near-orthogonal approach to resource allocation is, however, known to be inefficient compared to letting transmitted signals interfere in a controlled way [227].

In contrast to classical resource allocation with single-antenna transmitters [197, 267, 316], modern multi-antenna techniques enable resource allocation with precise spatial separation of users. By steering the data signals toward intended users, it is possible to increase the received signal power (called an array gain) and at the same time limit the interference caused to other non-intended users. The steering is tightly coupled with the concept of *beamforming* in classic array signal processing; that is, transmitting a signal from multiple antennas using different relative amplitudes and phases such that the components add up constructively in desired directions and destructively in undesired directions. Herein, steering basically means to form beams in the directions of users with line-of-sight propagation and to make multipath components add up coherently in the geographical area around non-line-of-sight users. The beamforming resolution depends on the propagation environment and typically improves with the number of transmit antennas [220]. The ability to steer signals toward intended users ideally enables global utilization of all spectral resources, thus

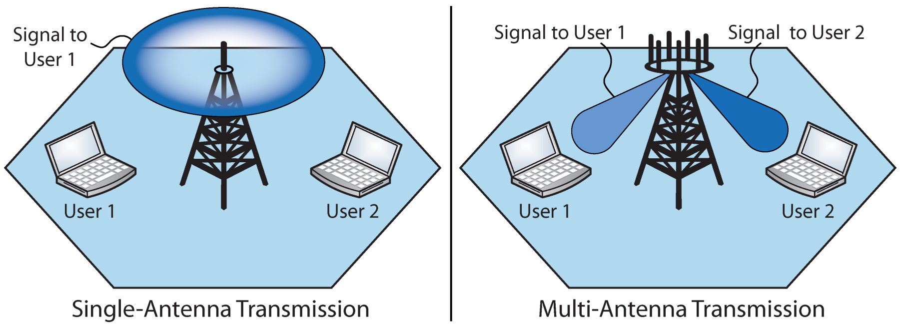


Fig. 1.1 Illustration of the difference between single-antenna and multi-antenna transmission. With a single antenna, the signal propagates according to a fixed antenna pattern (e.g., equally strong in all directions) and can create severe interference in directions where the intended user is not located. For example, interference is caused to User 2 when User 1 is served. With multiple antennas, the signal can be steered toward the intended user which enables simultaneous transmission to multiple spatially separated users with controlled inter-user interference.

removing the need for cell sectoring and fixed frequency reuse patterns; see Figure 1.1. This translates into a much higher spectral efficiency but also more complex implementation constraints — as described later in this section.

The seminal works of [74, 187, 261] provide a mathematical motivation behind multi-antenna communications; the spectral efficiency increases linearly with the number of antennas (if the receiver knows the channel and has at least as many antennas as the transmitter). The initial works considered *point-to-point* communication between two multi-antenna devices — a scenario that is fairly well-understood today [89, 165, 196, 269]. Encouraging results for the *single-cell downlink* where one multi-antenna device transmits to multiple user devices (also known as the broadcast channel) were initially derived in [46, 283]. The information-theoretic capacity region is now fully characterized, even under general conditions [295]. The optimal spectral efficiency is achieved by *nonlinear interference pre-cancelation* techniques, such as dirty paper coding [56]. The single-cell scenario is more challenging than point-to-point since the transmitter needs to know the channel directions of the intended users to perform nonlinear interference precancelation or any sensible linear transmission [84]. Thus, sufficient overhead signaling needs to be allocated for estimation and feedback of channel information [15, 44, 113]. On the other hand, high spectral efficiency can be achieved in single-cell scenarios while having low-cost single-antenna user devices and non-ideal channel conditions (e.g., high antenna correlation, keyhole-like propagation, and line-of-sight propagation) [84] — this is not possible in point-to-point communication.

The multi-cell downlink has attracted much attention, since the system-wide spectral efficiency can be further improved if the frequency reuse patterns are replaced by cooperation between transmitters. Ideally, this could make the whole network act as one large virtual cell that utilizes all available resources [81]. Such a setup actually exploits the existence of inter-cell interference, by allowing joint transmission from multiple cells to each and every user. Unlike the single-cell scenario, the optimal transmit strategy is unknown even for seemingly simple multi-cell scenarios, such as the *interference channel* where each transmitter serves a single unique user while interference is coordinated across all cells [69, 101, 157, 235]. Part of the explanation is that interference pre-cancelation, which is optimal in the single-cell case, cannot be applied between transmitters in the interference channel. Among the schemes that are suboptimal in the capacity-sense, *linear* transmission is practically appealing due to its low complexity, asymptotic optimality (in certain cases), and robustness to channel uncertainty. The best linear transmission scheme is generally difficult to obtain [157, 168], even in those single-cell scenarios where the capacity region is fully characterized. Recent works have however derived strong parameterizations [16, 180, 235, 325] and these will be described in Section 3.

This tutorial provides theoretical and conceptual insights on the optimization of general multi-cell systems with linear transmission. To this end, the tutorial first introduces a mathematical system model for the single-cell downlink. This model serves as the foundation when moving to the multi-cell downlink, which has many conceptual similarities but also important differences that should be properly addressed.

## 1.2 System Model: Single-Cell Downlink

Consider a single-cell scenario where a base station with $N$ antennas communicates with $K_r$ user devices, as illustrated in Figure 1.2. The

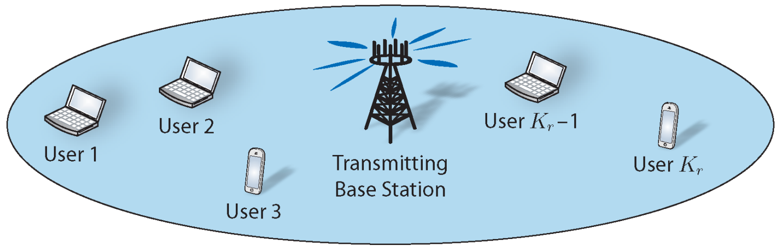


Fig. 1.2 Illustration of the downlink multi-user system in Section 1.2. A base station with $N$ antennas serves $K_r$ users.

$k$th user is denoted $\text{MS}_k$ (the abbreviation stands for mobile station) and is assumed to have a single effective antenna[1]; the case with multiple antennas per user is considered in Section 4.6. This scenario can be viewed as the superposition of several multiple-input single-output (MISO) links, thus it is also known as the *MISO broadcast channel* or *multi-user MISO communication* [46]. It is also frequently described as multi-user MIMO (multiple-input multiple-output) (cf. [84]), referring to that there are $K_r$ receive antennas in total, but we avoid this terminology as it creates confusion.

The channel to $\text{MS}_k$ is assumed to be flat-fading[2] and represented in the complex baseband by the dimensionless vector $\mathbf{h}_k \in \mathbb{C}^N$. The complex-valued element $[\mathbf{h}_k]_n$ describes the channel from the $n$th transmit antenna; its magnitude represents the gain (or rather the attenuation) of the channel, while its argument describes the phase-shift created by the channel. We assume that the channel vector is quasi-static; that is, constant for the duration of many transmission symbols, known as the *coherence time*. The collection of all channel vectors $\{\mathbf{h}_k\}_{k=1}^{K_r}$ is known as the *channel state information (CSI)* and is assumed perfectly known at the base station. We also assume that the transceiver hardware is ideal, without other impairments than can

[1] This means that $\text{MS}_k$ is equipped with either a single antenna or $M_k > 1$ antennas that are combined into a single effective antenna (e.g., using receive combining or antenna selection). There are several reasons for making this assumptions: it enables noniterative transmission design, put less hardware constraints on the user devices, requires less channel knowledge at the transmitter, and is close-to-optimal under realistic conditions [15, 28, 268].

[2] Flat-fading means that the frequency response is flat, which translates into a memoryless channel where the current output signal only depends on the current input signal.

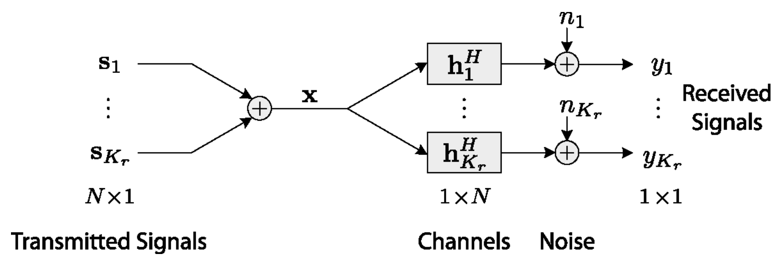


Fig. 1.3 Block diagram of the basic system model for downlink single-cell communications. $K_r$ single-antenna users are served by $N$ antennas.

be included in the channel vector and background noise. These assumptions are idealistic, but simplify the conceptual presentation in this and subsequent sections. It is generally impossible to find perfect models of reality, or as famously noted in [34]: "Remember that all system models are wrong." Therefore, the goal is to formulate a model that enables analysis and at the same time is accurate enough to provide valuable insights. Relaxations to more realistic conditions and assumptions are provided in Section 4.

Under these assumptions, the symbol-sampled complex-baseband received signal at $\text{MS}_k$ is $y_k \in \mathbb{C}$ and is given by the linear input–output model

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \tag{1.1}$$

where $n_k \in \mathbb{C}$ is the combined vector of additive noise and interference from surrounding systems. It is modeled as circularly symmetric complex Gaussian distributed, $n_k \sim \mathcal{CN}(0, \sigma^2)$, where $\sigma^2$ is the noise power. This input–output model is illustrated in Figure 1.3. In a multi-carrier system, for example, based on orthogonal frequency-division multiplexing (OFDM), the input–output model (1.1) could describe one of the subcarriers. For brevity, we concentrate on a single subcarrier in Sections 1–3, while the multi-carrier case is discussed in Section 4.5.

The transmitted signal $\mathbf{x} \in \mathbb{C}^N$ contains data signals intended for each of the users and is given by

$$\mathbf{x} = \sum_{k=1}^{K_r} \mathbf{s}_k, \tag{1.2}$$

where $\mathbf{s}_k \in \mathbb{C}^N$ is the signal intended for MS$_k$. These stochastic data signals are modeled as zero-mean with *signal correlation matrices*

$$\mathbf{S}_k = \mathbb{E}\{\mathbf{s}_k \mathbf{s}_k^H\} \in \mathbb{C}^{N \times N}. \tag{1.3}$$

This transmission approach is known as linear *multi-stream beamforming* (rank($\mathbf{S}_k$) is the number of streams) and the signal correlation matrices are important design parameters which will be used to optimize the performance/utility of the system.

---

**Definition 1.1.** Each selection of the signal correlation matrices $\mathbf{S}_1, \ldots, \mathbf{S}_{K_r}$ is called a *transmit strategy*. The average transmit power allocated to MS$_k$ is $\mathrm{tr}(\mathbf{S}_k)$.

---

The only transmit strategies of interest are those that satisfy the power constraints of the system, which are defined next.

### 1.2.1 Power Constraints

The power resources available for transmission need to be limited somehow to model the inherent restrictions of practical systems. The average transmit power $\mathrm{tr}(\mathbf{S}_k)$ and noise power $\sigma^2$ are normally measured in milliwatt [mW], with dBm as the corresponding unit in decibels. We assume that there are $L$ linear power constraints, which are defined as

$$\sum_{k=1}^{K_r} \mathrm{tr}(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_l \quad l = 1, \ldots, L, \tag{1.4}$$

where $\mathbf{Q}_{lk} \in \mathbb{C}^{N \times N}$ are Hermitian positive semi-definite weighting matrices and the limits $q_l \geq 0$ for all $l,k$. If $\mathbf{Q}_{lk}$ is normalized and dimensionless, then $q_l$ is measured in mW and serves as an upper bound on the allowed transmit power in the subspace spanned by $\mathbf{Q}_{lk}$. To ensure that the power is constrained in all spatial directions, these matrices satisfy $\sum_{l=1}^{L} \mathbf{Q}_{lk} \succ \mathbf{0}_N$ for every $k$. These constraints are given in advance and are based on, for example,

- physical limitations
  (e.g., to protect the dynamic range of power amplifiers);

- regulatory constraints
  (e.g., to limit the radiated power in certain directions);
- interference constraints
  (e.g., to control interference caused to certain users);
- economic decisions
  (e.g., to manage the long-term cost and revenue of running a base station).

Two simple examples are a total power constraint (i.e., $L = 1$ and $\mathbf{Q}_{1k} = \mathbf{I}_N$ for all $k$) and per-antenna constraints (i.e., $L = N$ and $\mathbf{Q}_{lk}$ is only nonzero at the $l$th diagonal element). While these examples can be viewed as two extremes, practical systems are typically limited in both respects.

The matrices $\mathbf{Q}_{lk}$ might be the same for all users, but can also be used to define subspaces where the transmit power should be kept below a certain threshold when transmitting to a specific user (or subset of users). The motivation is, for example, not to disturb neighboring systems and the corresponding constraints are called soft-shaping [107, 230], because the shape of the transmission is only affected if the power without the constraint would have exceeded the threshold $q_l$. For example, if the inter-user interference caused to $\text{MS}_k$ should not exceed $q_l$, then we can set $\mathbf{Q}_{li} = \mathbf{h}_k \mathbf{h}_k^H$ for all $i \neq k$ and $\mathbf{Q}_{lk} = \mathbf{0}_N$. This is relevant both to model so-called zero-forcing transmission (i.e., with zero inter-user interference) and in the area of cognitive radio, where a secondary system is allowed to use licensed spectrum if the interference caused to the system of the licensee is limited.

The $L$ linear sum power constraints introduced in (1.4) can be also decomposed into per-user power constraints as

$$\text{tr}(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_{lk} \quad k = 1,\ldots,K_r,\ l = 1,\ldots,L, \tag{1.5}$$

for some limits $q_{lk} \geq 0$ for all $l,k$. In order to fulfill (1.4), the per-user power limits need to satisfy the conditions

$$\sum_{k=1}^{K_r} q_{lk} \leq q_l \quad l = 1,\ldots,L. \tag{1.6}$$

This equivalent representation of the $L$ linear sum power constraints is useful to derive structural results on the optimal transmit strategies.

Selecting the limits $q_{lk}$ is part of the performance optimization and basically corresponds to the per-user power allocation.

### 1.2.2 Resource Allocation

The signal correlation matrices are important parameters that shape the transmission and ultimately decide what is received at the different users. Having defined the input–output model in (1.1) and the power constraints in (1.4), we are ready to give a first brief definition of the resource allocation problem considered in this tutorial.

**Definition 1.2.** Selecting a transmit strategy $\mathbf{S}_1,\ldots,\mathbf{S}_{K_r}$ in compliance with the power constraints is called *resource allocation.*

The selection should be based on some criterion on user satisfaction, which will be properly defined later in Section 1.4. Observe that resource allocation implicitly includes selecting which users to transmit to, the spatial directivity of the signals to selected users, and the power allocation. In principle, $\mathrm{tr}(\mathbf{S}_k)$ describes the power allocated for transmission to $\mathrm{MS}_k$, while the eigenvectors and eigenvalues of $\mathbf{S}_k$ describe the spatial distribution of this power. The rank of $\mathbf{S}_k$ equals the number of simultaneous data streams that are multiplexed to $\mathrm{MS}_k$. The general case when multiple users are served simultaneously is called *spatial division multiple access* (SDMA) [217], while the special case when only one user is allocated nonzero power at a time is known as *time division multiple access* (TDMA). The $N$ transmit antennas can be viewed as having $N$ spatial degrees-of-freedom in the resource allocation, which can be utilized for sending a total of $N$ simultaneous data streams in a controlled manner. The spectral efficiency is not always maximized by sending the maximum number of streams, since this might create much inter-user interference and can be very sensitive to CSI uncertainty — TDMA is the better choice in the absence of CSI [84].

SDMA is the main focus of this tutorial and we assume that there is an infinite queue of data to be sent to each user; thus, all users are available for transmission and are not upper-limited on how high performance they can achieve. The data is delivered to the base station through a *backhaul network*, which also will be used for base station coordination when we extend the single-cell model into a multi-cell model in Section 1.3.

---

**Remark 1.1 (Basic Channel Modeling).** The analysis in this tutorial is applicable under any channel conditions, noise power, and power constraints. Some intuition on typical system conditions (used in numerical simulations) might however aid the understanding.

The channel vector is often modeled as complex Gaussian, $\mathbf{h}_k \sim \mathcal{CN}(\bar{\mathbf{h}}_k, \mathbf{R}_k)$, where the mean value $\bar{\mathbf{h}}_k \in \mathbb{C}^N$ describes the line-of-sight propagation (if it exists) and the covariance matrix $\mathbf{R}_k \in \mathbb{C}^{N \times N}$ characterizes the varying nature of the channel. This model is called *Rician fading* or *Rayleigh fading* (if $\bar{\mathbf{h}}_k = \mathbf{0}$), since the magnitude of each channel element is Rice or Rayleigh distributed, respectively. Although simple, this model makes sense in rich multipath scenarios (e.g., based on the Lindeberg Central limit theorem [309]) and has been validated by measurements [54, 132, 288, 294, 306]. The spatial directivity is specified by the off-diagonal elements in $\mathbf{R}_k$ and the exponential correlation model in [162] provides a simple parametrization. The channel attenuation depends strongly on the distance between the transmitter and the receiver; this is modeled as $-128.1 - 37.6 \log_{10}(d)$ dB in 3GPP Long Term Evolution (LTE) [1], where $d$ is the separation in kilometers. Accordingly, $\frac{\mathrm{tr}(\mathbf{R}_k)}{N}$ lies in the range of $-70$ dB to $-140$ dB in cellular systems. Further reduction are introduced by signal penetration losses, while antenna gains improve the conditions.

The noise power $\sigma^2$ can be modeled as $-174 + 10 \log_{10}(b) + n_f$ dBm, where $b$ is the bandwidth in Hertz and $n_f$ is the noise figure caused by hardware components. For example, the noise power is $-127$ dBm for a 15 kHz subcarrier with a 5 dB noise figure. Furthermore, the transmit power (per flat-fading subcarrier) is typically in the range of 0–20 dBm. As the received signal power and the noise power are both very small quantities, normalization is often beneficial in numerical computations.

---

## 1.3 Extending Single-Cell Downlink to Multi-Cell Downlink

In traditional multi-cell systems, each user belongs to one cell at a time and resource allocation is performed unilaterally by its base station. This is enabled by having frequency reuse patterns such that cell sectors utilizing the same resources cause negligible interference to each other. The single-cell system model, defined in the previous section, can therefore be applied directly onto each cell sector — at least if the negligible interference from distant cell sectors is seen as part of the additive background noise. Accordingly, the base station can make autonomous resource allocation decisions and be sure that no uncoordinated interference appears within the cell.

A different story emerges in multi-cell multi-antenna scenarios where all base stations are simultaneously using the same frequency resources (to maximize the system-wide spectral efficiency). The counterpart of SDMA in multi-cell systems have been given many names, including *co-processing* [233], *cooperative processing* [321], *network MIMO* [279], *coordinated multi-point (CoMP)* [202], and *multi-cell processing* [81]. It is based on the same idea of exploiting the spatial dimensions for serving multiple users in parallel while controlling the interference. Network MIMO is particularly important for users that experience channel gains on the same order of magnitude from multiple base stations (e.g., cell edge users). The initial works in [125, 233, 321] assumed perfect co-processing at the base stations and modeled the whole network as one large multi-user MISO system where the transmit antennas happen to be distributed over a large area; all users were served by *joint transmission* from all base stations and the multi-cell characteristics were essentially reduced to just constraining the transmit power per antenna array or antenna, instead of the total transmit power (as traditionally assumed for single-cell systems). The optimal spectral efficiency under these ideal conditions can be obtained from the single-cell literature, in particular [295]. Although mathematically convenient, this approach leads to several implicit assumptions that are hard to justify in practice. First, global CSI and data sharing is required, which puts huge demands on the channel estimation, feedback links, and backhaul networks [122, 174, 175, 200, 247, 312, 313].

Second, coherent joint transmission (including joint interference cancelation) requires very accurate synchronization[3] between base stations [18, 262, 318] and increases the delay spread [322], potentially turning flat-fading channels into frequency-selective. Third, the complexity of centralized resource allocation algorithms is infeasible in terms of computations, delays, and scalability [21]. On the other hand, the early works on the multi-cell downlink provide (unattainable) upper bounds on the practical multi-cell performance.

Various alternative models have been proposed to capture multi-cell-specific characteristics. The CSI requirements were reduced in [191, 114, 246] by using the so-called Wyner model from [299] where interference only comes from immediate neighboring cells; see Example 1.1 for details. This enables relatively simple analysis, but the results can also be oversimplified [300]. Another approach is to divide base stations into *static disjoint cooperation clusters* as in Figure 1.4 [106, 174, 323]. Each cluster is basically operated as a single-cell system.

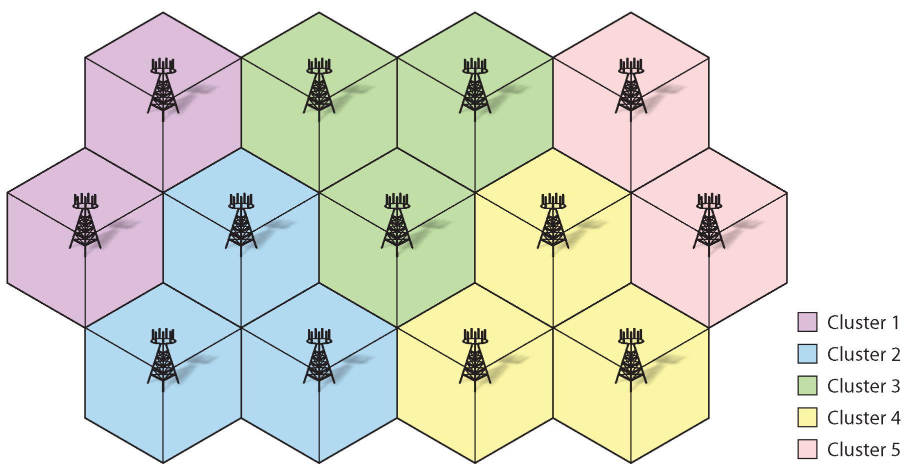


Fig. 1.4 Schematic illustration of static disjoint cooperation clusters.

[3] Synchronization is very important to enable signal contributions from different base stations to cancel out at nonintended users. Precise phase-synchronization can potentially be achieved and maintained by sending a common reference signal to the base stations from a master oscillator [8, 177], using reference clocks that are phase-locked to the GPS [124], or by estimating and feeding back the offset at the users [318].

If the clusters are sufficiently small (e.g., cell sectors connected to the same eNodeB in an LTE system), this approach enables practical channel acquisition, coordination, and synchronization within each cluster. Networks with static clusters unfortunately provide poor spectral efficiency when the user distribution is heterogeneous [173] and suffer from out-of-cluster interference [77]. The impact of these drawbacks can be reduced by having different static disjoint cooperation clusters on different frequency subcarriers [176], by increasing the cluster size and serve each user by a subset of its base stations [33], by having frequency reuse patterns in the cluster edge areas [146], and by changing the disjoint clusters over time [173, 199]. These approaches can however be viewed as treating the symptoms rather than the actual problem, namely the formation of clusters based on a base station-perspective. Steps toward more dynamic and flexible multi-cell coordination were taken in [18, 77, 109, 128, 129, 263] by creating clusters from a user-centric perspective. This means that the set of base stations that serve or reduce interference to a given user is based on the particular needs of this user. Consequently, each base station has its own unique set of users to coordinate interference toward and serves a subset of these users with data. Each base station coordinates its resource allocation decisions with exactly those base stations that affect the same users. This is very different from the disjointness mentioned above, because each base station basically cooperates with all of its neighbors and forms different cooperation clusters when serving different users. The geographical location of a user has a large impact on the clustering [109], but the desirable cooperation and coordination also change with time, for example, based on user activity levels, mobility of users, and macroscopic conditions such as congestion in certain areas. This tutorial considers *dynamic cooperation clusters* of this user-centric type and the framework includes the scenarios described above as special cases.

A seemingly different multi-cell setup arises in the area of *cognitive radio* [90, 102, 230]. Frequency spectrum is traditionally licensed to companies or agencies, which are given exclusive rights for utilization. Therefore, the licensee can unilaterally manage the transmissions and guarantee the service quality for its users. However, a major part of the licensed spectrum is under-utilized today, thus providing the opportunity for improvements in spectral efficiency [55]. The cognitive radio paradigm is based on having secondary systems that are allowed to use the spectrum if they are not disrupting the primary system (which owns the license). Three ways for the secondary system to achieve this are: interweave (detect and transmit when primary system is inactive), underlay (steer signals away from primary users to avoid interference), and overlay (compensate for the interference caused to primary users by participating in joint transmission of their intended signals). These cognitive radio scenarios can be modeled using the framework of this tutorial (see Section 4.8), and can naturally be extended for spectrum sharing between operators on equal terms.

### 1.3.1 Dynamic Cooperation Clusters

Next, we extend the downlink single-cell system model in Section 1.2 to a multi-cell scenario with $K_t$ base stations. The $j$th base station is denoted $\mathrm{BS}_j$ and is equipped with $N_j$ antennas. The antenna array can have any structure and we assume that $N_j$ is a fixed parameter.[4] Observe that the total number of transmit antennas is still denoted $N = \sum_{j=1}^{K_t} N_j$. Based on the discussion in the previous section and on [18], our general multi-cell system model will embrace the following observations:

- Each user is jointly served by a subset of all base stations;
- Some base stations and users are very far apart, making it impractical to estimate and separate the interference on these channels from the background noise.

Based on these observations, we make the following definition.

[4] The hardware design of antenna arrays has important implications on channel properties such as spatial correlation, mutual antenna coupling, and aperture — all of which are affecting the spatial resolution of beamforming. Release 9 of the LTE standard supports $N_j = 8$ antennas [1], but current research investigates the potential of having much larger arrays (up to several hundred of antennas). We refer to [220] for a recent survey on the challenges and opportunities of having unconventionally large numbers of antennas.

**Definition 1.3.** *Dynamic cooperation clusters (DCC)* means that:

- $\text{BS}_j$ has channel estimates to users in $\mathcal{C}_j \subseteq \{1, \dots, K_r\}$, while interference generated to users $i \notin \mathcal{C}_j$ is negligible and can be treated as part of the Gaussian background noise;
- $\text{BS}_j$ serves the users in $\mathcal{D}_j \subseteq \mathcal{C}_j$ with data.

This coordination framework is characterized by the sets $\mathcal{C}_j, \mathcal{D}_j \; \forall j$, which are illustrated in Figure 1.5. In this figure, the inner set $\mathcal{D}_j$ contains the users that $\text{BS}_j$ might serve with data. The larger outer set $\mathcal{C}_j$ contains all users that $\text{BS}_j$ should take into consideration and coordinate interference toward. The mnemonic rule is that $\mathcal{D}_j$ describes *data* from $\text{BS}_j$, while $\mathcal{C}_j$ describes *coordination* from $\text{BS}_j$. The membership of users in these sets changes dynamically during operation (e.g., based on individual user locations and the user density in different areas) and it should be noted that each base station may cooperate with different subsets of base stations for each of its users; in other words, the users can generally *not* be divided into disjoint groups served by disjoint groups of base stations.

How to select $\mathcal{C}_j, \mathcal{D}_j$ efficiently is a very important and complex problem [45]. On the one hand, joint transmission and interference coordination provide extra degrees-of-freedom to separate users spatially. This benefit comes, on the other hand, at the cost of spending

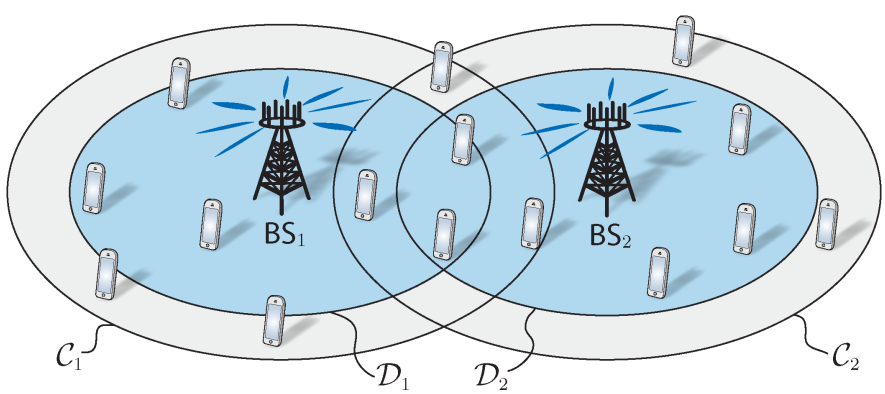


Fig. 1.5 Schematic intersection of two cells. $\text{BS}_j$ serves users in the inner circle ($\mathcal{D}_j$), while coordinating interference to users in the outer circle ($\mathcal{C}_j$). The interference caused to users outside both circles is negligible and included in the respective noise terms.

backhaul and overhead signaling on obtaining CSI, sharing data, and achieving base station synchronization. Increased expenditure is only well motivated if it leads to substantial improvements in spectral efficiency; joint transmission is more costly (it requires data sharing and tight synchronization) than interference coordination, thus we can generally expect $\mathcal{D}_j$ to be a much smaller set than $\mathcal{C}_j$. The clustering problem is discussed in Section 4.7, but for now we assume that the sets $\mathcal{C}_j, \mathcal{D}_j \, \forall j$ are given and known everywhere needed.

The reason for basing the tutorial on DCC is twofold. First, it enables joint analysis of different levels of multi-cell coordination (from the Wyner model or cognitive radio to global joint transmission). Second, it can resolve some of the issues that appear when the multi-cell downlink is viewed as a single-user system with a large distributed transmit antenna array and distributed power constraints. According to Definition 1.3, $\mathrm{BS}_j$ only needs to know its own channel to users that receive non-negligible interference from it — a natural assumption since these are the users for which $\mathrm{BS}_j$ can achieve reliable channel estimates.[5] In addition, only neighboring base stations need to be phase synchronized[6] and joint transmission only creates a small increase in delay-spread (which is easy to handle in OFDM systems by increasing the cyclic prefix [322]).

### 1.3.2 Extended System Model: Multi-Cell Downlink

In the multi-cell scenario, the channel from all base stations to $\mathrm{MS}_k$ is denoted $\mathbf{h}_k = [\mathbf{h}_{1k}^T \dots \mathbf{h}_{K_t k}^T]^T \in \mathbb{C}^N$, where $\mathbf{h}_{jk} \in \mathbb{C}^{N_j}$ is the channel from $\mathrm{BS}_j$. Based on the DCC in Definition 1.3, only certain channel elements of $\mathbf{h}_k$ will carry data and/or non-negligible interference. These can be selected by the diagonal matrices $\mathbf{D}_k \in \mathbb{C}^{N \times N}$ and $\mathbf{C}_k \in \mathbb{C}^{N \times N}$,

[5] There are two main system categories: Frequency division duplex (FDD) and Time division duplex (TDD). The main difference is that each frequency subcarrier in FDD is used for *either* downlink or uplink transmission, while each subcarrier in TDD switches between downlink and uplink transmission. TDD seems particularly useful for multi-cell coordination, because multiple base stations can exploit the same uplink pilot signal to estimate their respective channels (if channel reciprocity can be utilized [96]). The CSI acquisition is more demanding in FDD, since more resources are required for CSI feedback to the additional base stations (and possibly some extra backhaul signaling).

[6] Note that local phase synchronization does not imply global phase synchronization, because small deviations between neighboring base stations are acceptable but can grow into large deviation between distant base stations.

which are defined as

$$\mathbf{D}_k = \begin{bmatrix} \mathbf{D}_{1k} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{D}_{K_t k} \end{bmatrix} \quad \text{where } \mathbf{D}_{jk} = \begin{cases} \mathbf{I}_{N_j}, & \text{if } k \in \mathcal{D}_j, \\ \mathbf{0}_{N_j}, & \text{otherwise,} \end{cases} \tag{1.7}$$

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{C}_{1k} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{C}_{K_t k} \end{bmatrix} \quad \text{where } \mathbf{C}_{jk} = \begin{cases} \mathbf{I}_{N_j}, & \text{if } k \in \mathcal{C}_j, \\ \mathbf{0}_{N_j}, & \text{otherwise.} \end{cases} \tag{1.8}$$

Thus, $\mathbf{h}_k^H \mathbf{D}_k$ is the channel that carries data to $\text{MS}_k$ and $\mathbf{h}_k^H \mathbf{C}_k$ is the channel that carries non-negligible interference.[7] It is necessary to have both $\mathbf{D}_k$ and $\mathbf{C}_k$, to make sure that only the correct base stations transmit to $\text{MS}_k$ when optimizing the resource allocation.

Extending the single-cell input–output model in (1.1), the symbol-sampled complex-baseband received signal at $\text{MS}_k$ is

$$y_k = \mathbf{h}_k^H \mathbf{C}_k \sum_{i=1}^{K_r} \mathbf{D}_i \mathbf{s}_i + n_k \tag{1.9}$$

and is illustrated in Figure 1.6.[8] The additive term $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ is now assumed to model both noise and weak uncoordinated interference from all $\text{BS}_j$ with $k \notin \mathcal{C}_j$ (see Definition 1.3). This assumption limits the amount of CSI required to analyze the transmission and is reasonable if only users that would receive signals that are stronger than the background noise are included in $\mathcal{C}_j$. This might be satisfied if base stations coordinate interference to all cell edge users of adjacent cells (similar to the Wyner model [299]). The variance $\sigma_k^2$ is generally different among the users (representing how weak the uncoordinated interference is at

[7] The antennas that transmit to a certain user can, for simplicity, be thought of as being a single transmitter, although the antennas might belong to different base stations. The reality is however more complex, for example, due to base station-specific power constraints, separate channel acquisition, and distributed resource allocation; see Section 4.

[8] This tutorial considers transmission using linear beamforming over a single subcarrier and channel use. Higher spectral efficiency can potentially be achieved using nonlinear interference pre-subtraction at the base stations (e.g., dirty paper coding [56, 46, 283, 295]) or by extending the transmission over, for instance, a collection of channel realizations (e.g., interference alignment [41]). The truly optimal transmission scheme is unknown for general multi-cell systems, thus the linear beamforming considered in this tutorial should be viewed as a practically appealing transmission approach rather than the overall optimal strategy.

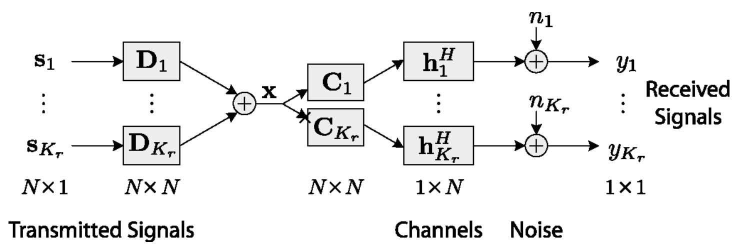


Fig. 1.6 Block diagram of the general system model for downlink multi-cell communications. $K_r$ single-antenna users are served by $N$ antennas.

a certain user) and is estimated and tracked using the received signals.[9] It is worth pointing out that $\sigma_k^2$ is implicitly coupled with the power constraints; if the system-wide power usage is increased, then the uncoordinated interference will also increase. This relationship has no particular impact on this tutorial since our power constraints are fixed, but is of paramount importance in any asymptotic analysis because multi-cell systems are fundamentally interference-limited in the high-SNR regime [164]. When nothing else is said, $\text{BS}_j$ is assumed to know the channels $\mathbf{h}_{jk}$ and variances $\sigma_k^2$ perfectly to all users $k \in \mathcal{C}_j$. The case with CSI uncertainty is considered in Section 4.

Just as in the single-cell scenario, the transmission is limited by the $L$ power constraints in (1.4). An important difference is that the actual transmitted signals are $\mathbf{D}_k\mathbf{s}_k$ (and not $\mathbf{s}_k$), thus each weighting matrix $\mathbf{Q}_{lk}$ should satisfy the additional condition that $\mathbf{Q}_{lk} - \mathbf{D}_k^H\mathbf{Q}_{lk}\mathbf{D}_k$ is diagonal for all $l,k$ (e.g., being zero). This technical assumption makes sure that power cannot be allocated to unallowed subspaces for the purpose of reducing the (measured) power in the subspaces used for transmission — which is only possible when $\mathbf{Q}_{lk}$ is nondiagonal.

It is frequently assumed in multi-cell scenarios (but not necessary) that each power constraint only affects the signals from one of the base stations; for example, per-transmitter power constraints is represented by having $L = K_t$ and the constraint affecting $\text{BS}_l$ is

$$\mathbf{Q}_{lk}^{\text{per-BS}} = \mathbf{D}_k^H \text{diag}\big(\mathbf{0}_{N_1+\cdots+N_{l-1}}, \mathbf{1}_{N_l}, \mathbf{0}_{N_{l+1}+\cdots+N_{K_t}}\big)\mathbf{D}_k \quad \forall l. \qquad (1.10)$$

[9] It is implicitly assumed that $n_k$ is an ergodic process, which is not necessarily satisfied if unknown communication systems with fast adaptive resource allocation strategies are creating the interference; a further discussion is available in [302].

The analysis in this tutorial is applicable to any feasible set of power constraints, when nothing else is stated.

### 1.3.3 Examples of Multi-Cell Scenarios

We conclude this section by illustrating that the proposed DCC can describe a variety of multi-cell scenarios. Different examples are given on the following pages.

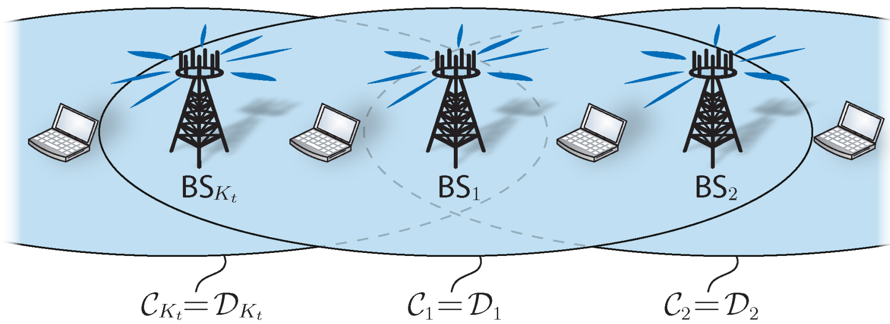


Fig. 1.7 Illustration of the multi-cell scenario called the *one-dimensional/linear Wyner model*. Users are jointly served by the closest base station and its two neighbors (in a cyclic manner), and only experience interference from these three base stations.

---

**Example 1.1 (Wyner model).** Based on an idea by A. Wyner [299], it can be assumed that users only receive signals from their own base station and the immediate neighboring base stations. This abstraction is supposed to capture the locality of interference. The one-dimensional (or linear) version of this model, where all devices are located on the boundary of a large circle, is illustrated in Figure 1.7. It is usually assumed that all users in the $j$th cell are jointly served by $\mathrm{BS}_{j-1}$, $\mathrm{BS}_j$, and $\mathrm{BS}_{j+1}$. This model was originally proposed for uplink transmission, but was used in [114, 191, 246] to analyze the ideal performance of joint downlink transmission.

Assume that there are $K_t$ base stations and $K_r$ users. If $\mathrm{MS}_k$ is geographically closest to $\mathrm{BS}_j$, then we have $\mathbf{D}_k = \mathbf{C}_k = \mathrm{diag}(\mathbf{0}_{N_1+\cdots+N_{j-2}}, \mathbf{I}_{N_{j-1}+N_j+N_{j+1}}, \mathbf{0}_{N_{j+2}+\cdots+N_{K_t}})$ since $\mathrm{MS}_k$ is served by $\mathrm{BS}_{j-1}$, $\mathrm{BS}_j$, and $\mathrm{BS}_{j+1}$ and only experiences interference from these base stations.

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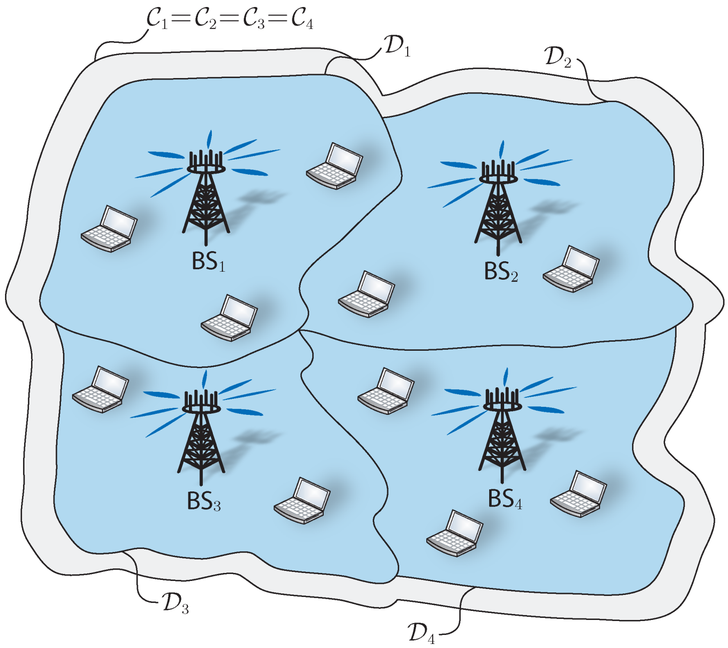


Fig. 1.8 Illustration of the multi-cell scenario of *coordinated beamforming*. Users are served by their own base station while interference is coordinated by joint resource allocation between all base stations.

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**Example 1.2 (Coordinated Beamforming).** Coordinated beamforming means that each base station has a disjoint set of users to serve with data, but selects transmit strategies jointly with all other base stations to reduce inter-cell interference [59, 82, 139, 211]; see Figure 1.8. There is an arbitrary number of users in each cell. The special case with only one user per cell is called the *interference channel* [69, 101, 157, 235].

Assume that there are $K_t = 2$ base stations and $K_r$ users. Then, $\mathbf{D}_k = \mathrm{diag}(\mathbf{I}_{N_1}, \mathbf{0}_{N_2})$ for all $\mathrm{MS}_k$ served by $\mathrm{BS}_1$, while $\mathbf{D}_k = \mathrm{diag}(\mathbf{0}_{N_1}, \mathbf{I}_{N_2})$ for all $\mathrm{MS}_k$ served by $\mathrm{BS}_2$. In addition, $\mathbf{C}_1 = \mathbf{C}_2 = \mathbf{I}_N$ due to the global interference coordination.

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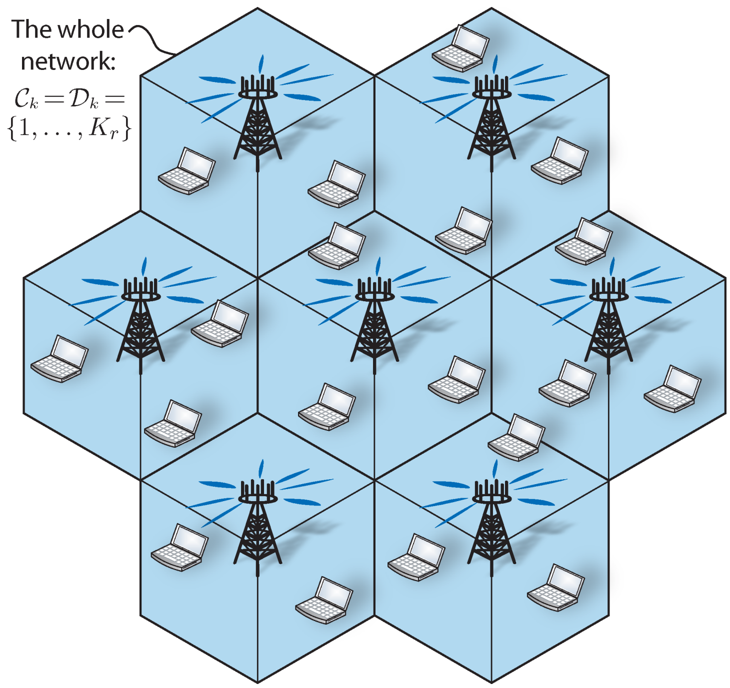


Fig. 1.9 Illustration of the *global joint transmission* scenario, where all cells and cell sectors are connected and perform joint transmission to all users in the whole network.

---

**Example 1.3 (Global Joint Transmission).** Ideally, all base stations can serve and coordinate interference to all users [125, 233, 321]. Even if the cellular network was originally built with many cells and cell sectors, this type of ideal/full CoMP turns the system into a single cell with distributed antenna arrays; see Figure 1.9. The main difference from the classic single-cell scenario might be the power constraints, which typically are defined per-antenna or per-transmitter.

This type of global joint transmission and interference coordination is represented by having $\mathbf{D}_k = \mathbf{C}_k = \mathbf{I}_N$ for all users $k$.

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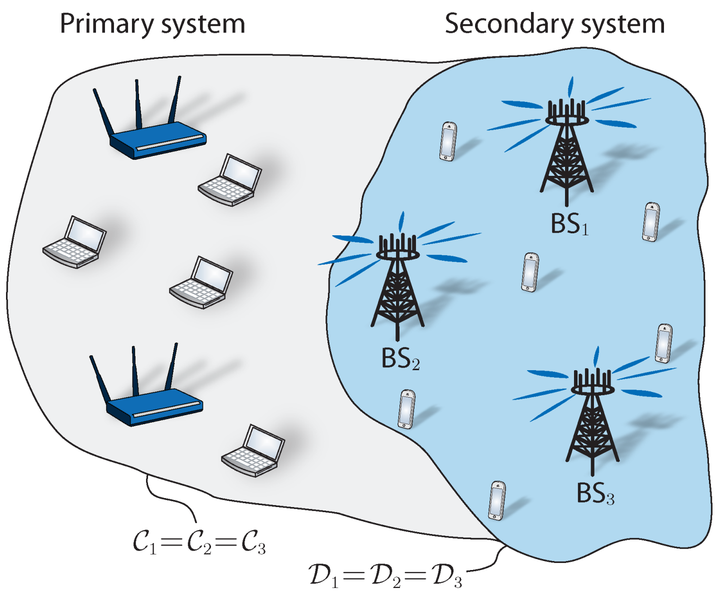


Fig. 1.10 Illustration of the scenario of *underlay cognitive radio*, where the secondary system is allowed to use frequency resources licensed by the primary system if the interference is kept below a threshold.

---

**Example 1.4 (Cognitive Radio).** *Underlay cognitive radio* is a scenario where a secondary system is allowed to use the licensed spectrum of a primary system if it causes mild interference on the primary system [90, 120, 230, 327]; see Figure 1.10. This scenario is particularly relevant when the primary system is not fully utilizing its spectrum.

Assume that users with indices in $\mathcal{K}_{\text{primary}} = \{1,\dots,K_{\text{primary}}\}$ belong to the primary systems, while users in $\mathcal{K}_{\text{secondary}} = \{K_{\text{primary}} + 1,\dots,K_r\}$ belong to the secondary system and are served by joint transmission. We then have $\mathbf{D}_k = \mathbf{0}_N$ for $k \in \mathcal{K}_{\text{primary}}$ and $\mathbf{D}_k = \mathbf{I}_N$ for $k \in \mathcal{K}_{\text{secondary}}$. We also have $\mathbf{C}_k = \mathbf{I}_N$ since interference is coordinated toward all users. Finally, we have $K_{\text{primary}}$ soft-shaping constraints of the form $\mathbf{Q}_{ki} = \mathbf{h}_i\mathbf{h}_i^H \; \forall k \in \mathcal{K}_{\text{secondary}}$ to limit the interference toward each primary user $i \in \mathcal{K}_{\text{primary}}$. The corresponding $q_i$ defines the maximal interference power that can be caused to user $i \in \mathcal{K}_{\text{primary}}$.

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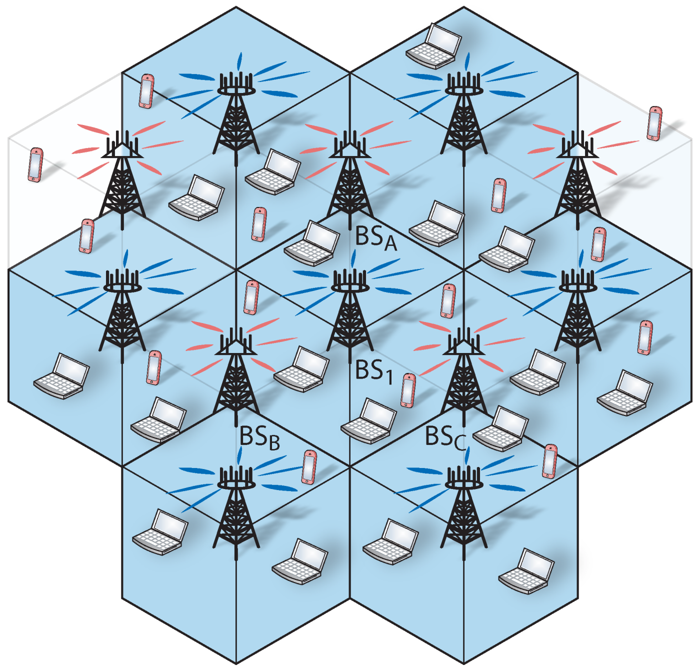



Fig. 1.11 Illustration of the scenario of *spectrum sharing* between two operators covering the same area, creating inter-operator interference.

---

**Example 1.5 (Spectrum Sharing Between two Operators).** *Spectrum sharing between operators* is a scenario where two operators agree to share some portion of their licensed frequency bands; see Figure 1.11 where Operator 1 has circular antenna arrays and serve laptops while Operator 2 has triangular arrays and serve smartphones.

Suppose $\text{MS}_k$ is served by $\text{BS}_1$ of Operator 1 with $\mathbf{D}_k = \text{diag}(\mathbf{I}_{N_1}, \mathbf{0}_{N_2}, \ldots)$. The signal received at $\text{MS}_k$ is a superposition of the signals from $\text{BS}_1$ of Operator 1 and $\text{BS}_\text{A}, \text{BS}_\text{B}, \text{BS}_\text{C}$ of Operator 2, thus $\mathbf{C}_k = \text{diag}(\underbrace{\mathbf{I}_N}_{\text{BS 1}}, \mathbf{0}, \ldots, \mathbf{0}, \underbrace{\mathbf{I}_{N_\text{A}}, \mathbf{I}_{N_\text{B}}, \mathbf{I}_{N_\text{C}}}_{\text{BS}_\text{A}, \text{BS}_\text{B}, \text{BS}_\text{C}}, \mathbf{0}, \ldots)$. This model is easily extended to the case in which inter-cell interference from the same operator is also considered (by modifying the matrix $\mathbf{C}_k$ accordingly). Another extension is to apply full joint transmission within one operator, which could be modeled by $\mathbf{D}_k = \text{diag}(\mathbf{I}_{N_1}, \mathbf{0}_{N_2}, \mathbf{I}_{N_3}, \mathbf{0}_{N_4}, \ldots)$.

---

## 1.4 Multi-Cell Performance Measures and Resource Allocation

In this section, we define a general way of measuring the performance in multi-cell systems. It is instructive to separate the performance into two parts: (1) the performance that each user experiences; and (2) the system utility which is a collection of simultaneously achievable user performances. These two parts are described and analyzed in the following subsections.

### 1.4.1 User Performance

To enable low-complexity and energy-efficient receivers, we assume *single user detection* meaning that a user is not attempting to decode and subtract interfering signals while decoding its own signals. This assumption is limiting in terms of spectral efficiency, except in the low-interference regime [4, 234], but requires less complex signal processing algorithms for reception. In principle, it also places the responsibility for interference control at the transmitter-side, where the computational resources are available. The corresponding SINR for $\text{MS}_k$ is

$$\begin{aligned}\text{SINR}_k(\mathbf{S}_1,\ldots,\mathbf{S}_{K_r}) &= \frac{\mathbf{h}_k^H\mathbf{C}_k\mathbf{D}_k\mathbf{S}_k\mathbf{D}_k^H\mathbf{C}_k^H\mathbf{h}_k}{\sigma_k^2+\mathbf{h}_k^H\mathbf{C}_k(\sum\limits_{i\neq k}\mathbf{D}_i\mathbf{S}_i\mathbf{D}_i^H)\mathbf{C}_k^H\mathbf{h}_k} \\ &= \frac{\mathbf{h}_k^H\mathbf{D}_k\mathbf{S}_k\mathbf{D}_k^H\mathbf{h}_k}{\sigma_k^2+\mathbf{h}_k^H\mathbf{C}_k(\sum\limits_{i\in\mathcal{I}_k}\mathbf{D}_i\mathbf{S}_i\mathbf{D}_i^H)\mathbf{C}_k^H\mathbf{h}_k},\end{aligned} \tag{1.11}$$

where the second equality follows from $\mathbf{C}_k\mathbf{D}_k = \mathbf{D}_k$ and $\mathbf{C}_k\mathbf{D}_i \neq \mathbf{0}$ only for users $i$ in

$$\mathcal{I}_k = \bigcup_{\{j\in\mathcal{J}:\,k\in\mathcal{C}_j\}} \mathcal{D}_j \setminus \{k\}. \tag{1.12}$$

This is the set of co-users being served by the same base stations that coordinate interference toward $\text{MS}_k$. Observe that the SINR is a dimensionless quantity, thus it does not matter if the transmit and noise powers are measured in milliwatt or watt. For brevity, we frequently write $\mathrm{SINR}_k$ instead of $\mathrm{SINR}_k(\mathbf{S}_1,\ldots,\mathbf{S}_{K_r})$ in this tutorial.

The signal-to-noise ratio (SNR) can be defined accordingly by removing the interference term in (1.11). We will however mostly use this term as an indication of the ideal signaling conditions to a given user: $q_j \frac{\|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k\|_2^2}{\sigma_k^2}$, where $q_j$ is the constraint that ultimately limits the transmit power. We show in Section 3.4 that the optimal transmission structure depends strongly on the SNR — roughly speaking, a low SNR is below 0 dB and a high SNR is above 20 dB.

Note that other channel gain based SINR expressions are possible. Consider the case in which $\mathrm{MS}_k$ receives two statistically independent data signals with correlation matrices $\mathbf{S}_k^{(1)}$ and $\mathbf{S}_k^{(2)}$, for example, from two different base stations. Then, the resulting SINR expression useful for information rate computation (after optimal receive processing with successive interference cancelation) is given by

$$\mathrm{SINR}_k^{\text{2-signals}}(\mathbf{S}_1,\ldots,\mathbf{S}_{K_r}) = \frac{\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k (\mathbf{S}_k^{(1)} + \mathbf{S}_k^{(2)}) \mathbf{D}_k^H \mathbf{C}_k^H \mathbf{h}_k}{\sigma_k^2 + \mathbf{h}_k^H \mathbf{C}_k (\sum_{i \neq k} \mathbf{D}_i \mathbf{S}_i \mathbf{D}_i^H) \mathbf{C}_k^H \mathbf{h}_k}. \quad (1.13)$$

This expression is equivalent to (1.11) if all data signals are independent.[10] However, if $\mathbf{S}_k^{(2)}$ represents a multi-cast signal meant for multiple users, then (1.13) cannot be written as (1.11). Multi-cast signals can, for example, be used for overhead signaling to different groups of users [127, 245]. This type of multi-cast scenario is further described in Section 4.

Each user $k$ has its own quality measure represented by the user performance function $g_k : \mathbb{R}_+ \to \mathbb{R}_+$ of the SINR. This function describes the satisfaction of the user and generally depends on the service currently used (e.g., its throughput and delay constraints[11]) and on the priority given by the subscription profile.

[10] This is can be seen by defining $\mathbf{S}_k = \mathbf{S}_k^{(1)} + \mathbf{S}_k^{(2)}$.

[11] Voice traffic is an *inelastic* service as the user requires short delays and that a minimum information rate is constantly available (while higher rates unnecessary). On the contrary, Internet traffic is *elastic* as it can accept long delays and variations in the information rate, while the satisfaction is strictly increasing with the information rate.

**Definition 1.4 (User Performance Function).** The performance of $\text{MS}_k$ is measured by an arbitrary continuous, differentiable, and *strictly monotonically increasing*[12] function $g_k(\text{SINR}_k)$ of the SINR. This function satisfies $g_k(0) = 0$, for notational convenience.

With this definition, it is preferable for $\text{MS}_k$ to have a large positive value on $g_k(\text{SINR}_k)$ because it corresponds to good performance.[13] Ideally, the function $g_k(\cdot)$ should be selected to quantify the performance quality in a way comprehensible to the user and the system provider. It is certainly difficult to summarize and connect the user expectations and final service quality with a physical entity such as the SINR. Nevertheless, Definition 1.4 gives a reasonable structure since improving the signal quality should always increase the performance [196], or at least not degrade it [40].

Most of the analytical results in this tutorial only requires the structural properties in Definition 1.4 and are indifferent to the actual choice of user performance functions, therefore we will only explicitly specify $g_k(\cdot)$ when needed. Furthermore, the functions only need to satisfy the continuity and monotonicity properties in Definition 1.4 in the SINR ranges supported by the power constraints in (1.4). The assumption $g_k(0) = 0$ is nonlimiting and always fulfilled after a simple variable transformation. Here follow some common examples on performance measures that satisfy our definition.

**Example 1.6 (Information Rate).** The *achievable information rate* (or mutual information) is $g_k(\text{SINR}_k) = \log_2(1 + \text{SINR}_k)$ and describes the number of bits that can be conveyed to user $k$ (per channel use) with an arbitrarily low probability of decoding error [57]. The underlying

[12] A function $g_k : \mathbb{R} \to \mathbb{R}$ is *strictly monotonically increasing* if it for any $x, x' \in \mathbb{R}$ such that $x > x'$ also follows that $f(x) > f(x')$.

[13] If we would like to minimize some kind of error $\breve{g}_k(\text{SINR}_k)$ that is strictly monotonically decreasing (e.g., mean square error or bit error rate), this can be reformulated into a maximization of the multiplicative inverse as $g_k(\text{SINR}_k) = \frac{1}{\breve{g}_k(\text{SINR}_k)} - \frac{1}{\breve{g}_k(0)}$ or maximization of the additive inverse as $g_k(\text{SINR}_k) = \breve{g}_k(0) - \breve{g}_k(\text{SINR}_k)$. Observe that both possibilities satisfy the condition of $g_k(0) = 0$ in Definition 1.4.

assumption is an infinite constellation $\mathbf{s}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{S}_k)$, error-control coding over very many channel uses, and ideal decoding [65].

---

**Example 1.7 (Mean Square Error).** The sum *mean square error (MSE)* is $\text{MSE}_k = \mathbb{E}\{\|\hat{\mathbf{s}}_k - \mathbf{s}_k\|_2^2\}$, where $\hat{\mathbf{s}}_k$ is an estimate of $\mathbf{s}_k$ obtained using the optimal Wiener filter [195] and noniterative reception. If $M$ data streams are intended for transmission to user $k$ (i.e., $\text{rank}(\mathbf{S}_k) \leq M$), then $\text{MSE}_k = M - \frac{\text{SINR}_k}{1+\text{SINR}_k}$. This error measure should be minimized, thus it is equivalent to maximizing $g_k(\text{SINR}_k) = \frac{\text{SINR}_k}{1+\text{SINR}_k}$.

---

**Example 1.8 (Bit Error Rate).** The *bit error rate (BER)* for Gray coded transmission of a 16-QAM constellation is

$$\begin{aligned} P_{k,\text{16-QAM}} = \frac{3}{8}\text{erfc}\left(\sqrt{\frac{1}{10}\text{SINR}_k}\right) + \frac{1}{4}\text{erfc}\left(\sqrt{\frac{9}{10}\text{SINR}_k}\right) \\ - \frac{1}{8}\text{erfc}\left(\sqrt{\frac{5}{2}\text{SINR}_k}\right), \end{aligned} \tag{1.14}$$

where $\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt$ is the complementary error function and $\text{rank}(\mathbf{S}_k) \leq 1$ [73, 189]. This error measure should be minimized, thus it is equivalent to maximizing $g_k(\text{SINR}_k) = 0.5 - P_{k,\text{16-QAM}}$.

---

In terms of merits and demerits, the information rate has a simple and marketable interpretation, but builds on idealized coding and signal processing assumptions. The MSE often gives simple expressions, but it can be argued that it is only vaguely connected to the user-experienced service quality. The BER is somewhat self-explanatory, but typically has complicated expressions (as seen from Example 1.8) and ignores channel coding which has a large impact on the effective error rate.

The actual throughput in modern communication systems, such as 3GPP LTE systems, can often be predicted as $\beta_1 \log_2(1 + \text{SINR}_k/\beta_2)$, for some parameters $\beta_1 \in [0.5, 0.75]$ and $\beta_2 \in [1, 2]$ that reflect the practical bandwidth and SNR efficiency, respectively [183]. This modified information rate expression is not perfect but is generally a good choice, because the parameters $\beta_1, \beta_2$ can be fitted to the output of a system-level simulator. However, there are certain practical situations in which adaptive coding and modulation is not possible (e.g., systems with very low-complexity devices) and BER/MSE measures are more appropriate.

The analysis and optimization procedure in this tutorial is applicable to any $g_k(\cdot)$ satisfying Definition 1.4; the particular choice will not affect the approach to achieve optimal resource allocation, but will certainly affect what is considered optimal.

Each transmitted data signal will in general affect all users and the impact is characterized by the channel gain region.

---

**Definition 1.5 (Channel Gain Region).** Consider the signal with correlation matrix $\mathbf{S}_k$. The received signal power at user $i$ is given by $x_{ki}(\mathbf{S}_k) = \mathbf{h}_i^H \mathbf{C}_i \mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H \mathbf{C}_i^H \mathbf{h}_i$. The *channel gain region* of this signal is defined as

$$\Omega_k = \left\{ (x_{k1}(\mathbf{S}_k), \ldots, x_{kK_r}(\mathbf{S}_k)) : \mathbf{S}_k \succeq \mathbf{0}_N, \mathrm{tr}(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_{lk} \quad \forall l \right\}. \tag{1.15}$$

---

The set $\Omega_k$ depends only on the signal correlation matrix $\mathbf{S}_k$ and on the per-user power constraints in (1.5). It describes the impact of the choice of $\mathbf{S}_k$ on the received channel gain at all users.

Note that the definition of the channel gain region in Definition 1.5 is different from the definition in [180] because of the feasible transmit strategies. Therefore, the next result which shows that $\Omega_k$ is compact and convex extends [180, Lemma 1].

---

**Definition 1.6.** A set $\mathcal{S} \subseteq \mathbb{R}^{K_r}$ is *compact* if it is closed and bounded. $\mathcal{S}$ is *convex* if $t\mathbf{r}_1 + (1-t)\mathbf{r}_2 \in \mathcal{S}$ whenever $\mathbf{r}_1, \mathbf{r}_2 \in \mathcal{S}$ and $t \in [0,1]$.

---

**Lemma 1.1.** The channel gain region $\Omega_k$ is compact and convex.

---

*Proof.* Define the vector with achievable channel gains as $\mathbf{x}_k(\mathbf{S}_k) = [x_{k1}(\mathbf{S}_k) \dots x_{kK_r}(\mathbf{S}_k)]^T$. The set of feasible signal correlation matrices is $\mathbb{S}_k = \left\{\mathbf{S}_k : \mathbf{S}_k \succeq \mathbf{0}_N, \operatorname{tr}(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_{lk} \quad \forall l\right\}$ and is compact and closed. Since $\Omega_k$ is achieved by a continuous mapping from the closed set $\mathbb{S}_k$, we can invoke [219, Theorem 4.14] to conclude that also $\Omega_k$ is a closed set.

It remains to show that $\Omega_k$ is convex: For any two points $\mathbf{x}_k(\mathbf{S}^{(1)}) \in \Omega_k$ and $\mathbf{x}_k(\mathbf{S}^{(2)}) \in \Omega_k$, we have to show that $\mathbf{x}_k(\mathbf{S}_z(t)) \in \Omega_k$ for $\mathbf{S}_z(t) = t\mathbf{S}^{(1)} + (1-t)\mathbf{S}^{(2)}$ and $t \in [0,1]$. It holds as

$$\begin{aligned} x_{ki}(\mathbf{S}_z(t)) &= \mathbf{h}_i^H \mathbf{C}_i \mathbf{D}_k \mathbf{S}_z(t) \mathbf{D}_k^H \mathbf{C}_i^H \mathbf{h}_i \\ &= \mathbf{h}_i^H \mathbf{C}_i \mathbf{D}_k \left(t\mathbf{S}^{(1)} + (1-t)\mathbf{S}^{(2)}\right) \mathbf{D}_k^H \mathbf{C}_i^H \mathbf{h}_i \\ &= t x_{ki}(\mathbf{S}^{(1)}) + (1-t) x_{ki}(\mathbf{S}^{(2)}). \end{aligned} \tag{1.16}$$

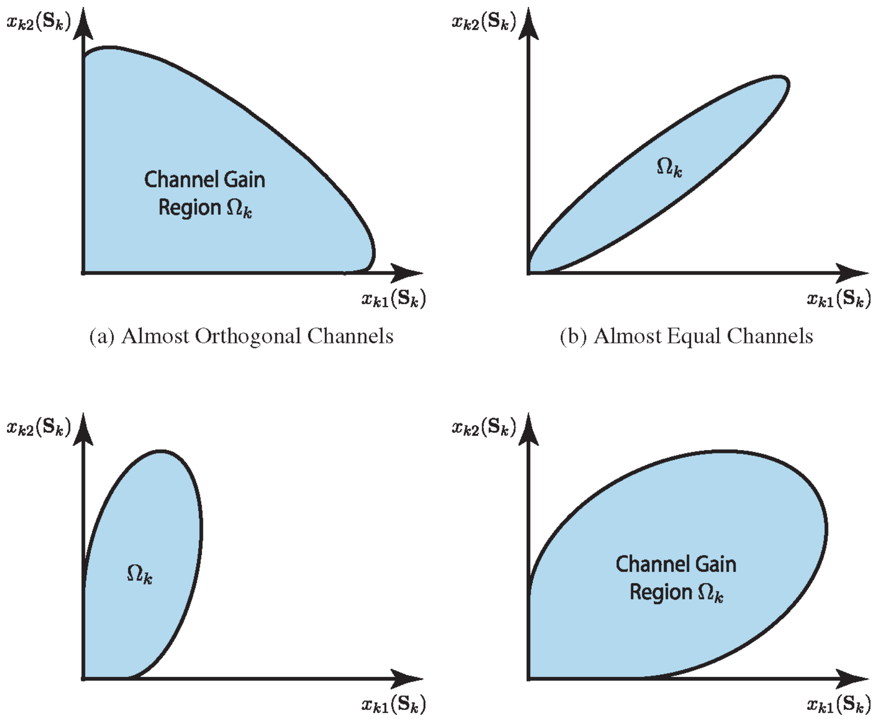
Furthermore, $\operatorname{tr}(\mathbf{Q}_{lk}\mathbf{S}_z(t)) \leq q_{lk}$ is satisfied because $\operatorname{tr}(\mathbf{Q}_{lk}\mathbf{S}_z(t)) = t\operatorname{tr}(\mathbf{Q}_{lk}\mathbf{S}^{(1)}) + (1-t)\operatorname{tr}(\mathbf{Q}_{lk}\mathbf{S}^{(2)}) \leq t q_{lk} + (1-t) q_{lk} = q_{lk}$. □

This lemma establishes the basic structure of the channel gain regions. The exact shape depends on the power constraints and the correlation between the channel vectors $\mathbf{C}_i^H \mathbf{h}_i$ of the users, as illustrated in Figure 1.12. If we consider a total power constraint, $\Omega_k$ resembles a triangle when the user channels are almost orthogonal (see Figure 1.12(a)), while it looks a line from the origin if the channels are almost parallel (see Figure 1.12(b)). Furthermore, the relative path losses $\|\mathbf{C}_i^H \mathbf{h}_i\|^2$ determine if the region looks thin or fat (see Figure 1.12(c)-(d)).

The relationship between individual user performance and channel gain regions is observed from the following SINR expression for $\mathrm{MS}_k$,

$$\mathrm{SINR}_k(x_{1k}(\mathbf{S}_1), \dots, x_{K_r k}(\mathbf{S}_{K_r})) = \frac{x_{kk}(\mathbf{S}_k)}{\sigma_k^2 + \sum\limits_{i \in \mathcal{I}_k} x_{ik}(\mathbf{S}_i)}. \tag{1.17}$$

From (1.17) the monotonicity of the SINR with respect to the different channel gains is easily observed. The SINR of $\mathrm{MS}_k$ is strictly monotonic increasing in $x_{kk}(\mathbf{S}_k)$ and strictly monotonic decreasing in $x_{ik}(\mathbf{S}_i)$ for all interfering links $i \in \mathcal{I}_k$. The conflict between the SINR expressions of different links becomes visible: increasing the own channel gain $x_{kk}$ might increase the channel gain $x_{ki}$ at some other user $i$ and thereby lower its SINR.




(a) Almost Orthogonal Channels

(b) Almost Equal Channels

(c) Unequal Path Losses

(d) Equal Path Losses

Fig. 1.12 Examples of channel gain regions with different shapes, but all being compact and convex. (a) and (b) illustrate the extremes of almost orthogonal and parallel channel vectors, respectively. (c) and (d) illustrate unequal and equal path losses $\|\mathbf{C}_i^H \mathbf{h}_i\|^2$, respectively.

The user performance function introduced in Definition 1.4 can also be expressed as a function of the channel gains,

$$g_k(\mathrm{SINR}_k) = g_k(x_{1k}(\mathbf{S}_1), \ldots, x_{K_r k}(\mathbf{S}_{K_r})). \tag{1.18}$$

By the monotonicity of the user performance function it follows that $g_k(\cdot)$ is also strictly monotonic increasing in $x_{kk}(\mathbf{S}_k)$ and strictly monotonic decreasing in $x_{ik}(\mathbf{S}_i)$ for all interfering links $i \in \mathcal{I}_k$.

### 1.4.2 Multi-Objective Resource Allocation

The channel gain regions highlight the inherent conflict and tradeoffs that appear when we want to maximize the performance of multiple users simultaneously. Each user has its own objective $g_k(\mathrm{SINR}_k)$ to be optimized, thus there are $K_r$ different objectives that typically are conflicting.

Optimization problems with multiple objectives appear naturally in many engineering fields to model tradeoffs between, for example, application performance, operational expenses, logistics, and environmental impacts. To analyze and obtain insights on such problems — without imposing any additional structure — it is common to formulate them mathematically as *multi-objective optimization problems* (MOPs). This tutorial will present and utilize some results and methods from the mathematical field of MOPs, but we refer to [38] for an in-depth survey.

Without loss of generality, our resource allocation problem is formulated as

$$\begin{aligned} \underset{\mathbf{S}_1 \succeq \mathbf{0}_N, \ldots, \mathbf{S}_{K_r} \succeq \mathbf{0}_N}{\text{maximize}} \quad & \{g_1(\mathrm{SINR}_1), \ldots, g_{K_r}(\mathrm{SINR}_{K_r})\} \\ \text{subject to} \quad & \sum_{k=1}^{K_r} \mathrm{tr}(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_l \quad \forall l. \end{aligned} \tag{1.19}$$

This MOP can be interpreted as searching for a transmit strategy $\mathbf{S}_1, \ldots, \mathbf{S}_{K_r}$ that satisfies the power constraints and maximizes the performance $g_k(\mathrm{SINR}_k)$ of all users [38]. Since the performance of different users are coupled by both power constraints and inter-user interference, there is generally not a *single* transmit strategy that simultaneously maximizes the performance of all users. For example, $\mathrm{SINR}_k$ in (1.11) improves if less interference is caused to $\mathrm{MS}_k$, but decreasing the interference at $\mathrm{MS}_k$ typically requires decreasing the useful signal power at other users and thereby degrading their SINRs. To study the conflicting objectives of a MOP it is instructive to consider the set of all feasible *operating points* $\mathbf{g} = [g_1 \ldots g_{K_r}]^T$ in (1.19) [38], which we call the performance region.[14]

---

**Definition 1.7.** The achievable *performance region* $\mathcal{R} \subseteq \mathbb{R}_+^{K_r}$ is

$$\mathcal{R} = \left\{ \left(g_1(\mathrm{SINR}_1), \ldots, g_{K_r}(\mathrm{SINR}_k)\right) : (\mathbf{S}_1, \ldots, \mathbf{S}_{K_r}) \in \mathbb{S} \right\} \tag{1.20}$$

where $\mathbb{S}$ is the set of feasible transmit strategies:

$$\mathbb{S} = \left\{ (\mathbf{S}_1, \ldots, \mathbf{S}_{K_r}) : \mathbf{S}_k \succeq \mathbf{0}_N, \quad \sum_{k=1}^{K_r} \mathrm{tr}(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_l \quad \forall l \right\}. \tag{1.21}$$

---

[14] The performance region can also be called the utility region or something that reflects the choice of user performance function (e.g., capacity region, rate region, or MSE region).

This region describes the performance that can be guaranteed to be simultaneously achievable by the users.[15] The $K_r$-dimensional performance region is nonempty as $\{\mathbf{0}_{K_r \times 1}\} \in \mathcal{R}$ and its shape depends strongly on the channel vectors, power constraints, and dynamic cooperation clusters. In general, $\mathcal{R}$ is not easily characterized and might be a nonconvex set, but we can prove that $\mathcal{R}$ is compact and normal [274].

**Definition 1.8.** A set $\mathcal{T}$ is called *normal on* $\mathcal{S} \subseteq \mathbb{R}^{K_r}$ if for any point $\mathbf{r} \in \mathcal{T}$, all $\mathbf{r}' \in \mathcal{S}$ with $\mathbf{r}' \leq \mathbf{r}$ also satisfy $\mathbf{r}' \in \mathcal{T}$ (componentwise inequality).

Normal sets are also known as comprehensive sets [39, 193].

**Lemma 1.2.** The achievable performance region $\mathcal{R}$ is compact and normal on $\mathbb{R}_+^{K_r}$.

*Proof.* To prove that $\mathcal{R}$ is a compact set, observe that the set of feasible transmit strategies $\mathbb{S}$ in (1.21) is compact. Next, observe that $g_k(\mathrm{SINR}_k)$ are continuous functions of $\mathbf{S}_1,\ldots,\mathbf{S}_{K_r}$ by definition. The compactness of $\mathcal{R}$ follows by invoking [219, Theorem 4.14], which says that the continuous mapping of a compact set is also a compact set. Since $\mathcal{R}$ is the image of a continuous mapping from $\mathbb{S}$, it is compact.

Proving that $\mathcal{R}$ is normal on $\mathbb{R}_+^{K_r}$ is a bit involved, although this property is quite intuitive. We outline the proof from [14, Lemma 5.1]. For any given $\mathbf{r} = (r_1,\ldots,r_{K_r}) \in \mathcal{R}$, we need to show that any $\mathbf{r}' = (r_1',\ldots,r_{K_r}') \in \mathbb{R}_+^{K_r}$ with $\mathbf{r}' \leq \mathbf{r}$ also belongs to $\mathcal{R}$. To this end, let $\mathbf{S}_1^*,\ldots,\mathbf{S}_{K_r}^*$ be a feasible transmit strategy that attains $\mathbf{r}$ and consider the alternative transmit strategy $p_1\mathbf{S}_1^*,\ldots,p_{K_r}\mathbf{S}_{K_r}^*$, where $p_1,\ldots,p_{K_r}$ is a set of power allocation coefficients that should belong to

$$\mathcal{A} = \left\{ (p_1,\ldots,p_{K_r})\colon \sum_{k=1}^{K_r} p_k \mathrm{tr}(\mathbf{Q}_{lk}\mathbf{S}_k^*) \leq q_l \quad \forall l \right\} \tag{1.22}$$

[15] Nonconvex performance regions can be increased by allowing for time-sharing between multiple operating points. This approach gives a region that equals the convex hull of $\mathcal{R}$, but the corresponding resource allocation problems are very complicated and not considered in this tutorial. The general framework for time-sharing in [39] can however be combined with the results in this tutorial. We also note that time-sharing can be viewed as part of the scheduling; see Section 4.7.

to make the strategy feasible. Obviously, the point $\mathbf{r}$ is achieved by selecting $(p_1^*,\dots,p_{K_r}^*) = (1,\dots,1)$. To prove that a given $\mathbf{r}' \leq \mathbf{r}$ also belongs to $\mathcal{R}$, we need to find $(p_1,\dots,p_{K_r}) \in \mathcal{A}$ that gives this point. This corresponds to the conditions $\mathrm{SINR}_k = g_k^{-1}(r'_k)\ \forall k$, which can be formulated as $K_r$ linear equations and solved using the approach in [205]. Finally, the existence of a $(p_1,\dots,p_{K_r}) \in \mathcal{A}$ for any $\mathbf{r}' \leq \mathbf{r}$ can be proved using interference functions, see [227, Theorem 3.5]. □

This means that for any point $\mathbf{g} \in \mathcal{R}$, all points that give weaker performance than $\mathbf{g}$ are also in $\mathcal{R}$. This property is very natural and rational. In fact, if a region is not normal it looks very unnormal; see the illustrations in Figure 1.13 where only (b)–(f) are possible shapes for a performance region, while (a) is not a simply-connected set (i.e., contains holes) and has a strange boundary. Figure 1.13 also illustrates how the interference coupling and power constraints affect the region: (b) represents the degenerate case when the user have orthogonal channels and individual power constraints, while (c)–(f) describe a gradually increasing coupling between the users. Roughly speaking, $\mathcal{R}$ is convex when the users are weakly coupled and concave under strong coupling, while practical performance regions are hybrids of these extremes.

Apart from being compact, the performance region can also be upper bounded by a certain box.

**Definition 1.9.** A *box* is denoted $[\mathbf{a},\mathbf{b}]$, for some $\mathbf{a},\mathbf{b} \in \mathbb{R}^{K_r}$, and is the set of all $\mathbf{g} \in \mathbb{R}^{K_r}$ such that $\mathbf{a} \leq \mathbf{g} \leq \mathbf{b}$ (componentwise inequality).

**Lemma 1.3.** The performance region $\mathcal{R}$ satisfies $\mathcal{R} \subseteq [\mathbf{0},\mathbf{u}]$, where $\mathbf{u} = [u_1 \dots u_{K_r}]^T$ is called the *utopia point*. The element $u_k$ is the optimum of the single-user optimization problem

$$\begin{aligned} \underset{\mathbf{S}_k \succeq \mathbf{0}_N}{\text{maximize}} \quad & g_k\left(\frac{\mathbf{h}_k^H \mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H \mathbf{h}_k}{\sigma_k^2}\right) \\ \text{subject to} \quad & \mathrm{tr}(\mathbf{Q}_{lk}\mathbf{S}_k) \leq q_l \quad \forall l. \end{aligned} \tag{1.23}$$

*Proof.* The single-user problem in (1.23) is achieved from the MOP in (1.19) by setting $\mathbf{S}_i = \mathbf{0}_N$ for all $i \neq k$. As inter-user interference

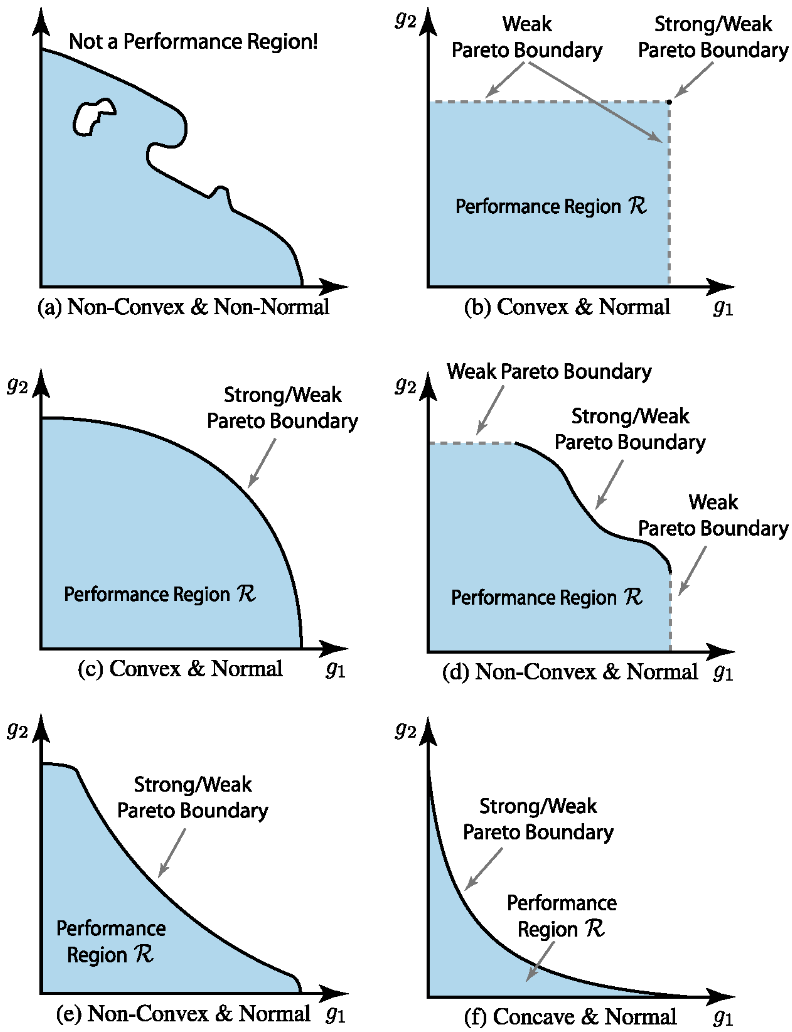



Fig. 1.13 Examples of compact regions with different shapes. Only (b)–(f) are normal and can thus be performance regions. The outer boundaries of (c), (e), (f) satisfy the conditions for both weak and strong Pareto optimality, while the horizontal and vertical parts of the outermost boundaries in (b) and (d) only satisfy weak Pareto optimality.

only can reduce $\text{SINR}_k$, (1.23) provides an achievable upper bound on the performance of $\text{MS}_k$ and it follows that $\mathcal{R} \subseteq [\mathbf{0}, \mathbf{u}]$. $\square$

The utopia point $\mathbf{u}$ is the unique solution to (1.19) in degenerate scenarios (when the optimization decouples and all users can achieve

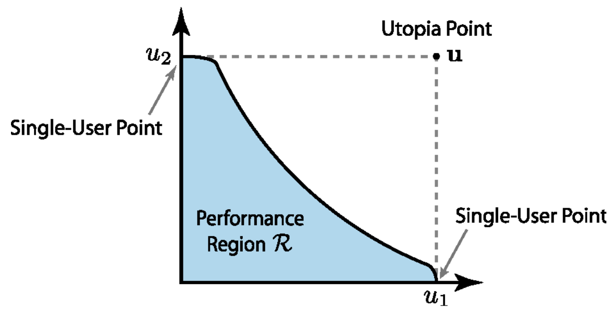



Fig. 1.14 Example of a performance region. The utopia point is shown, along with the single-user points achieved by solving (1.23).

maximal performance simultaneously, see Figure 1.13(b)). In general, $\mathbf{u} \notin \mathcal{R}$ and represents an unattainable upper bound on performance; see Figure 1.14. Since there is no total order of vectors in $\mathbb{R}_+^{K_r}$, we can only achieve a set of tentative vector solutions to (1.19) which are mutually unordered. These tentative solutions are all operating points in $\mathcal{R}$ that are not dominated by any other feasible point. These points are called *Pareto optimal* and are such that the performance cannot be improved for any user without deteriorating for at least one other user.

---

**Definition 1.10.** A point $\mathbf{y} \in \mathbb{R}_+^n$ is a *strong* Pareto optimal point of a compact normal set $\mathcal{T} \subseteq \mathbb{R}_+^n$, if $\mathbf{y} \in \mathcal{T}$ while $\{\mathbf{y}' \in \mathbb{R}_+^n : \mathbf{y}' \geq \mathbf{y}\} \cap \mathcal{T} \setminus \{\mathbf{y}\} = \emptyset$. The set of all strong Pareto optimal points is called the *strong Pareto boundary of* $\mathcal{T}$ and is denoted $\partial\mathcal{T}$.

In addition, a point $\mathbf{y} \in \mathbb{R}_+^n$ is a *weak* Pareto optimal point of a compact normal set $\mathcal{T} \subseteq \mathbb{R}_+^n$, if $\mathbf{y} \in \mathcal{T}$ while $\{\mathbf{y}' \in \mathbb{R}_+^n : \mathbf{y}' > \mathbf{y}\} \cap \mathcal{T} = \emptyset$. The set of all weak Pareto optimal points is called the *weak Pareto boundary of* $\mathcal{T}$ and is denoted $\partial^+\mathcal{T}$.

---

This definition distinguishes between (a) the strong Pareto boundary $\partial\mathcal{R}$ where the performance cannot be unilaterally improved for *any* user and (b) the weak Pareto boundary $\partial^+\mathcal{R}$ where we might be able to improve performance for some of the users but not simultaneously for *all* users. The strong Pareto boundary can be seen as the proper definition of the tentative solutions to a MOP, but we will see that the weak definition has better structural and analytical properties. The strong Pareto boundary is always a subset of the weak Pareto boundary: $\partial\mathcal{R} \subseteq \partial^+\mathcal{R}$. The difference is visualized in Figure 1.13(b),(d), where the weak Pareto boundary contains the whole outermost boundary (including the vertical and horizontal parts) while the strong Pareto boundary only contains a subset of it. The single-user points $[0\dots 0\, u_k\, 0\dots 0]^T$ are always Pareto optimal, but might only satisfy the conditions for weak Pareto optimality.

Knowing that $\mathcal{R}$ is a normal, compact, and contained in $[\mathbf{0},\mathbf{u}]$ simplifies the search for weak Pareto optimal points, particularly since these properties imply that $\mathcal{R}$ is simply-connected (i.e., contains no holes). We have the following result.

**Lemma 1.4.** The weak Pareto boundary $\partial^+\mathcal{R}$ of the performance region $\mathcal{R}$ is a compact and simply-connected set.

*Proof.* The compactness follows from that $\mathcal{R}$ is bounded and that the limit of any sequence of weak Pareto points must be contained in $\partial^+\mathcal{R}$ (easily shown by contradiction, see [40, Proposition A.3.4]). $\partial^+\mathcal{R}$ is simply-connected if there is a path in the set between any two points $\mathbf{r}_1,\mathbf{r}_2 \in \partial^+\mathcal{R}$. As $\mathcal{R}$ is normal there will always be a path between $\mathbf{r}_1$ and $\mathbf{r}_2$ that goes through the interior of $\mathcal{R}$, and every point on this path can be replaced by a dominating weak Pareto point to construct a Pareto optimal path; thus, $\partial^+\mathcal{R}$ is simply-connected. □

In comparison, the strong Pareto boundary $\partial\mathcal{R}$ need not be simply-connected, but can be a disconnected subset of the weak Pareto boundary. Therefore, it is easier to search for and characterize the weak Pareto boundary. This is mainly an academic limitation, because $\partial\mathcal{R} = \partial^+\mathcal{R}$ in most realistic scenarios. The explanation is that there are no truly orthogonal channels or resources in practice, thus there will always be some interference leakage that prevents unilateral improvements. As all properties of $\partial^+\mathcal{R}$ also hold for $\partial\mathcal{R}$, we sometimes refer to both as simply the *Pareto boundary*. We will later describe different algorithms for solving MOPs and as the Pareto boundary contains all tentative solutions, searching for Pareto optimal points is always an important part of such algorithms.

By the monotonicity of the user performance functions $g_k(\cdot)$ on the channel gains $x_{ki}(\mathbf{S}_k)$, there is a tight connection between the Pareto boundary of $\mathcal{R}$ and certain parts of the channel gain regions $\Omega_k$. Since the channel gain regions are not normal, we need to make a few definitions before specifying this relationship.

---

**Definition 1.11.** A vector $\mathbf{x}$ *dominates* a vector $\mathbf{y}$ *in direction* $\mathbf{e} \in \{-1,+1\}^n$, written as $\mathbf{x} \geq^{\mathbf{e}} \mathbf{y}$, if $x_i e_i \geq y_i e_i$ for all $i = 1,\ldots,n$ and there is at least one strict inequality.

---

Using this terminology, it is possible to describe the part of the boundary of a compact convex set we are interested in.

---

**Definition 1.12.** A point $\mathbf{y} \in \mathbb{R}_+^n$ is called an *upper boundary point* of a compact convex set $\mathcal{C} \subseteq \mathbb{R}_+^n$ *in direction* $\mathbf{e} \in \{-1,+1\}^n$ if $\mathbf{y} \in \mathcal{C}$ while the set $\{\mathbf{y}' \in \mathbb{R}_+^n : \mathbf{y}' \geq^{\mathbf{e}} \mathbf{y}\} \subseteq \mathbb{R}_+^n \setminus \mathcal{C}$. We denote the set of upper boundary points in direction $\mathbf{e}$ as $\partial^{\mathbf{e}}\mathcal{C}$.

---

An illustration of the definition is shown in Figure 1.15. The upper boundaries in the three directions $\mathbf{e}_1 = [+1 \ +1]^T$, $\mathbf{e}_2 = [+1 \ -1]^T$, and $\mathbf{e}_3 = [-1 \ +1]^T$ are shown by the arrows. Note that the direction vector with all components equal to $-1$ is typically not of interest, as the

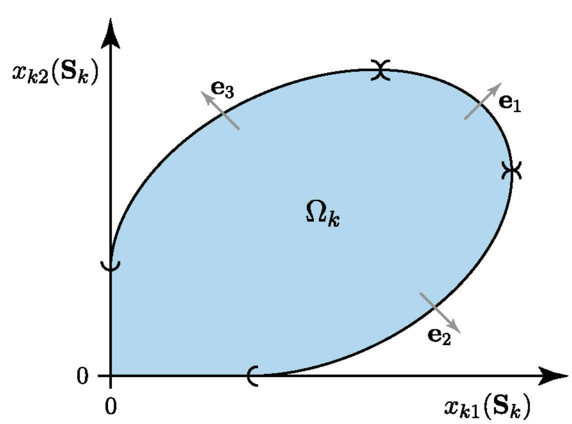



Fig. 1.15 Example of a channel gain region with upper boundary in direction $\mathbf{e}_1 = [+1 \ +1]^T$, $\mathbf{e}_2 = [+1 \ -1]^T$, and $\mathbf{e}_3 = [-1 \ +1]^T$.

corresponding upper boundary is the origin. Also note that the upper boundary in direction $\mathbf{e}_1$ coincides with the usual Pareto boundary.

**Lemma 1.5.** Suppose the strong Pareto boundary of the performance region $\mathcal{R}$ is achieved by a transmit strategy $\mathbf{S}_1,\ldots,\mathbf{S}_{K_r}$. For each $k$, the matrix $\mathbf{S}_k$ also achieves the upper boundary of the channel gain region $\Omega_k$ in the direction $\mathbf{e}_k = [-1 \ldots -1\ +1\ -1 \ldots -1]^T$, where only the $k$th component is positive.

*Proof.* The proof works by contradiction. Assume that $\mathbf{S}_1,\ldots,\mathbf{S}_{K_r}$ achieve the strong Pareto boundary of $\mathcal{R}$ but there is a user $k$ that does not achieve the upper boundary of $\Omega_k$ in direction $\mathbf{e}_k$. Then, it is possible to shift the operating point $\mathbf{x}_k(\mathbf{S}_k)$ in $\Omega_k$ in the direction of the $k$th component without changing the other $K_r - 1$ components; that is, we can find $\mathbf{x}'_k \in \Omega_k$ with increased channel gain $x'_{kk} > x_{kk}$ for the intended user and the same channel gains $x'_{ki} = x_{ki}$ for all other users $i \neq k$. Since this new $\mathbf{x}'_k \in \Omega_k$ there exists a corresponding $\mathbf{S}'_k$ which achieves this point. Using the same set of signal correlation matrices for all other users but replacing $\mathbf{S}_k$ with $\mathbf{S}'_k$ leads to improved performance of user $k$ and unchanged performance for all other users. This is a contradiction to the assumption that $\mathbf{S}_1,\ldots,\mathbf{S}_{K_r}$ achieved the strong Pareto boundary of $\mathcal{R}$. $\square$

The directions in Lemma 1.5 correspond to the monotonicity of the user performance functions on the channel gains. The performance function of user $k$ is monotonically increasing in $x_{kk}$ and monotonically decreasing in all other channel gains, therefore we want to maximize the channel gain $x_{kk}$ and minimize all other channel gains. This corresponds to a direction $\mathbf{e}_k = [-1 \ldots -1\ +1\ -1 \ldots -1]^T$ with $[\mathbf{e}_k]_k = 1$.

## 1.5 Basic Properties of Optimal Resource Allocation

Having defined the user performance functions and the concepts of performance region and channel gain regions, we have sufficient structure to derive two fundamental properties of the optimal multi-objective resource allocation:

- Sufficiency of single-stream beamforming;
- Conditions for full power usage.

These optimality properties are derived in this subsection. Taking these properties into account when solving (1.19) will greatly reduce the search space for optimal solutions. We will utilize the derived properties for simplified resource allocation in the remainder of this tutorial.

### 1.5.1 Sufficiency of Single-Stream Beamforming

The first property is the sufficiency of having signal correlation matrices $\mathbf{S}_k$ that are rank one. This might seem intuitive when each user only has a single (effective) receive antenna and is often assumed in resource allocation without discussion (see e.g., [59, 263, 264, 280, 308, 329]). In general, high-rank solutions might be necessary for optimality — it depends on the type of user performance functions and receive processing that is considered. In this tutorial, we assume single-user detection and $g_k(\cdot)$ of the type in Definition 1.4. We will show that it is sufficient (but not always necessary) to consider signal correlation matrices with rank one under these conditions. As the rank equals the number of data streams, this is called *single-stream beamforming*. First, we give a toy example from [18] showing that high-rank solutions sometimes can give the same performance (but never better) than the rank-one solutions.

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**Example 1.9 (Rank of Optimal Strategy).** Consider a point-to-point system ($K_t = K_r = 1$) with $N = 2$ transmit antennas, the channel vector $\mathbf{h}_1 = [1\ 0]^T$, and per-antenna power constraints

$$\mathrm{tr}\left(\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\mathbf{S}_1\right) \leq 1, \quad \mathrm{tr}\left(\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\mathbf{S}_1\right) \leq 1. \tag{1.24}$$

The MOP in (1.19) reduces to a single-objective resource allocation problem which is solved optimally by both the rank-one matrix $\mathbf{S}_1 = \left[\begin{smallmatrix}1 & 0\\ 0 & 0\end{smallmatrix}\right]$ and by the rank-two matrix $\mathbf{S}_1 = \left[\begin{smallmatrix}1 & 0\\ 0 & 1\end{smallmatrix}\right]$.

---

To prove the sufficiency of rank-one signal correlation matrices, we will make use of some basic results in optimization theory (see Section 2.1 for an introduction to this topic). We start with a lemma.

---

**Lemma 1.6.** Consider the optimization problem

$$\begin{aligned} \underset{\mathbf{V} \succeq 0}{\text{maximize}} \;\; & \mathrm{tr}(\mathbf{A}\mathbf{V}) \\ \text{subject to} \;\; & \mathrm{tr}(\mathbf{B}_m \mathbf{V}) \leq b_m \quad m = 1, \dots, M, \end{aligned} \tag{1.25}$$

with an arbitrary Hermitian matrix $\mathbf{A}$, Hermitian matrices $\mathbf{B}_m \succeq 0$ that satisfy $\sum_{m=1}^{M} \mathbf{B}_m \succ \mathbf{0}$, and scalars $b_m \geq 0 \, \forall m$.

This problem is linear in $\mathbf{V}$ (and hence convex) and always has optimal solutions with $\mathrm{rank}(\mathbf{V}) \leq 1$.

---

*Proof.* This is a linear optimization problem in $\mathbf{V}$ (see Section 2.1). The Lagrangian function is $\mathcal{L}(\mathbf{V}, \boldsymbol{\lambda}) = -\mathrm{tr}(\mathbf{A}\mathbf{V}) + \sum_{m=1}^{M} \lambda_m (\mathrm{tr}(\mathbf{B}_m \mathbf{V}) - b_m)$ and the dual problem is

$$\begin{aligned} \underset{\lambda_m \geq 0}{\text{minimize}} \;\; & \sum_{m=1}^{M} \lambda_m b_m \\ \text{subject to} \;\; & \sum_{m=1}^{M} \lambda_m \mathbf{B}_m - \mathbf{A} \succeq \mathbf{0}. \end{aligned} \tag{1.26}$$

Observe that (1.25) and (1.26) are always feasible because $\mathbf{V} = \mathbf{0}$ satisfies all primal constraints and $\sum_{m=1}^{M} \mathbf{B}_m \succ \mathbf{0}$ implies dual feasibility. Therefore, strong duality holds (see Lemma 2.4) and the KKT conditions are necessary and sufficient for any optimal solution to (1.25):

$$\mathrm{tr}(\mathbf{B}_m \mathbf{V}) \leq b_m \quad \forall m, \tag{1.27}$$

$$\sum_{m=1}^{M} \lambda_m \mathbf{B}_m - \mathbf{A} \succeq \mathbf{0}, \tag{1.28}$$

$$\lambda_m (\mathrm{tr}(\mathbf{B}_m \mathbf{V}) - b_m) = 0 \quad \forall m, \tag{1.29}$$

$$\mathrm{tr}\left( \mathbf{V} \left( \sum_{m=1}^{M} \lambda_m \mathbf{B}_m - \mathbf{A} \right) \right) = 0, \tag{1.30}$$

$$\mathbf{V} \succeq 0, \quad \lambda_m \geq 0 \quad \forall m. \tag{1.31}$$

To prove the sufficiency of rank-one solutions $\mathbf{V} = \mathbf{v}\mathbf{v}^H$, we consider the following alternative optimization problem

$$\begin{aligned} &\underset{\mathbf{v}}{\text{maximize}}\ \ \mathbf{v}^H\mathbf{A}\mathbf{v} \\ &\text{subject to}\ \ \mathbf{v}^H\mathbf{B}_m\mathbf{v} \le b_m \quad \forall m. \end{aligned} \tag{1.32}$$

We want to show that every optimal solution $\mathbf{v}^*$ to (1.32) also satisfies (1.27)–(1.31) for $\mathbf{V} = \mathbf{v}^*(\mathbf{v}^*)^H$ and thus is optimal for (1.25). Although the cost function in (1.32) is generally nonconvex, the constraint functions are convex and thus the KKT conditions are necessary for $\mathbf{v}^*$ (see Lemma 2.2). Now, observe that (1.26) is also the dual problem of (1.32), therefore the feasibility is ensured by the same argument as above. Furthermore, (1.27) and (1.28) are satisfied by $\mathbf{v}^*$ and its corresponding Lagrange multipliers $\mu_m^*$. Next, (1.29) follows from the corresponding complementarity condition $\mu_m^*(\mathbf{v}^H\mathbf{B}_m\mathbf{v} - b_m) = 0$. Finally, (1.30) follows from multiplying the stationarity condition of (1.32), $\big(\sum_{m=1}^M \lambda_m\mathbf{B}_m - \mathbf{A}\big)\mathbf{v} = \mathbf{0}$, with $\mathbf{v}^H$ from the right-hand side. □

Before we show the sufficiency of rank-one signal correlation matrices for the performance region $\mathcal{R}$, we show the corresponding sufficiency for the channel gain regions $\Omega_1,\ldots,\Omega_{K_r}$.

**Lemma 1.7.** All upper boundary points of the channel gain region $\Omega_k$ in some arbitrary direction $\mathbf{e} \in \{-1,+1\}^{K_r}$ can be achieved by signal correlation matrices with $\text{rank}(\mathbf{S}_k) \le 1$.

*Proof.* Since $\Omega_k$ is convex and compact, the boundary can be achieved using the Supporting hyperplane theorem [273, Theorem 1.5] by the following optimization problem

$$\begin{aligned} &\underset{\mathbf{S}_k \succeq 0}{\text{maximize}}\ \ \sum_{i=1}^{K_r} \lambda_i x_{ki}(\mathbf{S}_k) \\ &\text{subject to}\ \ \text{tr}(\mathbf{Q}_{lk}\mathbf{S}_k) \le q_{lk} \quad \forall l. \end{aligned} \tag{1.33}$$

The objective function in (1.33) can be rewritten as

$$\begin{aligned}\sum_{i=1}^{K_r} \lambda_i x_{ki}(\mathbf{S}_k) &= \sum_{i=1}^{K_r} \lambda_i \mathbf{h}_i^H \mathbf{C}_i \mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H \mathbf{C}_i^H \mathbf{h}_i \\ &= \sum_{i=1}^{K_r} \lambda_i \mathrm{tr}(\mathbf{D}_k^H \mathbf{C}_i^H \mathbf{h}_i \mathbf{h}_i^H \mathbf{C}_i \mathbf{D}_k \mathbf{S}_k) \\ &= \mathrm{tr}\Bigg( \underbrace{\sum_{i=1}^{K_r} \lambda_i \mathbf{D}_k^H \mathbf{C}_i^H \mathbf{h}_i \mathbf{h}_i^H \mathbf{C}_i \mathbf{D}_k}_{\mathbf{A}_k} \mathbf{S}_k \Bigg).\end{aligned} \tag{1.34}$$

This is an optimization problem of the form (1.25) and thus the existence of solutions with $\mathrm{rank}(\mathbf{S}_k) \leq 1$ follows from Lemma 1.6. □

Note that $\mathrm{rank}(\mathbf{S}_k) \leq 1$ implies that the signal correlation matrix $\mathbf{S}_k$ is either rank one or identically zero; $\mathbf{S}_k = \mathbf{0}_N$ means no transmission.

By Lemma 1.7, the sufficiency of single-stream beamforming follows immediately for the performance region.

**Theorem 1.8.** Every point in the performance region $\mathcal{R}$ (including the weak Pareto boundary) can be achieved using single-stream beamforming (i.e., $\mathrm{rank}(\mathbf{S}_k) \leq 1 \; \forall k$).

*Proof.* Lemma 1.7 shows that the boundary of each channel gain region $\Omega_k$ is obtained by $\mathbf{S}_k$ with $\mathrm{rank}(\mathbf{S}_k) \leq 1$. Since the strong Pareto boundary of the performance region is achieved by transmit strategies which achieve also the boundary of the channel gain regions (see Lemma 1.5), sufficiency of $\mathrm{rank}(\mathbf{S}_k) \leq 1$ follows. To show that also points on the *weak* Pareto boundary (and all other points in $\mathcal{R}$) are achievable by rank-one solutions, we can simply repeat the approach in the proof of Lemma 1.2 (which showed that $\mathcal{R}$ is normal by fixing the beamforming directions and changing the power allocation). □

The implication of Theorem 1.8 is that any operating point in $\mathcal{R}$ (and particularly Pareto optimal points) can be achieved using single-stream beamforming, thus all tentative solutions to the MOP in (1.19) are achievable by $\mathbf{S}_k = \mathbf{v}_k \mathbf{v}_k^H$ for some *beamforming vectors* $\mathbf{v}_k \in \mathbb{C}^{N \times 1}$ $\forall k$. Without loss of generality, we can reformulate (1.19) as

$$\begin{aligned} \underset{\mathbf{v}_1,\dots,\mathbf{v}_{K_r}}{\text{maximize}} \; & \{g_1(\text{SINR}_1),\dots,g_{K_r}(\text{SINR}_{K_r})\} \\ \text{subject to } & \text{SINR}_k = \frac{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2}{\sigma_k^2 + \sum\limits_{i \neq k} |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2} \quad \forall k, \\ & \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \quad \forall l. \end{aligned} \tag{1.35}$$

Considering (1.35) instead of (1.19) greatly reduces the search space for optimal solutions and makes the solution easier to implement in practice, because vector coding or successive interference cancelation are required if $\text{rank}(\mathbf{S}_k) > 1$ [89]. The problem formulation in (1.35) will be used as the starting point in the remainder of this tutorial.

### 1.5.2 Conditions for Full Power Usage

If only the total transmit power over all base stations is constrained, it is trivial to prove that any Pareto optimal solution to (1.19) and (1.35) will use all available power. Under general power constraints, it may be better not to use full power at each transmitter or antenna; there is a balance between increasing channel gains of useful signals and limiting the interference. This is illustrated by the following toy example, which is based on [18].

---

**Example 1.10 (Limited Power Usage).** Consider a two-user interference channel with single-antenna base stations ($K_t = K_r = 2$, $N_1 = N_2 = 1$) and the channel vectors $\mathbf{h}_1 = [1 \; \sqrt{1/10}]^T$ and $\mathbf{h}_2 = [\sqrt{1/2} \; 1]^T$. $\text{BS}_j$ transmits to $\text{MS}_j$ and coordinates interference to both users, meaning that $\mathbf{D}_1 = [\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}]$, $\mathbf{D}_2 = [\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}]$, and $\mathbf{C}_1 = \mathbf{C}_2 = \mathbf{I}_2$. The per-transmitter power is constrained as $\text{tr}(\mathbf{D}_j \mathbf{S}_j) \leq 20 \; \forall j$.

The single-user point of $\text{MS}_1$ is achieved by $\mathbf{S}_1 = 20\mathbf{D}_1$ and $\mathbf{S}_2 = \mathbf{0}_2$, while the corresponding point for $\text{MS}_2$ is achieved by $\mathbf{S}_1 = \mathbf{0}_2$ and $\mathbf{S}_2 = 20\mathbf{D}_2$. Observe that only the base station associated with the active user is satisfying its power constraint with equality.

Furthermore, the operating point where both users have exactly the same SINR is achieved by $\mathbf{S}_1 = 10\mathbf{D}_1$ and $\mathbf{S}_2 = 20\mathbf{D}_2$. This transmit strategy gives $\text{SINR}_1 = \text{SINR}_2 = \frac{10}{3}$. Observe that only $\text{BS}_2$ uses full power and if $\text{BS}_1$ would increase its power then $\text{SINR}_2$ decreases. This shows that this is a strong Pareto optimal point.

---

In principle, knowing that a certain constraint is active (i.e., satisfied with equality at the optimal solution) removes one dimension from the resource allocation problem. The following theorem provides conditions for when full power should be used in general multi-cell systems.

---

**Theorem 1.9.** The following holds for the multi-objective resource allocation problems (1.19) and (1.35):

- Every weak Pareto optimal point can be achieved by a transmit strategy that satisfies at least one power constraint with equality.
- If only the total power per transmitter is constrained, then every strong Pareto optimal point requires that $\text{BS}_j$ uses full power if $\mathcal{D}_j \neq \emptyset$ and the channels $\mathbf{h}_{jk}$ for all users $k \in \mathcal{C}_j$ are linearly independent.

---

*Proof.* If $q_l = 0$ for some $l$, the first part of the theorem is always satisfied. Now assume that $q_l > 0 \, \forall l$. Let $\mathbf{S}_1^*, \dots, \mathbf{S}_{K_r}^*$ be a transmit strategy that achieves the weak Pareto boundary and assume that all power constraints in (1.4) are inactive. We define

$$\varsigma = \max_{1 \leq l \leq L} \sum_{k=1}^{K_r} \frac{\text{tr}(\mathbf{Q}_{lk}\mathbf{S}_k^*)}{q_l} \tag{1.36}$$

and note that $\varsigma > 1$ since all constraints are inactive. The alternative strategy $\varsigma\mathbf{S}_1^*, \dots, \varsigma\mathbf{S}_{K_r}^*$ will satisfy all constraints and at least one of them will be active. The performance is not decreased since $\varsigma$ can be seen as decreasing the relative noise power in each SINR in (1.11). Thus, there always exists a solution with at least one active constraint.

The second part is proved by contradiction. Suppose $\widetilde{\mathbf{S}}_1,\ldots,\widetilde{\mathbf{S}}_{K_r}$ achieves a strong Pareto optimal point and that $\mathrm{BS}_j$ is not using full power (but satisfies the conditions in the theorem); that is,

$$\sum_{k=1}^{K_r} \mathrm{tr}\left(\mathbf{Q}_{jk}^{\text{per-BS}}\widetilde{\mathbf{S}}_k\right) < q_j \tag{1.37}$$

where $\mathbf{Q}_{jk}^{\text{per-BS}}$ was defined in (1.10). The assumption of linear independence means that it exists $k \in \mathcal{D}_j$ with

$$\mathbf{h}_{jk} \notin \mathrm{span}\left(\bigcup_{i\in\mathcal{C}_j\setminus\{k\}} \{\mathbf{h}_{ji}\}\right). \tag{1.38}$$

Therefore, it exists a unit-norm vector $\mathbf{v} \neq \mathbf{0}_{N_j\times 1}$ such that $\mathbf{h}_{jk}^H\mathbf{v} \neq 0$ and $\mathbf{h}_{ji}^H\mathbf{v} = 0$ for all $i \in \mathcal{C}_j \setminus \{k\}$ (i.e., a zero-forcing vector). Then, the alternative signal correlation matrix $\mathbf{S}_k = \widetilde{\mathbf{S}}_k + \widetilde{\mathbf{v}}\widetilde{\mathbf{v}}^H$ with

$$\widetilde{\mathbf{v}} = \left[\mathbf{0}_{1\times N_1+\cdots+N_{j-1}} \sqrt{q_j - \sum_k \mathrm{tr}(\mathbf{Q}_{jk}^{\text{per-BS}}\widetilde{\mathbf{S}}_k)}\mathbf{v}^T \; \mathbf{0}_{1\times N_{j+1}+\cdots+\times N_{K_t}}\right]^T \tag{1.39}$$

will strictly increase the signal power and cause exactly the same inter-user interference as $\widetilde{\mathbf{S}}_k$. As $g_k(\cdot)$ is strictly increasing we have unilaterally improved the performance of $\mathrm{MS}_k$ which is a contradiction to the strong Pareto optimality. □

The first implication from Theorem 1.9 is that at least one power constraint should be active at any Pareto optimal point. Second, observe that the linear independence of user channels is a very mild condition when $|\mathcal{C}_j| \leq N_j$ (e.g., satisfied with probability one when the channel realizations are drawn from a stochastic distribution with non-singular covariance matrices). Roughly speaking, the fewer users that a base station coordinates interference to, the more power is used at this base station at strong Pareto optimal points. The condition on linear independence can be relaxed to the existence of (at least) one user in $\mathcal{D}_j$ with a channel linearly independent to all other users in $\mathcal{C}_j$ that are actually scheduled (i.e., receive nonzero signal power).

## 1.6 Subjective Solutions to Resource Allocation

Recall that the Pareto boundary of the performance region contains all tentative solutions to the MOP in (1.35), each representing a certain tradeoff between the users' performance. Whenever the utopia point is outside of the performance region, there is no objectively optimal resource allocation — there are multiple strong Pareto optimal points and none of these are distinctly better than the others. To actually compare the merits of different Pareto optimal points, the *system designer* (or decision maker) needs to bring in its own subjective perspective on system utility. Different methods to obtain subjectively optimal solutions are outlined in this section and will be the subject of the subsequent sections of this tutorial.

A common approach is to let the system designer describe its preferences as an aggregate system utility function $f : \mathcal{R} \to \mathbb{R}$ that takes any point in $\mathcal{R}$ as input and produces a scalar value describing how preferable this point is (large output means high preference).

---

**Definition 1.13 (System Utility Function).** A *system utility function* is denoted $f(g_1(\mathrm{SINR}_1), \dots, g_{K_r}(\mathrm{SINR}_{K_r}))$ and is Lipschitz continuous[16] and *monotonically increasing*[17] on $[\mathbf{0}, \mathbf{u}]$.

---

This definition incorporates most system utility functions that appear in literature. In fact, many frequently used functions are *strictly* increasing functions, as seen in the following example [130, 168].

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**Example 1.11 (System Utility Functions).** For a given operating point $\mathbf{g} = (g_1, \dots, g_{K_r}) \in \mathcal{R}$, the following system utility functions

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[16] A function $f : [\mathbf{a}, \mathbf{b}] \to \mathbb{R}$ is *Lipschitz continuous with Lipschitz constant* $L_f$ if $|f(\mathbf{g}) - f(\mathbf{g}')| \leq L_f \|\mathbf{g} - \mathbf{g}'\|_1$ for all $\mathbf{g}, \mathbf{g}' \in [\mathbf{a}, \mathbf{b}]$.

[17] A function $f : \mathbb{R}^n \to \mathbb{R}$ is *monotonically increasing* if for any $\mathbf{g}, \mathbf{g}' \in \mathbb{R}^n$ such that $\mathbf{g} \geq \mathbf{g}'$ it follows that $f(\mathbf{g}) \geq f(\mathbf{g}')$. The function is *strictly* monotonically increasing if for any $\mathbf{g}, \mathbf{g}'' \in \mathbb{R}^n$ such that $\mathbf{g} > \mathbf{g}''$, it also follows that $f(\mathbf{g}) > f(\mathbf{g}'')$.

satisfy[18] Definition 1.13:

- Weighted arithmetic mean: $f(\mathbf{g}) = \sum_k w_k g_k$ (also known as weighted sum utility);
- Weighted geometric mean: $f(\mathbf{g}) = \prod_k g_k^{w_k}$ (also known as weighted proportional fairness [130]);
- Weighted harmonic mean: $f(\mathbf{g}) = \left(\sum_k \frac{w_k}{g_k}\right)^{-1}$;
- Weighted max-min fairness: $f(\mathbf{g}) = \min_k \frac{g_k}{w_k}$ (also known as weighted worst-user performance);
- Weighted compromise: $f(\mathbf{g}) = -(\sum_k (w_k(r_k^* - g_k))^p)^{1/p}$ (for some reference point $\mathbf{r}^* \in \mathbb{R}_+^n \backslash \mathcal{R}$ and $1 \leq p \leq \infty$).

The weighting factors $w_k \geq 0$ can be taken to have unit sum, $\sum_{k=1}^{K_r} w_k = 1$, without loss of generality. In case of equal weighting factors, the arithmetic mean maximizes the aggregate system utility $\sum_k g_k$, while the geometric mean, harmonic mean, and max-min fairness gradually sacrifice aggregate utility to achieve more fairness among the users. For a given type of system utility function, the weighting factors can compensate for heterogeneous user channel conditions, handle delay constraints, enforce subscription profiles, etc.

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There are other system utility functions, for example, the $\alpha$-proportional fairness in [179] that bridges the gap between proportional fairness and max-min fairness by varying a parameter (the arithmetic and harmonic means are also represented by certain parameter values). Weighted utilities for best-effort users are given in [112].

Based on a system utility function, the multi-objective optimization problem in (1.35) can be converted (called *scalarization*) to the

[18] Every continuously differentiable function is locally Lipschitz continuous, but some functions are not globally Lipschitz since the first derivative becomes infinite when approaching the origin. The weighted geometric mean $\prod_k g_k^{w_k}$ has such problems, but this can be resolved by optimizing $\prod_k g_k^{c w_k}$ instead where $c$ is selected to make $c w_k > 1 \forall k$. The weighted harmonic mean also needs additional treatment.

following single-objective optimization problem

$$\begin{aligned} \underset{\mathbf{v}_1,\ldots,\mathbf{v}_{K_r}}{\text{maximize}} \quad & f\left(g_1(\mathrm{SINR}_1),\ldots,g_{K_r}(\mathrm{SINR}_{K_r})\right) \\ \text{subject to} \quad & \mathrm{SINR}_k = \frac{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2}{\sigma_k^2 + \sum_{i \neq k} |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2} \quad \forall k, \\ & \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \quad \forall l. \end{aligned} \tag{1.40}$$

This problem has a single (nonunique) solution, because the system utility function resolves the conflicting interests in the MOP. The selection of $f(\cdot)$ is therefore very important and should be based on a profound knowledge of $\mathcal{R}$ — the alternative of just selecting $f(\cdot)$ out of the blue corresponds to making decisions without knowing the alternatives. Two of the main objectives of this tutorial is to characterize the performance region and develop a framework for solving any single-objective resource allocation problem of the form (1.40). The latter can be viewed as a *network utility maximization* [40, 53, 131, 194], thus we can utilize many of the results on distributed optimization that has been developed under this umbrella; see Section 4.2.

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**Remark 1.2 (All Utility Functions are Subjective).** Observe that all utility functions are subjective by nature, because each function imposes a certain order of vectors in the performance region and in $\mathbb{R}_+^{K_r}$. Although this transforms the resource allocation into the tractable form (1.40) where there is a single solution, this is only because all other Pareto optimal points are discarded by the choice of $f(\cdot)$. Therefore, we stress that the particular choice of $f(\cdot)$ should always be clearly motivated in research papers and not considered as given beforehand.

---

The basic connection between $\mathcal{R}$ and $f(\cdot)$ is given by the following important result.

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**Lemma 1.10.** If $f(\cdot)$ is an increasing function, then the global optimum to (1.40) is attained on $\partial^+\mathcal{R}$. In addition, for any $\tilde{\mathbf{g}} \in \partial^+\mathcal{R}$ there exists a (strictly) increasing $f(\cdot)$ for which (1.40) has $\tilde{\mathbf{g}}$ as global optimum.

---

*Proof.* For the first statement, assume that $\bar{\mathbf{g}} \notin \partial^+\mathcal{R}$ is a global optimum to (1.40). By the definition of the weak Pareto boundary and using that $f(\cdot)$ is increasing, there exist a point $\mathbf{g}' \in \partial^+\mathcal{R}$ with $\mathbf{g}' \geq \bar{\mathbf{g}}$. This point satisfies $f(\mathbf{g}') \geq f(\bar{\mathbf{g}})$ and therefore also solves (1.40).

The second statement is proved using the weighted max-min fairness function $f(\mathbf{g}) = \min_{\{k:\tilde{g}_k>0\}} g_k/\tilde{g}_k$ for given $\tilde{\mathbf{g}} = (\tilde{g}_1,\dots,\tilde{g}_{K_r}) \in \partial^+\mathcal{R}$. Obviously, $\max_{\mathbf{g}\in\mathcal{R}} f(\mathbf{g}) \geq f(\tilde{\mathbf{g}}) = 1$ and assume for the purpose of contradiction that there exists $\mathbf{g}^* \in \mathcal{R}$ that achieves strict inequality. This means that $\mathbf{g}^* > \tilde{\mathbf{g}}$ and thus $\tilde{\mathbf{g}}$ cannot be a weak Pareto optimal point since it requires $\{\mathbf{y}' \in \mathbb{R}_+^n : \mathbf{y}' > \tilde{\mathbf{g}}\} \cap \mathcal{R} \neq \emptyset$ (see Definition 1.10). This contradiction yields $\max_{\mathbf{g}\in\mathcal{R}} f(\mathbf{g}) = f(\tilde{\mathbf{g}})$ and thus $\tilde{\mathbf{g}}$ is the (nonunique) global optimum. □

Based on this lemma, we only need to search the weak Pareto boundary of $\mathcal{R}$ to solve any resource allocation problem of the form (1.40). Unfortunately, this is not as simple as it seems; we will show in Section 2 that (1.40) can only be solved in an efficient manner in certain special cases (e.g., depending on $f(\cdot)$, the number of transmit antennas, and the structure of the power constraints).

Similar to Lemma 1.10, there is an important connection between (1.40) and the channel gain regions.

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**Corollary 1.11.** Suppose the solution to the optimization problem in (1.40) is achieved by signal correlation matrices $\mathbf{S}_1,\dots,\mathbf{S}_{K_r}$ (with $\text{rank}(\mathbf{S}_k) \leq 1 \forall k$). Each $\mathbf{S}_k$ achieves a point on the upper boundary of the corresponding channel gain region $\Omega_k$ in direction $\mathbf{e}_k$ for all $k$.

---

*Proof.* The corollary follows from the monotonicity of $f(\cdot)$, Lemma 1.5, and Lemma 1.10. □

It is important to note that the set of transmit strategies that achieve points on the upper boundaries of the channel gain regions is much larger than the set of transmit strategies that achieves operating points on the Pareto boundary of $\mathcal{R}$, which again is much larger than the set of transmit strategies that maximizes $f(\cdot)$ in (1.40). The reason is that the upper boundary of *each* of the $K_r$ channel gain regions has dimension $K_r - 1$ whereas the Pareto boundary of $\mathcal{R}$ has only dimension $K_r - 1$.

### 1.6.1 Four Methods to Solve Resource Allocation Problems

We have shown how scalarization converts the MOP in (1.35) into a single-objective problem (1.40) with a single solution. There are different ways of utilizing scalarization for finding a Pareto optimal point that makes the system designer satisfied. The preferable approach depends on how well the system designer can specify its subjective views in mathematical terms, and whether the system designer is taking an active or passive part in the optimization. The different methods can be categorized as follows [38, 324]:

(1) *No-preference methods* are applied when the system designer has no subjective preference on the final solution. To emphasize neutrality, (1.40) can be solved using a weighted system utility function (see Example 1.11) where the weighting factors are used for normalization (i.e., using the utopia point for weighting as $w_k = \frac{u_k}{\sum_{i=1}^{K_r} u_i}$).

(2) *A priori methods* are used when the system designer has a clear invariable goal, corresponding to a certain $f(\cdot)$. For instance, an optimistic reference point $\mathbf{r}^*$ might be given in advance and the optimal solution minimizes the distance to this point as $f(\mathbf{g}) = -\|\mathbf{r}^* - \mathbf{g}\|_p$ in the $L_p$-norm (i.e., a compromise problem). Maximizing the sum utility is another example. Any prior knowledge of the performance region and system-wide preference on the final solution should be taken into account when selecting $f(\cdot)$.

(3) *A posteriori methods* generate a set of sample points on the Pareto boundary (the whole set is infinite and nontrivial to characterize) and let the system designer select among these points. Based on Lemma 1.10, sample points are achieved by solving (1.40) for a set of different system utility functions. For example, a certain type of function can be selected from Example 1.11 and the weighting factors are then varied over

a grid. Keep in mind that the whole Pareto boundary cannot be reached by all types of functions (see Remark 1.3).

(4) *Interactive methods* can be viewed as an iterative combination of *a priori* and *a posteriori* methods, where each iteration generates new sample points on the Pareto boundary based on previous suggestions from the system designer. The advantage of this approach is that the preference of the system designer can be modified as the shape of Pareto boundary (i.e., the different alternatives) is learned, thus giving a kind of psychological convergence to the final solution.

All of these methods involve one or multiple scalarizations of the MOP into SOPs of the form (1.40). Section 2 will therefore be devoted to solving SOP for any choice of $f(\cdot)$. Section 3 derives structure on the optimal transmit strategies and parameterizes the Pareto boundary. Based on the knowledge and experience from these sections, we will return to the aforementioned four methods in Section 3.5. We will then shed light on how these methods can be formulated and implemented efficiently for practical resource allocation.

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**Remark 1.3 (Shortcomings of Weighted Arithmetic Mean).** It has become a common practice to optimize the weighted arithmetic mean (e.g., the weighted sum information rate) in the area of communications. This could make sense when $\mathcal{R}$ is convex, which holds for the ideal capacity region but not necessarily in other scenarios. Even if all possible weights are considered, the weighted arithmetic mean only finds Pareto optimal points that coincide with the convex hull of $\mathcal{R}$; this is illustrated in Figure 1.16(a). The weights are often viewed as the relative priority of different users, but the coupling is complicated and can in general be misleading. First, the notion of priority makes sense in a local area of the performance region, but the global interpretation of the weighting is not easily characterized [216]. This is particularly evident for nonconvex performance regions, because a small perturbation in the weights can greatly affect the optimal operating point; see Figure 1.16(b). Second, the physical setup makes it easier to simultaneously serve spatially separated users (rather than co-located users) and

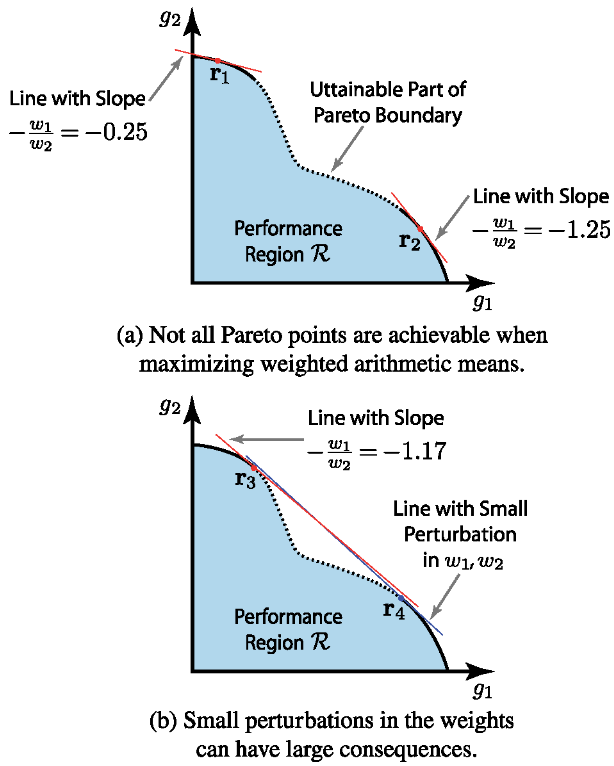


Fig. 1.16 Example of maximization of the weighted arithmetic mean $w_1 g_1 + w_2 g_2$ for a nonconvex performance region. The weights $w_1, w_2$ define a line (or hyperplane of dimension $K_r - 1$) that is moved away from the origin until it leaves the performance region; the final intersection with the Pareto boundary gives the optimal operating point. (a) shows that certain points of the Pareto boundary can never be attained by maximizing a weighted arithmetic mean; (b) shows that a small perturbation in the weights can move the optimal solution from one side of the gap to the other side (i.e., from $r_3$ to $r_4$).

thus promotes unbalanced allocation of resources; see further examples on inter-criteria correlation in [258]. Third, the linearity of $f(\cdot)$ implicitly assumes that degrading the performance of one user can be fully compensated by improving for other users, which might not be reasonable in practice [38]. In fact, the *law of diminishing marginal utility* suggests that $f(\cdot)$ should be nonlinear since users become increasingly satisfied with their current performance and less interested in further improvements [223]. Nevertheless, maximizing the weighted arithmetic mean guarantees Pareto optimality and has a simple geometric interpretation (see Figure 1.16), but the system designer should be aware of the limitations and select the weights carefully.

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**Remark 1.4 (Game Theoretic Approaches).** Game theory provides an alternative approach to MOPs where the users are seen as players that compete for resources. The game can be formulated in a variety of ways, but the Pareto boundary describes the efficient outcomes for any cooperative game. This approach makes particular sense for ad hoc networks in unlicensed bands and cognitive radio, where there is no joint decision-making and users are indeed competing for spectrum. We refer to [68, 140, 171, 230] and references therein for further details.

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## 1.7 Numerical Examples

In this section, we provide a numerical example that illustrates various concepts defined in this section. We consider a simple scenario with $K_r = 2$ users, $N = 3$ transmit antennas, and global joint transmission (as in Example 1.3). The channel vectors are generated as $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ (i.e., uncorrelated Rayleigh fading) and we assume per-antenna power constraints with $q_l = 10$ (i.e., 10 dBm). The average single-user SNR $\frac{\mathbb{E}\{q_l \|\mathbf{h}_k\|_2^2\}}{\sigma_k^2}$ is $q_l N$ for User 1 and $q_l \frac{N}{4}$ for User 2, creating an asymmetry that will highlight properties of different system utility functions.

Figure 1.17 shows the performance regions for a single random channel realization for different user performance functions. In Figure 1.17(a), the additive inverse of the MSE is considered (i.e., $g_k(\mathrm{SINR}_k) = \frac{\mathrm{SINR}_k}{1+\mathrm{SINR}_k}$ to make $g_k(0) = 0$), but the figure axes show MSEs to enhance viewing. The information rate $g_k(\mathrm{SINR}_k) = \log_2(1 + \mathrm{SINR}_k)$ is the user performance function in Figure 1.17(b). In both cases, the optimal operating points are shown for the five functions in Example 1.11: arithmetic mean (sum utility), geometric mean (proportional fairness), harmonic mean, max-min fairness, and distance to the utopia point. The weighting factors are $w_1 = w_2 = \frac{1}{2}$.

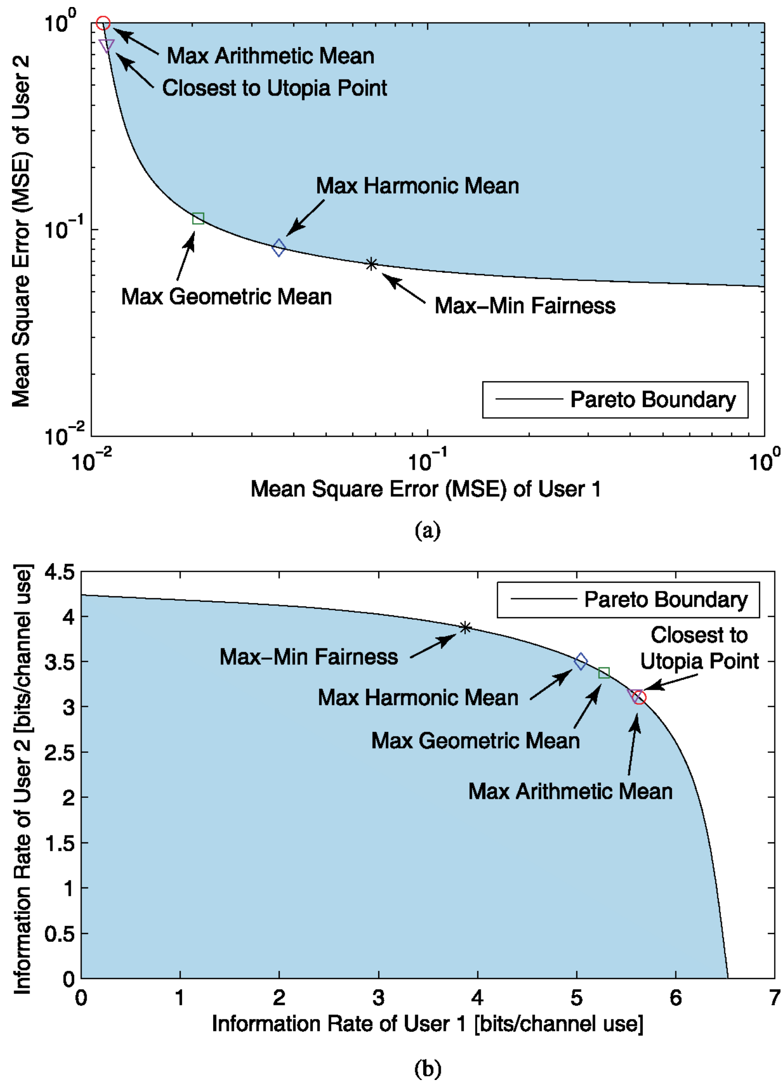


Fig. 1.17 Performance regions for a single channel realizations for different user performance functions: (a) the inverse MSE; and (b) information rate. The Pareto boundary is indicated along with the optimal operating points for different system utility functions.

It is clear that the optimal operating points for these system utility functions are on the Pareto boundary (confirming Lemma 1.10), but at quite different places. As noted in Example 1.11, the arithmetic mean only cares about the aggregate system utility and ignores which user who gets the performance, while max-min fairness makes sure that all users get exactly the same performance. The geometric mean and harmonic mean are in between these extremes, taking both aggregate system utility and user fairness into account. Searching for the point with the smallest Euclidean distance to the utopia point is similar to maximizing the arithmetic mean. By changing the weighting factors in Example 1.11, the optimal point for a certain type of system utility function can be moved around on the Pareto boundary; in fact, the Pareto boundaries in Figure 1.17 were generated by solving weighted max-min fairness problems over a fine grid of weighting factors.

## 1.8 Summary and Outline

Coordinated multi-cell multi-antenna communication provides an opportunity to increase the system-wide spectral efficiency, as compared to traditional multi-cell setups built on strict interference avoidance. There are many similarities between the single-cell and multi-cell downlink, which can be utilized to bring insights from one case to the other. However, there are also important differences that need to be modeled and managed properly. In this tutorial, we defined a general system model based on *dynamic cooperation clusters* and arbitrary linear power constraints. The main idea behind such clusters is that each base station coordinates interference to exactly those users whom it causes non-negligible interference, while only sending data to a subset of them. As exemplified in this section, this framework can jointly describe many important multi-cell scenarios, including the Wyner model, interference channel, coordinated beamforming, global joint transmission, cognitive radio, and spectrum sharing.

The user performance depends on functions of the SINRs (e.g., information rate, MSE, or error probability), which in turn depends on the selection of signal correlation matrices. Each signal correlation matrix will generally affect all users, which can be illustrated by channel gain regions. These regions were proved to be convex and compact, and the upper boundaries in different directions represent maximization of the received signal power at different users. The joint selection of signal correlation matrices is called resource allocation and can be formulated as a multi-objective optimization problem. There is not a single solution to such a problem, but many possible tradeoffs between maximizing performance for individual users and maximizing the aggregate utility of the whole system. This tradeoff is illustrated by the performance region $\mathcal{R}$, which was proved to be compact and normal. The Pareto boundary of $\mathcal{R}$ contains all resource allocations that can be regarded optimal. Furthermore, it was shown that all Pareto optimal points can be achieved using single-stream beamforming and optimality conditions for using full transmit power was derived.

To solve the multi-objective resource allocation problem it is necessary to conclude which Pareto optimal points that are preferable for the system. There are different categories of methods and most of them include the selection of a system utility function that assigns a value to each point in the performance region indicating the subjective preference of the system designer. This function can, for example, be the sum utility or max-min fairness. This scalarizes the multi-objective problem to a single-objective problem with a single solution.

### 1.8.1 Outline

Section 2 shows how to solve any single-objective optimization problem. It becomes clear that some problem formulations enable practically efficient algorithms while others can only be optimally solved for offline benchmarking. Section 3 reduces the search-space by parameterizing the optimal transmit strategies and thereby characterizing the Pareto boundary. Section 3 also provides guidelines for formulating and solving multi-objective resource allocation problem in computationally efficient manners.

Finally, Section 4 generalizes the system model to include practical nonidealities, such as CSI uncertainty, hardware impairments, and limited backhaul signaling. It will be shown which results on optimal resource allocation in Sections 2 and 3 that can be easily generalized, and which become intractable. The design of dynamic cooperation clusters and multi-cell scheduling is also discussed. Furthermore. we describe extensions to multi-cast transmission, multi-carrier systems, multi-antenna users, cognitive radio, and physical layer security.

# References


[1] *Further advancements for E-UTRA physical layer aspects (Release 9)*. 3GPP TS 36.814, March 2010.

[2] A. Aggarwal and T. Meng, "Minimizing the peak-to-average power ratio of OFDM signals using convex optimization," *IEEE Transactions on Signal Processing*, vol. 54, no. 8, pp. 3099–3110, 2006.

[3] H. Al-Shatri and T. Weber, "Achieving the maximum sum rate using D.C. programming in cellular networks," *IEEE Transactions on Signal Processing*, vol. 30, no. 3, pp. 1331–1341, 2012.

[4] V. Annapureddy and V. Veeravalli, "Sum capacity of MIMO interference channels in the low interference regime," *IEEE Transactions on Information Theory*, vol. 57, no. 5, pp. 2565–2581, 2011.

[5] M. Ariaudo, I. Fijalkow, J.-L. Gautier, M. Brandon, B. Aziz, and B. Milevsky, "Green radio despite "dirty RF" front-end," *EURASIP Journal on Wireless Communications and Networking*, 2012.

[6] G. Bagherikaram, A. S. Motahari, and A. K. Khandani, "The secrecy capacity region of the Gaussian MIMO broadcast channel," *IEEE Transactions on Information Theory*, Submitted, arXiv:0903.3261, 2009.

[7] V. Balakrishnan, S. Boyd, and S. Balemi, "Robust downlink beamforming based on outage probability specifications," *International Journal on Robust and Nonlinear Control*, vol. 1, no. 4, pp. 295–317, 1991.

[8] H. Balan, R. Rogalin, A. Michaloliakos, K. Psounis, and G. Caire, "AirSync: Enabling distributed multiuser MIMO with full spatial multiplexing," *IEEE/ACM Transactions on Networking*, Submitted, arXiv:1205.6862.

[9] A. Ben-Tal and A. Nemirovski, "Robust convex optimization," *Mathematics Of Operations Research*, vol. 23, no. 4, pp. 769–805, 1998.

[10] A. Ben-Tal and A. Nemirovski, *Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications*. SIAM, 2001.
[11] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antennas in Wireless Communications*, (L. C. Godara, ed.), CRC Press, 2001.
[12] D. Bertsekas, *Nonlinear Programming*. Athena Scientific, 2nd ed., 1999.
[13] M. Biguesh and A. Gershman, "Downlink channel estimation in cellular systems with antenna arrays at base stations using channel probing with feedback," *EURASIP Journal on Applied Signal Processing*, no. 9, no. 9, pp. 1330–1339, 2004.
[14] E. Björnson, "Multiantenna cellular communications: Channel estimation, feedback, and resource allocation," PhD thesis, KTH Royal Institute of Technology, 2011.
[15] E. Björnson, M. Bengtsson, and B. Ottersten, "Receive combining vs. multistream multiplexing in multiuser MIMO systems," in *Proceedings of IEEE Swedish-Communication Technologies Workshop*, pp. 109–114, 2011.
[16] E. Björnson, M. Bengtsson, and B. Ottersten, "Pareto characterization of the multicell MIMO performance region with simple receivers," *IEEE Transactions on Signal Processing*, vol. 60, no. 8, pp. 4464–4469, 2012.
[17] E. Björnson, M. Bengtsson, G. Zheng, and B. Ottersten, "Computational framework for optimal robust beamforming in coordinated multicell systems," in *Proceedings of IEEE Computational Advance in Multi-Sensor Adaptive Processing*, 2011.
[18] E. Björnson, N. Jaldén, M. Bengtsson, and B. Ottersten, "Optimality properties, distributed strategies, and measurement-based evaluation of coordinated multicell OFDMA transmission," *IEEE Transactions on Signal Processing*, vol. 59, no. 12, pp. 6086–6101, 2011.
[19] E. Björnson and E. Jorswieck, "Optimal resource allocation in coordinated multi-cell systems: Matlab code," Technical Report, doi: http://dx.doi.org/10.1561/0100000069_supp.
[20] E. Björnson, M. Kountouris, M. Bengtsson, and B. Ottersten, "Receive combining vs. multi-stream multiplexing in downlink systems with multi-antenna users," *IEEE Transactions on Signal Processing*, Submitted, arXiv:1207.2776.
[21] E. Björnson and B. Ottersten, "On the principles of multicell precoding with centralized and distributed cooperation," in *Proceedings of International Conference on Wireless Communication and Signal Processing*, 2009.
[22] E. Björnson and B. Ottersten, "A framework for training-based estimation in arbitrarily correlated Rician MIMO channels with Rician disturbance," *IEEE Transactions on Signal Processing*, vol. 58, no. 3, pp. 1807–1820, 2010.
[23] E. Björnson, R. Zakhour, D. Gesbert, and B. Ottersten, "Cooperative multicell precoding: Rate region characterization and distributed strategies with instantaneous and statistical CSI," *IEEE Transactions on Signal Processing*, vol. 58, no. 8, pp. 4298–4310, 2010.
[24] E. Björnson, P. Zetterberg, and M. Bengtsson, "Optimal coordinated beamforming in the multicell downlink with transceiver impairments," in *Proceedings of IEEE Global Communication Conference*, 2012.

[25] E. Björnson, P. Zetterberg, M. Bengtsson, and B. Ottersten, "Capacity limits and multiplexing gains of MIMO channels with transceiver impairments," *IEEE Communications Letters*, To appear, 2013.

[26] E. Björnson, G. Zheng, M. Bengtsson, and B. Ottersten, "Robust monotonic optimization framework for multicell MISO systems," *IEEE Transactions on Signal Processing*, vol. 60, no. 5, pp. 2508–2523, 2012.

[27] B. Bloch and J. Barros, *Physical Layer Security — From Information Theory to Security Engineering*. Cambridge University Press, 2011.

[28] F. Boccardi and H. Huang, "A near-optimum technique using linear precoding for the MIMO broadcast channel," in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, 2007.

[29] H. Boche, S. Naik, and M. Schubert, "Characterization of convex and concave resource allocation problems in interference coupled wireless systems," *IEEE Transactions on Signal Processing*, vol. 59, no. 5, pp. 2382–2394, 2011.

[30] H. Boche and M. Schubert, "A general duality theory for uplink and downlink beamforming," in *Proceedings of IEEE Vehicular Technology Conference-Fall*, pp. 87–91, 2002.

[31] H. Boche and M. Schubert, "A calculus for log-convex interference functions," *IEEE Transactions on Information Theory*, vol. 54, no. 12, pp. 5469–5490, 2008.

[32] F. Bock and B. Ebstein, "Assignment of transmitter powers by linear programming," *IEEE Transactions on Electromagnetic Compatibility*, vol. 6, no. 2, pp. 36–44, 1964.

[33] M. Boldi, A. Tölli, M. Olsson, E. Hardouin, T. Svensson, F. Boccardi, L. Thiele, and V. Jungnickel, "Coordinated multipoint (CoMP) systems," in *Mobile and Wireless Communications for IMT-Advanced and Beyond*, (A. Osseiran, J. Monserrat, and W. Mohr, eds.), pp. 121–155, Wiley, 2011.

[34] G. Box and N. Draper, *Empirical Model-building and Response Surfaces*. Wiley, 1987.

[35] S. Boyd, E. F. L. El Ghaoui, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Society for Industrial and Applied Mathematics (SIAM), 1994.

[36] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2010.

[37] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[38] J. Branke, K. Deb, K. Miettinen, and R. S. (Eds.), *Multiobjective Optimization: Interactive and Evolutionary Approaches*. Springer, 2008.

[39] J. Brehmer, *Utility Maximization in Nonconvex Wireless Systems*. Springer, 2012.

[40] J. Brehmer and W. Utschick, "Optimal interference management in multi-antenna, multi-cell systems," in *Proceedings of International Zurich Seminar on Communications*, pp. 134–137, 2010.

[41] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the $k$-user interference channel," *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3425–3441, 2008.
[42] D. Cai, T. Quek, and C. Tan, "A unified analysis of max-min weighted SINR for MIMO downlink system," *IEEE Transactions on Signal Processing*, vol. 59, no. 8, pp. 5384–5395, 2011.
[43] D. Cai, T. Quek, C. Tan, and S. Low, "Max-min SINR coordinated multipoint downlink transmission — duality and algorithms," *IEEE Transactions on Signal Processing*, vol. 60, no. 10, pp. 5384–5395, 2012.
[44] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, "Multiuser MIMO achievable rates with downlink training and channel state feedback," *IEEE Transactions on Information Theory*, vol. 56, no. 6, pp. 2845–2866, 2010.
[45] G. Caire, S. Ramprashad, and H. Papadopoulos, "Rethinking network MIMO: Cost of CSIT, performance analysis, and architecture comparisons," in *Proceedings of Information Theory and Applications Workshop*, 2010.
[46] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Transactions on Information Theory*, vol. 49, no. 7, pp. 1691–1706, 2003.
[47] P. Cao, S. Shi, and E. Jorswieck, "Efficient computation of the Pareto boundary for the two-user MIMO interference channel," in *Proceedings of IEEE Workshop on Signal Processing Advances in Wireless Communications*, 2012.
[48] P. Cao, S. Shi, and E. Jorswieck, "On the Pareto boundary for the two-user single-beam MIMO interference channel," *IEEE Transactions on Signal Processing*, Submitted, arXiv:1202.5474, 2012.
[49] C.-B. Chae, I. Hwang, R. W. Heath, and V. Tarokh, "Interference aware-coordinated beamforming system in a multi-cell environment," *IEEE Transactions on Wireless Communications*, vol. 11, no. 10, pp. 3692–3703, 2012.
[50] B. Chalise, S. Shahbazpanahi, A. Czylwik, and A. Gershman, "Robust downlink beamforming based on outage probability specifications," *IEEE Transactions on Wireless Communications*, vol. 6, no. 10, pp. 3498–3503, 2007.
[51] T.-H. Chang, W.-K. Ma, and C.-Y. Chi, "Worst-case robust multiuser transmit beamforming using semidefinite relaxation: Duality and implications," in *Proceedings of Asilomar Conference on Signals, Systems, and Computers*, 2011.
[52] M. Chiang, P. Hande, T. Lan, and C. Tan, "Power control in wireless cellular networks," *Foundations and Trends in Networking*, vol. 2, no. 4, pp. 355–580, 2008.
[53] M. Chiang, S. Low, R. Calderbank, and J. Doyle, "Layering as optimization decomposition: A mathematical theory of network architectures," *Proceedings of Institute of Electrical and Electronics Engineers*, vol. 95, no. 1, pp. 255–312, 2007.
[54] D. Chizhik, J. Ling, P. Wolniansky, R. Valenzuela, N. Costa, and K. Huber, "Multiple-input-multiple-output measurements and modeling in Manhattan," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 3, pp. 321–331, 2003.

[55] S. S. Company, "General survey of radio frequency bands (30 MHz to 3 GHz): Vienna, Virginia, September 1–5," Technical Report, version 2.0, 2010.

[56] M. Costa, "Writing on dirty-paper," *IEEE Transactions on Information Theory*, vol. 29, no. 3, pp. 439–441, 1983.

[57] T. Cover and J. Thomas, *Elements of Information Theory*. Wiley, 1991.

[58] I. Csiszár and J. Körner, "Broadcast channels with confidential messages," *IEEE Transactions on Information Theory*, vol. 24, no. 3, pp. 339–348, 1978.

[59] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Transactions on Wireless Communications*, vol. 9, no. 5, pp. 1748–1759, 2010.

[60] D. Dardari, V. Tralli, and A. Vaccari, "A theoretical characterization of nonlinear distortion effects in OFDM systems," *IEEE Transactions on Communications*, vol. 48, no. 10, pp. 1755–1764, 2000.

[61] P. de Kerret and D. Gesbert, "The multiplexing gain of the network MIMO channel with distributed CSI," *IEEE Transactions on Information Theory*, To appear, arXiv:1108.3742.

[62] P. de Kerret and D. Gesbert, "Towards optimal CSI allocation in multicell MIMO channels," in *Proceedings of IEEE International Conference on Communications*, 2012.

[63] P. Dighe, R. Mallik, and S. Jamuar, "Analysis of transmit-receive diversity in rayleigh fading," *IEEE Transactions on Communications*, vol. 51, no. 4, pp. 694–703, 2003.

[64] G. Dimić and N. Sidiropoulos, "On downlink beamforming with greedy user selection: Performance analysis and a simple new algorithm," *IEEE Transactions on Signal Processing*, vol. 53, no. 10, pp. 3857–3868, 2005.

[65] M. Dohler, R. Heath, A. Lozano, C. Papadias, and R. Valenzuela, "Is the PHY layer dead?," *IEEE Communications Magazine*, vol. 49, no. 4, pp. 159–165, 2011.

[66] Y. Eldar and N. Merhav, "A competitive minimax approach to robust estimation of random parameters," *IEEE Transactions on Signal Processing*, vol. 52, no. 7, pp. 1931–1946, 2004.

[67] K. Eriksson, S. Shi, N. Vučić, M. Schubert, and E. Larsson, "Globally optimal resource allocation for achieving maximum weighted sum rate," in *Proceedings of IEEE Global Communications Conference*, 2010.

[68] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 3, pp. 517–528, 2007.

[69] R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Transactions on Information Theory*, vol. 54, no. 12, pp. 5534–5562, 2008.

[70] K. Fan, "Minimax theorems," *Proceedings of National Academic Society*, vol. 39, no. 1, pp. 42–47, 1953.

[71] C. Farsakh and J. Nossek, "Channel allocation and downlink beamforming in an SDMA mobile radio system," in *Proceedings of IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, pp. 687–691, 1995.

[72] G. Fettweis, M. Löhning, D. Petrovic, M. Windisch, P. Zillmann, and W. Rave, "Dirty RF: A new paradigm," in *Proceedings of IEEE International Symposium on Personal Indoor and Mobile Radio Communications*, pp. 2347–2355, 2005.

[73] M. Fitz and J. Seymour, "On the bit error probability of QAM modulation," *International Journal on Wireless Information Networks*, vol. 1, no. 2, pp. 131–139, 1994.

[74] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311–335, 1998.

[75] M. Fuchs, G. D. Galdo, and M. Haardt, "Low-complexity space-time-frequency scheduling for MIMO systems with SDMA," *IEEE Transactions on Vehicular Technology*, vol. 56, no. 5, pp. 2775–2784, 2007.

[76] C. Galiotto, Y. Huang, N. Marchetti, and M. Zorzi, "Performance evaluation of non-ideal RF transmitter in LTE/LTE-advanced systems," in *Proceedings of European Wireless*, pp. 266–270, 2009.

[77] I. Garcia, N. Kusashima, K. Sakaguchi, and K. Araki, "Dynamic cooperation set clustering on base station cooperation cellular networks," in *Proceedings of IEEE International Symposium on Personal Indoor and Mobile Radio Communications*, pp. 2127–2132, 2010.

[78] L. Georgiadis, M. Neely, and L. Tassiulas, "Power Control in Wireless Cellular Networks," *Foundations and Trends in Networking*, vol. 1, no. 1, pp. 1–144, 2006.

[79] D. Gerlach and A. Paulraj, "Base station transmitting antenna arrays for multipath environments," *Signal Processing*, vol. 54, no. 1, pp. 59–73, 1996.

[80] A. Gershman, N. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 62–75, 2010.

[81] D. Gesbert, S. Hanly, H. Huang, S. Shamai, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE Journal on Selected Areas in Communications*, vol. 28, no. 9, pp. 1380–1408, 2010.

[82] D. Gesbert, S. Kiani, A. Gjendemsjø, and G. Øien, "Adaptation, coordination, and distributed resource allocation in interference-limited wireless networks," *Proceedings of Institute of Electrical and Electronics Engineers*, vol. 95, no. 12, pp. 2393–2409, 2007.

[83] D. Gesbert and M. Kountouris, "Rate scaling laws in multicell networks under distributed power control and user scheduling," *IEEE Transactions on Information Theory*, vol. 57, no. 1, pp. 234–244, 2011.

[84] D. Gesbert, M. Kountouris, R. Heath, C.-B. Chae, and T. Sälzer, "Shifting the MIMO paradigm," *IEEE Signal Processing Magazine*, vol. 24, no. 5, pp. 36–46, 2007.

[85] G. Ginis and J. Cioffi, "A multi-user precoding scheme achieving crosstalk cancellation with application to DSL systems," in *Proceedings of Asilomer Conference on Signals, Systems, and Computers*, 2000.

[86] A. Giovanidis, J. Krolikowski, and S. Brueck, "A 0-1 program to form minimum cost clusters in the downlink of cooperating base stations," in *Proceedings of IEEE Wireless Communication and Networking Conference*, 2012.

[87] S. Goel and R. Negi, "Guaranteeing secrecy using artificial noise," *IEEE Transactions on Wireless Communications*, vol. 7, no. 6, pp. 2180–2189, 2008.

[88] J. Goldberg and J. Fonollosa, "Downlink beamforming for spatially distributed sources in cellular mobile communications," *Signal Processing*, vol. 65, no. 2, pp. 181–197, 1998.

[89] A. Goldsmith, S. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 5, pp. 684–702, 2003.

[90] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proceedings of Institute of Electrical and Electronics Engineers*, vol. 97, no. 5, pp. 894–914, 2009.

[91] K. Gomadam, V. Cadambe, and S. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," in *Proceedings of IEEE Global Communications Conference*, 2008.

[92] J. Gong, S. Zhou, Z. Niu, L. Geng, and M. Zheng, "Joint scheduling and dynamic clustering in downlink cellular networks," in *Proceedings of IEEE Global Communications Conference*, 2011.

[93] J. González-Coma, P. Castro, and L. Castedo, "Impact of transmit impairments on multiuser MIMO non-linear transceivers," in *Proceedings of ITG Workshop on Smart Antennas (WSA)*, 2011.

[94] B. Göransson, S. Grant, E. Larsson, and Z. Feng, "Effect of transmitter and receiver impairments on the performance of MIMO in HSDPA," in *Proceedings of IEEE Workshop on Signal Processing Advances in Wireless Communications*, 2008.

[95] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming," http://cvxr.com/cvx, April 2011.

[96] M. Guillaud, D. Slock, and R. Knopp, "A practical method for wireless channel reciprocity exploitation through relative calibration," in *Proceedings of International Symposium on Signal Processing and Its Applications*, pp. 403–406, 2005.

[97] C. Guthy, W. Utschick, and G. Dietl, "Low-complexity linear zero-forcing for the MIMO broadcast channel," *IEEE Journal of Selected Topics in Signal Processing*, vol. 3, no. 6, pp. 1106–1117, 2009.

[98] Y. Haimes, L. Lasdon, and D. Wismer, "On a bicriterion formulation of the problems of integrated system identification and system optimization," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 1, no. 3, pp. 296–297, 1971.

[99] D. Hammarwall, M. Bengtsson, and B. Ottersten, "Utilizing the spatial information provided by channel norm feedback in SDMA systems," *IEEE Transactions on Signal Processing*, vol. 56, no. 7, pp. 3278–3293, 2008.

[100] S. Han, Q. Zhang, and C. Yang, "Distributed coordinated multi-point downlink transmission with over-the-air communication," in *Proceedings of International Conference on Communications and Networking in China*, 2010.

[101] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Transactions on Information Theory*, vol. 27, no. 1, pp. 49–60, 1981.

[102] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–219, 2005.
[103] H. Holma and A. Toskala, *LTE for UMTS: Evolution to LTE-Advanced*. Wiley, 2nd ed., 2011.
[104] M. Hong and Z.-Q. Luo, "E-reference signal processing," ch. *Signal Processing and Optimal Resource Allocation for the Interference Channel.* Elsevier, 2013.
[105] X. Hou, E. Björnson, C. Yang, and M. Bengtsson, "Cell-grouping based distributed beamforming and scheduling for multi-cell cooperative transmission," in *Proceedings of IEEE International Symposium on Personal Indoor and Mobile Radio Communications*, 2011.
[106] H. Huang, M. Trivellato, A. Hottinen, M. Shafi, P. Smith, and R. Valenzuela, "Increasing downlink cellular throughput with limited network MIMO coordination," *IEEE Transactions on Wireless Communications*, vol. 8, no. 6, pp. 2983–2989, 2009.
[107] Y. Huang and D. Palomar, "Rank-constrained separable semidefinite program with applications to optimal beamforming," *IEEE Transactions on Signal Processing*, vol. 58, no. 2, pp. 664–678, 2010.
[108] Y. Huang, G. Zheng, M. Bengtsson, K. Wong, L. Yang, and B. Ottersten, "Distributed multicell beamforming design approaching Pareto boundary with max-min fairness," *IEEE Transactions on Wireless Communications*, vol. 11, no. 8, pp. 2921–2933, 2012.
[109] H. Huh, G. Caire, H. Papadopoulos, and S. Ramprashad, "Achieving "massive MIMO spectral efficiency with a not-so-large number of antennas," *IEEE Transactions on Wireless Communications*, vol. 11, no. 9, pp. 3226–3239, 2012.
[110] T. Hungerford, *Abstract Algebra: An Introduction.* Brooks Cole, 1996.
[111] N. Jaldén, P. Zetterberg, B. Ottersten, and L. Garcia, "Inter- and intrasite correlations of large-scale parameters from macrocellular measurements at 1800 MHz," *EURASIP Journal on Wireless Communications and Networking*, 2007.
[112] Z. Jiang, Y. Ge, and Y. Li, "Max-utility wireless resource management for best-effort traffic," *IEEE Transactions on Wireless Communications*, vol. 4, no. 1, pp. 100–111, 2005.
[113] N. Jindal, "MIMO broadcast channels with finite-rate feedback," *IEEE Transactions on Information Theory*, vol. 52, no. 11, pp. 5045–5060, 2006.
[114] S. Jing, D. Tse, J. Soriaga, J. Hou, J. Smee, and R. Padovani, "Multicell downlink capacity with coordinated processing," *EURASIP Journal on Wireless Communications and Networking*, 2008.
[115] M. Joham, W. Utschick, and J. Nossek, "Linear transmit processing in MIMO communications systems," *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 2700–2712, 2005.
[116] E. Jorswieck, "Secrecy capacity of single- and multi-antenna channels with simple helpers," in *Proceedings of ITG Conference on Source and Channel Coding*, 2010.

[117] E. Jorswieck and E. Larsson, "The MISO interference channel from a game-theoretic perspective: A combination of selfishness and altruism achieves Pareto optimality," in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 5364–5367, 2008.
[118] E. Jorswieck and E. Larsson, "Monotonic optimization framework for the two-user MISO interference channel," *IEEE Transactions on Communications*, vol. 58, no. 7, pp. 2159–2168, 2010.
[119] E. Jorswieck, E. Larsson, and D. Danev, "Complete characterization of the Pareto boundary for the MISO interference channel," *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 5292–5296, 2008.
[120] E. Jorswieck and R. Mochaourab, "Beamforming in underlay cognitive radio: Null-shaping constraints and greedy user selection," in *Proceedings of IEEE CrownCom*, pp. 1–5, 2010.
[121] E. Jorswieck, A. Wolf, and S. Gerbracht, "Trends in Telecommunications Technologies," ch. *Secrecy on the Physical Layer in Wireless Networks.* IN-TECH Education and Publishing, 2010.
[122] J. Jose, A. Ashikhmin, T. Marzetta, and S. Vishwanath, "Pilot contamination and precoding in multi-cell TDD systems," *IEEE Transactions on Communications*, vol. 10, no. 8, pp. 2640–2651, 2011.
[123] S. Joshi, P. Weeraddana, M. Codreanu, and M. Latva-aho, "Coordinated beamforming for MISO interference channel: Complexity analysis and efficient algorithms," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 2090–2095, 2012.
[124] V. Jungnickel, T. Wirth, M. Schellmann, T. Haustein, and W. Zirwas, "Synchronization of cooperative base stations," in *Proceedings of IEEE International Symposium on Wireless Communication Systems*, pp. 329–334, 2008.
[125] M. Karakayali, G. Foschini, and R. Valenzuela, "Network coordination for spectrally efficient communications in cellular systems," *IEEE Wireless Communications Magazine*, vol. 13, no. 4, pp. 56–61, 2006.
[126] E. Karipidis and E. Larsson, "Efficient computation of the Pareto boundary for the MISO interference channel with perfect CSI," in *Proceedings of International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks*, pp. 573–577, 2010.
[127] E. Karipidis, N. Sidiropoulos, and Z.-Q. Luo, "Quality of service and max-min fair transmit beamforming to multiple cochannel multicast groups," *IEEE Transactions on Signal Processing*, vol. 56, no. 3, pp. 1268–1279, 2008.
[128] S. Kaviani and W. Krzymien, "Multicell scheduling in network MIMO," in *Proceedings of IEEE Global Communications Conference*, 2010.
[129] S. Kaviani, O. Simeone, W. Krzymien, and S. Shamai, "Linear precoding and equalization for network MIMO with partial cooperation," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 5, pp. 2083–2095, 2012.
[130] F. Kelly, "Charging and rate control for elastic traffic," *European Transactions on Telecommunication*, vol. 8, pp. 33–37, 1997.
[131] F. Kelly, A. Maulloo, and D. Tan, "Rate control for communication networks: shadow prices, proportional fairness and stability," *Journal on Operational Research Society*, vol. 49, no. 3, pp. 237–252, 1997.

[132] J. Kermoal, L. Schumacher, K. Pedersen, P. Mogensen, and F. Frederiksen, "A stochastic MIMO radio channel model with experimental validation," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 6, pp. 1211–1226, 2002.

[133] H. Kha, H. Tuan, and H. Nguyen, "Fast global optimal power allocation in wireless networks by local D.C. programming," *IEEE Transactions on Wireless Communications*, vol. 11, no. 2, pp. 510–515, 2012.

[134] S. Kiani and D. Gesbert, "Optimal and distributed scheduling for multicell capacity maximization," *IEEE Transactions on Wireless Communications*, vol. 7, no. 1, pp. 288–297, 2008.

[135] M. Kobayashi, M. Debbah, and J. Belfiore, "Outage efficient strategies in network MIMO with partial CSIT," in *Proceedings of IEEE International Symposium on Information Theory*, pp. 249–253, 2009.

[136] B. Kolman and R. Beck, *Elementary Linear Programming with Applications*. Academic Press, 1995.

[137] S. Koskie and Z. Gajic, "Signal-to-interference-based power control for wireless networks: A survey, 1992–2005," *Dynamics of Continuous, Discrete and Impulsive Systems B*, vol. 13, no. 2, pp. 187–220, 2006.

[138] J. Kotecha and A. Sayeed, "Transmit signal design for optimal estimation of correlated MIMO channels," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 546–557, 2004.

[139] M. Ku and D. Kim, "Tx-Rx beamforming with multiuser MIMO channels in multiple-cell systems," in *Proceedings of International Conference on Advanced Communication Technology*, pp. 1767–1771, 2008.

[140] E. Larsson, E. Jorswieck, J. Lindblom, and R. Mochaourab, "Game theory and the flat-fading Gaussian interference channel," *IEEE Signal Processing Magazine*, vol. 26, no. 5, pp. 18–27, 2009.

[141] V. Lau, "Proportional fair spacetime scheduling for wireless communications," *IEEE Transactions on Communications*, vol. 53, no. 8, pp. 1353–1360, 2005.

[142] B. Lee, H. W. Je, I. Sohn, O.-S. Shin, and K. B. Lee, "Interference-aware decentralized precoding for multicell MIMO TDD systems," in *Proceedings of IEEE Global Communications Conference*, 2008.

[143] G. Lee, J. Park, Y. Sung, and M. Yukawa, "Coordinated beamforming with relaxed zero forcing," in *Proceedings of International Conference on Wireless Communications and Signal Processing*, 2011.

[144] J. Lee and N. Jindal, "Symmetric capacity of MIMO downlink channels," in *Proceedings of IEEE International Symposium on Information Theory*, pp. 1031–1035, 2006.

[145] J.-W. Lee, R. Mazumdar, and N. Shroff, "Non-convex optimization and rate control for multi-class services in the internet," *IEEE/ACM Transactions on Networking*, vol. 13, no. 4, pp. 827–840, 2005.

[146] J. Li, H. Zhang, X. Xu, X. Tao, T. Svensson, C. Botella, and B. Liu, "A novel frequency reuse scheme for coordinated multi-point transmission," in *Proceedings of IEEE Vehicular Technology Conference-Spring*, 2010.

[147] Y. Liang, H. V. Poor, and S. Shamai (Shitz), "Information Theoretic Security," *Foundations and Trends in Communications and Information Theory*, vol. 5, no. 4–5, pp. 355–580, 2009.

[148] J. Lindblom, E. Karipidis, and E. Larsson, "Selfishness and altruism on the MISO interference channel: The case of partial transmitter CSI," *IEEE Communications Letters*, vol. 13, no. 9, pp. 667–669, 2009.

[149] J. Lindblom, E. Karipidis, and E. Larsson, "Closed-form parameterization of the Pareto boundary for the two-user MISO interference channel," in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 3372–3375, 2011.

[150] A. Liu, Y. Liu, H. Xiang, and W. Luo, "Polite water-filling for weighted sum-rate maximization in MIMO B-MAC networks under multiple linear constraints," *IEEE Transactions on Signal Processing*, vol. 60, no. 2, pp. 834–847, 2012.

[151] H. Liu and G. Xu, "Multiuser blind channel estimation and spatial channel pre-equalization," in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 1756–1759, 1995.

[152] J. Liu, T. Hou, and H. D. Sherali, "Optimal power allocation for achieving perfect secrecy capacity in MIMO wire-tap channels," in *Proceedings of IEEE International Conference on Communications*, 2009.

[153] L. Liu, R. Zhang, and K. Chua, "Achieving global optimality for weighted sum-rate maximization in the K-user Gaussian interference channel with multiple antennas," *IEEE Transactions on Wireless Communications*, vol. 11, no. 5, pp. 1933–1945, 2012.

[154] Q. Liu, R. Baxley, X. Ma, and G. Zhou, "Error vector magnitude optimization for OFDM systems with a deterministic peak-to-average power ratio constraint," *IEEE Journal of Selected Topics in Signal Processing*, vol. 3, no. 3, pp. 418–429, 2009.

[155] R. Liu, T. Liu., H. Poor, and S. Shamai (Shitz), "Multiple-input multiple-output Gaussian broadcast channels with confidential messages," *IEEE Transactions on Information Theory*, vol. 56, no. 9, pp. 4215–4227, 2010.

[156] R. Liu and W. Trappe, *Securing Wireless Communications at the Physical Layer*. Springer Berlin Heidelberg, 2010.

[157] Y.-F. Liu, Y.-H. Dai, and Z.-Q. Luo, "Coordinated beamforming for MISO interference channel: Complexity analysis and efficient algorithms," *IEEE Transactions on Signal Processing*, vol. 59, no. 3, pp. 1142–1157, 2011.

[158] Y.-F. Liu, Y.-H. Dai, and Z.-Q. Luo, "Max-min fairness linear transceiver design for a multi-user MIMO interference channel," in *Proceedings of IEEE International Conference on Communications*, 2011.

[159] T. Lo, "Maximum ratio transmission," *IEEE Transactions on Communications*, vol. 47, no. 10, pp. 1458–1461, 1999.

[160] M. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret, "Applications of second-order cone programming," *Linear Algebra and its Applications*, vol. 284, no. 1–3, pp. 193–228, 1998.

[161] J. Löfberg, "YALMIP: A toolbox for modeling and optimization in MATLAB," in *Proceedings of IEEE International Symposium on Computer Aided Control System Design*, pp. 284–289, 2004.

[162] S. Loyka, "Channel capacity of MIMO architecture using the exponential correlation matrix," *IEEE Communications Letters*, vol. 5, no. 9, pp. 369–371, 2001.

[163] S. Loyka, V. Kostina, and F. Gagnon, "Error rates of the maximum-likelihood detector for arbitrary constellations: Convex/concave behavior and applications," *IEEE Transactions on Information Theory*, vol. 56, no. 4, pp. 1948–1960, 2010.

[164] A. Lozano, R. Heath, and J. Andrews, "Fundamental limits of cooperation," *IEEE Transactions on Information Theory*, Submitted, arXiv:1204.0011.

[165] A. Lozano and N. Jindal, "Transmit diversity vs. spatial multiplexing in modern MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 9, no. 1, pp. 186–197, 2010.

[166] Z.-Q. Luo, W.-K. Ma, A.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 20–34, 2010.

[167] Z.-Q. Luo and W. Yu, "An introduction to convex optimization for communications and signal processing," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1426–1438, 2006.

[168] Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 57–73, 2008.

[169] J. Lv, R. Blasco-Serrano, E. Jorswieck, R. Thobaben, and A. Kliks, "Optimal Beamforming in MISO Cognitive Channels with Degraded Message Sets," in *IEEE Wireless Communications and Networking Conference*, 2012.

[170] J. Lv and E. Jorswieck, "Spatial shaping in cognitive system with coded legacy transmission," in *Proceedings of ITG Workshop on Smart Antennas (WSA)*, 2011.

[171] A. MacKenzie and L. DaSilva, *Game Theory for Wireless Engineers*. Morgan & Claypool, 2006.

[172] M. Maddah-Ali, A. Mobasher, and A. Khandani, "Fairness in multiuser systems with polymatroid capacity region," *IEEE Transactions on Information Theory*, vol. 55, no. 5, pp. 2128–2138, 2009.

[173] P. Marsch, S. Brück, A. Garavaglia, M. Schulist, R. Weber, and A. Dekorsy, "Clustering," in *Coordinated Multi-Point in Mobile Communications: From Theory to Practice*, ch. 7, (P. Marsch and G. Fettweis, eds.), pp. 139–159, Cambridge, 2011.

[174] P. Marsch and G. Fettweis, "On multicell cooperative transmission in backhaul-constrained cellular systems," *Annals of Telecommunication*, vol. 63, pp. 253–269, 2008.

[175] P. Marsch and G. Fettweis, "On downlink network MIMO under a constrained backhaul and imperfect channel knowledge," in *Proceedings of IEEE Global Communications Conference*, 2009.

[176] P. Marsch and G. Fettweis, "Static clustering for cooperative multi-point (CoMP) in mobile communications," in *Proceedings of IEEE International Conference on Communications*, 2011.

[177] H.-P. Mayer and H. Schlesinger, "Antenna synchronization for coherent network MIMO," US Patent, 20120002967, 2010.

[178] M. McKay, A. Grant, and I. Collings, "Performance analysis of MIMO-MRC in double-correlated rayleigh environments," *IEEE Transactions on Communications*, vol. 55, no. 3, pp. 497–507, 2007.

[179] J. Mo and J. Walrand, "Fair end-to-end window-based congestion control," *IEEE/ACM Transactions on Networking*, vol. 8, no. 5, pp. 556–567, 2000.

[180] R. Mochaourab and E. Jorswieck, "Optimal beamforming in interference networks with perfect local channel information," *IEEE Transactions on Signal Processing*, vol. 59, no. 3, pp. 1128–1141, 2011.

[181] R. Mochaourab and E. Jorswieck, "Walrasian equilibrium in two-user multiple-input single-output interference channels," in *Proceedings of IEEE International Conference on Communications*, 2011.

[182] R. Mochaourab and E. Jorswieck, "Robust beamforming in interference channels with imperfect transmitter channel information," *Signal Processing*, vol. 92, no. 11, pp. 2509–2518, 2012.

[183] P. Mogensen, W. Na, I. Kovács, F. Frederiksen, A. Pokhariyal, K. Pedersen, T. Kolding, K. Hugl, and M. Kuusela, "LTE capacity compared to the Shannon bound," in *Proceedings of IEEE Vehicular Technology Conference-Spring*, 2007.

[184] N. Moghadam, P. Zetterberg, P. Händel, and H. Hjalmarsson, "Correlation of distortion noise between the branches of MIMO transmit antennas," in *Proceedings of IEEE International Symposium on Personal Indoor and Mobile Radio Communications*, 2012.

[185] M. Mohseni, R. Zhang, and J. Cioffi, "Optimized transmission for fading multiple-access and broadcast channels with multiple antennas," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1627–1639, 2006.

[186] G. Montalbano, I. Ghauri, and D. Slock, "Multiuser blind channel estimation and spatial channel pre-equalization," in *Proceedings of Asilomer Conference on Signals, Systems, and Computers*, pp. 1337–1341, 1998.

[187] R. Mueller and G. Foschini, "The capacity of linear channels with additive Gaussian noise," *The Bell System Technical Journal*, pp. 81–94, 1970.

[188] C. Nader, P. Händel, and N. Björsell, "Peak-to-average power reduction of OFDM signals by convex optimization: Experimental validation and performance optimization," *IEEE Transactions on Instrumentation and Measurement*, vol. 60, no. 2, pp. 473–479, 2011.

[189] M. Naeem and D. Lee, "On convexity of MQAM's and MPAM's bit error probability functions," *International Journal on Communication Systems*, vol. 22, pp. 1465–1477, 2009.

[190] R. Nettleton and H. Alavi, "Power control for a spread spectrum cellular mobile radio system," in *Proceedings of IEEE Vehicular Technology Conference*, pp. 242–246, 1983.

[191] B. L. Ng, J. Evans, S. Hanly, and D. Aktas, "Distributed downlink beamforming with cooperative base stations," *IEEE Transactions on Information Theory*, vol. 54, no. 12, pp. 5491–5499, 2008.

[192] H. Nikaido, *Convex Structures and Economic Theory*. Academic Press, 1968.

[193] M. Nokleby and A. Swindlehurst, "Bargaining and the MISO interference channel," *EURASIP Journal on Advances in Signal Process*, 2009.

[194] D. Palomar and M. Chiang, "A tutorial on decomposition methods for network utility maximization," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1439–1451, 2006.

[195] D. Palomar, J. Cioffi, and M. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization," *IEEE Transactions on Signal Processing*, vol. 51, no. 9, pp. 2381–2401, 2003.

[196] D. Palomar and Y. Jiang, "MIMO transceiver design via majorization theory," *Foundations and Trends in Communications and Information Theory*, vol. 3, no. 4–5, pp. 331–551, 2006.

[197] A. Pantelidou and A. Ephremides, "Scheduling in wireless networks," *Foundations and Trends in Communications and Information Theory*, vol. 4, no. 4, pp. 421–511, 2010.

[198] A. Papadogiannis and G. Alexandropoulos, "A dynamic clustering approach in wireless networks with multi-cell cooperative processing," in *Proceedings of IEEE International Conference on Fuzzy Systems*, 2010.

[199] A. Papadogiannis, D. Gesbert, and E. Hardouin, "A dynamic clustering approach in wireless networks with multi-cell cooperative processing," in *Proceedings of IEEE International Conference on Communications*, 2008.

[200] A. Papadogiannis, E. Hardouin, and D. Gesbert, "Decentralising multicell cooperative processing: A novel robust framework," *EURASIP Journal on Wireless Communications and Networking*, 2009.

[201] S.-H. Park, H. Parkand, H. Kong, and I. Lee, "New beamforming techniques based on virtual SINR maximization for coordinated multi-cell transmission," *IEEE Transactions on Wireless Communications*, vol. 11, no. 3, pp. 1034–1044, 2012.

[202] S. Parkvall, E. Dahlman, A. Furuskär, Y. Jading, M. Olsson, S. Wänstedt, and K. Zangi, "LTE-advanced — evolving LTE towards IMT-advanced," in *Proceedings of IEEE Vehicular Technology Conference-Fall*, 2008.

[203] C. Peel, B. Hochwald, and A. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication — part I: Channel inversion and regularization," *IEEE Transactions on Communications*, vol. 53, no. 1, pp. 195–202, 2005.

[204] H. Pennanen, A. Tölli, and M. Latva-aho, "Decentralized coordinated downlink beamforming via primal decomposition," *IEEE Signal Processing Letters*, vol. 18, no. 11, pp. 647–650, 2011.

[205] S. Pillai, T. Suel, and S. Cha, "The Perron-Frobenius theorem: Some of its applications," *IEEE Signal Processing Magazine*, vol. 22, no. 2, pp. 62–75, 2005.

[206] L. Qian, Y. Zhang, and J. Huang, "MAPEL: Achieving global optimality for a non-convex wireless power control problem," *IEEE Transactions on Wireless Communications*, vol. 8, no. 3, pp. 1553–1563, 2009.

[207] J. Qiu, R. Zhang, Z.-Q. Luo, and S. Cui, "Optimal distributed beamforming for MISO interference channels," *IEEE Transactions on Signal Processing*, vol. 59, no. 11, pp. 5638–5643, 2011.

[208] F. Rashid-Farrokhi, K. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1437–1450, 1998.

[209] F. Rashid-Farrokhi, L. Tassiulas, and K. Liu, “Joint optimal power control and beamforming in wireless networks using antenna arrays,” *IEEE Transactions on Communications*, vol. 46, no. 10, pp. 1313–1324, 1998.
[210] M. Razaviyayn, M. Hong, and Z.-Q. Luo, “Linear transceiver design for a MIMO interfering broadcast channel achieving max-min fairness,” in *Proceedings of Asilomar Conference on Signals, Systems, and Computers*, pp. 1309–1313, 2011.
[211] T. Ren and R. La, “Downlink beamforming algorithms with inter-cell interference in cellular networks,” *IEEE Transactions on Wireless Communications*, vol. 5, no. 10, pp. 2814–2823, 2006.
[212] W. Rhee and J. Cioffi, “Increase in capacity of multiuser OFDM system using dynamic subchannel allocation,” in *Proceedings of IEEE Vehicular Technology Conference-Spring*, pp. 1085–1089, 2000.
[213] R. Rockafellar, “Lagrange multipliers and optimality,” *SIAM Review*, vol. 35, no. 2, pp. 183–238, 1993.
[214] J. Roh and B. Rao, “Multiple antenna channels with partial channel state information at the transmitter,” *IEEE Transactions on Wireless Communications*, vol. 3, no. 2, pp. 677–687, 2004.
[215] M. Rossi, A. Tulino, O. Simeone, and A. Haimovich, “Non-convex utility maximization in Gaussian MISO broadcast and interference channels,” in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 2960–2963, 2011.
[216] B. Roy and V. Mousseau, “A theoretical framework for analysing the notion of relative importance of criteria,” *Journal on Multi-Criteria Decision Analysis*, vol. 5, pp. 145–159, 1996.
[217] R. Roy and B. Ottersten, “Spatial division multiple access wireless communication systems,” US Patent, 5515378, 1991.
[218] A. Rubinov, H. Tuy, and H. Mays, “An algorithm for monotonic global optimization problems,” *Optimization*, vol. 49, pp. 205–221, 2001.
[219] W. Rudin, *Principles of Mathematical Analysis*. McGraw-Hill, 1976.
[220] F. Rusek, D. Persson, B. Lau, E. Larsson, T. Marzetta, O. Edfors, and F. Tufvesson, “Scaling up MIMO: Opportunities and challenges with very large arrays,” *IEEE Signal Processing Magazine*, vol. 30, no. 1, pp. 40–60, 2013.
[221] M. Sadek, A. Tarighat, and A. Sayed, “A leakage-based precoding scheme for downlink multi-user MIMO channels,” *IEEE Transactions on Wireless Communications*, vol. 6, no. 5, pp. 1711–1721, 2007.
[222] S. Sarkar and L. Tassiulas, “Fair allocation of discrete bandwidth layers in multicast networks,” in *Proceedings of IEEE INFOCOM*, 2000.
[223] L. Savage, *The Foundations of Statistics*. Courier Dover Publications, 1972.
[224] T. Schenk, *RF Imperfections in High-Rate Wireless Systems: Impact and Digital Compensation*. Springer, 2008.
[225] D. Schmidt, C. Shi, R. Berry, M. Honig, and W. Utschick, “Distributed resource allocation schemes,” *IEEE Signal Processing Magazine*, vol. 26, no. 5, pp. 53–63, 2009.

[226] M. Schubert and H. Boche, “Solution of the multiuser downlink beamforming problem with individual SINR constraints,” *IEEE Transactions on Vehicular Technology*, vol. 53, no. 1, pp. 18–28, 2004.
[227] M. Schubert and H. Boche, “QoS-based resource allocation and transceiver optimization,” *Foundations and Trends in Communications and Information Theory*, vol. 2, no. 6, pp. 383–529, 2005.
[228] M. Schubert and H. Boche, *Interference Calculus: A General Framework for Interference Management and Network Utility Optimization*. Springer, 2012.
[229] G. Scutari, D. Palomar, J.-S. Pang, and F. Facchinei, “Flexible design of cognitive radio wireless systems,” *IEEE Signal Processing Magazine*, vol. 26, no. 5, pp. 107–123, 2009.
[230] G. Scutari, D. P. Palomar, and S. Barbarossa, “Cognitive MIMO radio,” *IEEE Signal Processing Magazine*, vol. 25, no. 6, pp. 46–59, 2008.
[231] K. Seong, M. Mohseni, and J. Cioff, “Optimal resource allocation for OFDMA downlink systems,” in *Proceedings of IEEE International Symposium on Information Theory*, 2006.
[232] S. Shafiee and S. Ulukus, “Achievable rates in Gaussian MISO channels with secrecy constraints,” in *Proceedings of IEEE International Symposium on Information Theory*, 2007.
[233] S. Shamai and B. Zaidel, “Enhancing the cellular downlink capacity via co-processing at the transmitting end,” in *Proceedings of IEEE Vehicular Technology Conference-Spring*, pp. 1745–1749, 2001.
[234] X. Shang, B. Chen, G. Kramer, and H. V. Poor, “Noisy-interference sum-rate capacity of parallel Gaussian interference channels,” *IEEE Transactions on Information Theory*, vol. 57, no. 1, pp. 210–226, 2011.
[235] X. Shang, B. Chen, and H. V. Poor, “Multiuser MISO interference channels with single-user detection: Optimality of beamforming and the achievable rate region,” *IEEE Transactions on Information Theory*, vol. 57, no. 7, pp. 4255–4273, 2011.
[236] C. Shannon, “A mathematical theory of communication,” *Bell System Technical Journal*, vol. 27, pp. 379–423, 623–656, 1948.
[237] C. Shannon, “Communication theory of secrecy systems,” *Bell System Technical Journal*, vol. 28, pp. 656–715, 1949.
[238] M. Sharif and B. Hassibi, “On the capacity of MIMO broadcast channels with partial side information,” *IEEE Transactions on Information Theory*, vol. 51, no. 2, pp. 506–522, 2005.
[239] C. Shen, T.-H. Chang, K.-Y. Wang, Z. Qiu, and C.-Y. Chi, “Distributed robust multicell coordinated beamforming with imperfect CSI: An ADMM approach,” *IEEE Transactions on Signal Processing*, vol. 60, no. 6, pp. 2988–3003, 2012.
[240] Z. Shen, R. Chen, J. Andrews, R. Heath, and B. Evans, “Low complexity user selection algorithms for multiuser MIMO systems with block diagonalization,” *IEEE Transactions on Signal Processing*, vol. 54, no. 9, pp. 3658–3663, 2006.
[241] M. B. Shenouda and T. Davidson, “Probabilistically-constrained approaches to the design of the multiple antenna downlink,” in *Proceedings of Asilomar Conference on Signals, Systems, and Computers*, pp. 1120–1124, 2008.

[242] M. B. Shenouda and T. Davidson, "Nonlinear and linear broadcasting with QoS requirements: Tractable approaches for bounded channel uncertainties," *IEEE Transactions on Signal Processing*, vol. 57, no. 5, pp. 1936–1947, 2009.

[243] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel," *IEEE Transactions on Signal Processing*, vol. 59, no. 9, pp. 4331–4340, 2011.

[244] H. Shirani-Mehr, G. Caire, and M. Neely, "MIMO downlink scheduling with non-perfect channel state knowledge," *IEEE Transactions on Communications*, vol. 58, no. 7, pp. 2055–2066, 2010.

[245] N. Sidiropoulos, T. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Transactions on Signal Processing*, vol. 54, no. 6, pp. 2239–2251, 2006.

[246] O. Simeone, N. Levy, A. Sanderovich, O. Somekh, B. Zaidel, H. Poor, and S. Shamai, "Cooperative wireless cellular systems: An information-theoretic view," *Foundations and Trends in Communications and Information Theory*, vol. 8, no. 1–2, pp. 1–177, 2012.

[247] O. Simeone, O. Somekh, H. V. Poor, and S. Shamai, "Downlink multicell processing with limited-backhaul capacity," *EURASIP Journal on Advances in Signal Processing*, 2009.

[248] H. Skjevling, D. Gesbert, and A. Hjørungnes, "Low-complexity distributed multibase transmission and scheduling," *EURASIP Journal on Advances in Signal Processing*, 2008.

[249] M. Slater, "Lagrange multipliers revisited," Technical Report 403, Cowles Commission Discussion Paper, Mathematics, 1950.

[250] B. Song, Y.-H. Lin, and R. Cruz, "Weighted max-min fair beamforming, power control, and scheduling for a MISO downlink," *IEEE Transactions on Signal Processing*, vol. 7, no. 2, pp. 464–469, 2008.

[251] E. Song, Q. Shi, M. Sanjabi, R. Sun, and Z.-Q. Luo, "Robust SINR-constrained MISO downlink beamforming: When is semidefinite programming relaxation tight?," in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, 2011.

[252] Q. Spencer, A. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 461–471, 2004.

[253] R. Stridh, M. Bengtsson, and B. Ottersten, "System evaluation of optimal downlink beamforming with congestion control in wireless communication," *IEEE Transactions on Wireless Communications*, vol. 5, no. 4, pp. 743–751, 2006.

[254] C. Studer, M. Wenk, and A. Burg, "MIMO transmission with residual transmit-RF impairments," in *Proceedings of ITG/IEEE Workshop on Smart Antennas (WSA)*, 2010.

[255] C. Studer, M. Wenk, and A. Burg, "System-level implications of residual transmit-RF impairments in MIMO systems," in *Proceedings of European Conference on Antennas and Propagation*, 2011.

[256] J. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optimization Methods and Software*, vol. 11–12, pp. 625–653, 1999.
[257] A. Tajer, N. Prasad, and X. Wang, "Robust linear precoder design for multi-cell downlink transmission," *IEEE Transactions on Signal Processing*, vol. 59, no. 1, pp. 235–251, 2011.
[258] L. Tanner, "Selecting a text-processing system as a qualitative multiple criteria problem," *European Journal on Operational Research*, vol. 50, no. 2, pp. 179–187, 1991.
[259] A. Tarighat, M. Sadek, and A. Sayed, "A multi user beamforming scheme for downlink MIMO channels based on maximizing signal-to-leakage ratios," in *Proceedings of International Conference on Acoustics, Speech, and Signal Processing*, pp. 1129–1132, 2005.
[260] A. Tarighat and A. Sayed, "Joint compensation of transmitter and receiver impairments in OFDM systems," *IEEE Transactions on Wireless Communications*, vol. 6, no. 1, pp. 240–247, 2007.
[261] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585–595, 1999.
[262] A. Tölli, M. Codreanu, and M. Juntti, "Soft handover in adaptive MMIMO-OFDM cellular system with cooperative processing," in *Proceedings of IEEE International Symposium on Personal Indoor and Mobile Radio Communications*, 2006.
[263] A. Tölli, M. Codreanu, and M. Juntti, "Cooperative MIMO-OFDM cellular system with soft handover between distributed base station antennas," *IEEE Transactions on Wireless Communications*, vol. 7, no. 4, pp. 1428–1440, 2008.
[264] A. Tölli, H. Pennanen, and P. Komulainen, "On the value of coherent and coordinated multi-cell transmission," in *Proceedings of IEEE International Conference on Communications*, 2009.
[265] A. Tölli, H. Pennanen, and P. Komulainen, "Decentralized minimum power multi-cell beamforming with limited backhaul signaling," *IEEE Transactions on Wireless Communications*, vol. 10, no. 2, pp. 570–580, 2011.
[266] D. Tomecki and S. Stanczak, "On feasible SNR region for multicast downlink channel: Two user case," in *Proceedings of International Conference on Acoustics, Speech, and Signal Processing*, 2010.
[267] N. Tripathi, J. H. Reed, and H. Vanlandingham, *Radio Resource Management in Cellular Systems*. Springer, 2001.
[268] M. Trivellato, F. Boccardi, and H. Huang, "On transceiver design and channel quantization for downlink multiuser MIMO systems with limited feedback," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 8, pp. 1494–1504, 2008.
[269] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.
[270] T.-L. Tung and K. Yao, "Optimal downlink power-control design methodology for a mobile radio DS-CDMA system," in *Proceedings of IEEE Workshop on Signal Processing Systems*, pp. 165–170, 2002.

[271] R. Tütüncü, K. Toh, and M. Todd, "Solving semidefinite-quadratic-linear programs using SDPT3," *Mathematical Programming*, vol. 95, no. 2, pp. 189–217, 2003.
[272] H. Tuy, "Global minimization of a difference of two convex functions," *Nonlinear Analysis and Optimization*, vol. 30, pp. 150–182, 1987.
[273] H. Tuy, *Convex Analysis and Global Optimization*. Kluwer Academic Publishers, 1998.
[274] H. Tuy, "Monotonic optimization: Problems and solution approaches," *SIAM Journal of Optimization*, vol. 11, no. 2, pp. 464–494, 2000.
[275] H. Tuy, F. Al-Khayyal, and P. Thach, "Monotonic optimization: Branch and cut methods," in *Essays and Surveys in Global Optimization*, (C. Audet, P. Hansen, and G. Savard, eds.), Springer US, 2005.
[276] W. Utschick and J. Brehmer, "Monotonic optimization framework for coordinated beamforming in multicell networks," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 1899–1909, 2012.
[277] S. Vavasis, "Complexity issues in global optimization: A survey," in *Handbook of Global Optimization*, pp. 27–41, Kluwer, 1995.
[278] M. Vázquez, A. Pérez-Neira, and M. Lagunas, "A unifying approach to transmit beamforming for the MISO interference channel," in *Proceedings of ITG Workshop on Smart Antennas (WSA)*, 2012.
[279] S. Venkatesan, A. Lozano, and R. Valenzuela, "Network MIMO: Overcoming intercell interference in indoor wireless systems," in *Proceedings of IEEE Asilomar Conference on Signals, Systems, and Computers*, pp. 83–87, 2007.
[280] L. Venturino, N. Prasad, and X. Wang, "Coordinated linear beamforming in downlink multi-cell wireless networks," *IEEE Transactions on Wireless Communications*, vol. 9, no. 4, pp. 1451–1461, 2010.
[281] S. Verdú, *Multiuser Detection*. Cambridge University Press, 1998.
[282] E. Visotsky and U. Madhow, "Optimum beamforming using transmit antenna arrays," in *Proceedings of IEEE Vehicular Technology Conference*, pp. 851–856, 1999.
[283] P. Viswanath and D. Tse, "Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality," *IEEE Transactions on Information Theory*, vol. 49, no. 8, pp. 1912–1921, 2003.
[284] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1277–1294, 2002.
[285] B. Vojčić and W. Jang, "Transmitter precoding in synchronous multiuser communications," *IEEE Transactions on Communications*, vol. 46, no. 10, pp. 1346–1355, 1998.
[286] N. Vučić and H. Boche, "Robust QoS-constrained optimization of downlink multiuser MISO systems," *IEEE Transactions on Signal Processing*, vol. 57, no. 2, pp. 714–725, 2009.
[287] S. Wagner, R. Couillet, M. Debbah, and D. Slock, "Large system analysis of linear precoding in MISO broadcast channels with limited feedback," *IEEE Transactions on Information Theory*, vol. 58, no. 7, pp. 4509–4537, 2012.

[288] J. Wallace and M. Jensen, “Measured characteristics of the MIMO wireless channel,” in *Proceedings on IEEE Vehicular Technology Conference-Fall*, pp. 2038–2042, 2001.

[289] K.-Y. Wang, T.-H. Chang, C.-Y. C. W.-K. Ma, and A. So, “Probabilistic SINR constrained robust transmit beamforming: A Bernstein-type inequality based conservative approach,” in *Proceedings on IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 3080–3083, 2011.

[290] R. Weber, A. Garavaglia, M. Schulist, S. Brueck, and A. Dekorsy, “Self-organizing adaptive clustering for cooperative multipoint transmission,” in *Proceedings of IEEE Vehicular Technology Conference-Spring*, 2011.

[291] P. Weeraddana, M. Codreanu, S. Joshi, and M. Latva-aho, “Multicell downlink weighted sum-rate maximization: A distributed approach,” in *Proceedings of Asilomar Conference on Signals, Systems, and Computers*, pp. 1368–1375, 2011.

[292] P. Weeraddana, M. Codreanu, M. Latva-aho, and A. Ephremides, “Weighted sum-rate maximization for a set of interfering links via branch and bound,” *IEEE Transactions on Signal Processing*, vol. 59, no. 8, pp. 3977–3996, 2011.

[293] P. Weeraddana, M. Codreanu, M. Latva-aho, A. Ephremides, and C. Fischione, “Weighted Sum-Rate Maximization in Wireless Networks: A Review,” *Foundations and Trends in Networking*, vol. 6, no. 1–2, pp. 1–163, 2012.

[294] W. Weichselberger, M. Herdin, H. Özcelik, and E. Bonek, “A stochastic MIMO channel model with joint correlation of both link ends,” *IEEE Transactions on Wireless Communications*, vol. 5, no. 1, pp. 90–100, 2006.

[295] H. Weingarten, Y. Steinberg, and S. Shamai, “The capacity region of the Gaussian multiple-input multiple-output broadcast channel,” *IEEE Transactions on Information Theory*, vol. 52, no. 9, pp. 3936–3964, 2006.

[296] A. Wiesel, Y. Eldar, and S. Shamai, “Linear precoding via conic optimization for fixed MIMO receivers,” *IEEE Transactions on Signal Processing*, vol. 54, no. 1, pp. 161–176, 2006.

[297] A. Wiesel, Y. Eldar, and S. Shamai, “Zero-forcing precoding and generalized inverses,” *IEEE Transactions on Signal Processing*, vol. 56, no. 9, pp. 4409–4418, 2008.

[298] A. Wyner, “The wire-tap channel,” *Bell System Technical Journal*, vol. 54, pp. 1355–1387, 1975.

[299] A. Wyner, “Shannon-theoretic approach to a Gaussian cellular multiple-access channel,” *IEEE Transactions on Information Theory*, vol. 40, no. 6, pp. 1713–1727, 1994.

[300] J. Xu, J. Zhang, and J. Andrews, “On the accuracy of the Wyner model in cellular networks,” *IEEE Transactions on Wireless Communications*, vol. 10, no. 9, pp. 3098–3109, 2011.

[301] Y. Xu, T. Le-Ngoc, and S. Panigrahi, “Global concave minimization for optimal spectrum balancing in multi-user DSL networks,” *IEEE Transactions on Signal Processing*, vol. 56, no. 7, pp. 2875–2885, 2008.

[302] J. Yang, E. Björnson, and M. Bengtsson, “Receive beamforming design based on a multiple-state interference model,” in *Proceedings of IEEE International Conference on Communications*, 2011.

[303] W. Yang and G. Xu, "Optimal downlink power assignment for smart antenna systems," in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 3337–3340, 1998.

[304] R. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 7, pp. 1341–1347, 1995.

[305] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 528–541, 2006.

[306] K. Yu, M. Bengtsson, B. Ottersten, D. McNamara, P. Karlsson, and M. Beach, "Modeling of wide-band MIMO radio channels based on NLoS indoor measurements," *IEEE Transactions on Vehicular Technology*, vol. 53, no. 3, pp. 655–665, 2004.

[307] W. Yu, T. Kwon, and C. Shin, "Multicell coordination via joint scheduling, beamforming and power spectrum adaptation," in *Proceedings of IEEE INFOCOM*, 2011.

[308] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 2646–2660, 2007.

[309] S. Zabell, "Alan Turing and the central limit theorem," *The American Mathematical Monthly*, vol. 102, no. 6, pp. 483–494, 1995.

[310] R. Zakhour and D. Gesbert, "Coordination on the MISO interference channel using the virtual SINR framework," in *Proceedings of ITG Workshop on Smart Antennas (WSA)*, 2009.

[311] R. Zakhour and D. Gesbert, "Distributed multicell-MISO precoding using the layered virtual SINR framework," *IEEE Transactions on Wireless Communications*, vol. 9, no. 8, pp. 2444–2448, 2010.

[312] R. Zakhour and D. Gesbert, "Team decision for the cooperative MIMO channel with imperfect CSIT sharing," in *Proceedings of Information Theory and Applications Workshop*, 2010.

[313] R. Zakhour and D. Gesbert, "Optimized data sharing in multicell MIMO with finite backhaul capacity," *IEEE Transactions on Signal Processing*, vol. 59, no. 12, pp. 6102–6111, 2011.

[314] J. Zander, "Performance of optimum transmitter power control in cellular radio systems," *IEEE Transactions on Vehicular Technology*, vol. 41, no. 1, pp. 57–62, 1992.

[315] J. Zander and M. Frodigh, "Comment on "Performance of optimum transmitter power control in cellular radio systems"," *IEEE Transactions on Vehicular Technology*, vol. 43, no. 3, p. 636, 1994.

[316] J. Zander and S.-L. Kim, *Radio Resource Management for Wireless Networks*. Artech House, 2001.

[317] A. Zanella, M. Chiani, and M. Win, "Performance of MIMO MRC in correlated rayleigh fading environments," in *Proceedings of IEEE Vehicular Technology Conference-Spring*, pp. 1633–1637, 2005.

[318] B. Zarikoff and J. Cavers, "Coordinated multi-cell systems: Carrier frequency offset estimation and correction," *IEEE Journal on Selected Areas in Communications*, vol. 28, no. 9, pp. 1490–1501, 2010.

[319] P. Zetterberg, "Experimental investigation of TDD reciprocity-based zero-forcing transmit precoding," *EURASIP Journal on Advances in Signal Processing*, January 2011.
[320] P. Zetterberg and B. Ottersten, "The spectrum efficiency of a base station antenna array system for spatially selective transmission," *IEEE Transactions on Vehicular Technology*, vol. 44, no. 3, pp. 651–660, 1995.
[321] H. Zhang and H. Dai, "Cochannel interference mitigation and cooperative processing in downlink multicell multiuser MIMO networks," *EURASIP Journal on Wireless Communications and Networking*, vol. 2, pp. 222–235, 2004.
[322] H. Zhang, N. Mehta, A. Molisch, J. Zhang, and H. Dai, "Asynchronous interference mitigation in cooperative base station systems," *IEEE Transactions on Wireless Communications*, vol. 7, no. 1, pp. 155–165, 2008.
[323] J. Zhang, R. Chen, J. Andrews, A. Ghosh, and R. Heath, "Networked MIMO with clustered linear precoding," *IEEE Transactions on Wireless Communications*, vol. 8, no. 4, pp. 1910–1921, 2009.
[324] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Transactions on Evolutionary Computation*, vol. 11, no. 6, pp. 712–731, 2007.
[325] R. Zhang and S. Cui, "Cooperative interference management with MISO beamforming," *IEEE Transactions on Signal Processing*, vol. 58, no. 10, pp. 5450–5458, 2010.
[326] R. Zhang, Y.-C. Liang, and S. Cui, "Dynamic resource allocation in cognitive radio networks," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 102–114, 2010.
[327] X. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 88–102, 2008.
[328] G. Zheng, K.-K. Wong, and T.-S. Ng, "Robust linear MIMO in the downlink: A worst-case optimization with ellipsoidal uncertainty regions," *EURASIP Journal on Advances in Signal Processing*, 2008.
[329] G. Zheng, K.-K. Wong, and T.-S. Ng, "Throughput maximization in linear multiuser MIMO-OFDM downlink systems," *IEEE Transactions on Vehicular Communications*, vol. 57, no. 3, pp. 1993–1998, 2008.
[330] J. Zyren and W. McCoyh, "Overview of the 3GPP long term evolution physical layer," in *Freescale Semiconductor*, 2007.