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# Information-Theoretic Foundations of Mismatched Decoding

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# Foundations and Trends® in Communications and Information Theory

*Published, sold and distributed by:*

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The preferred citation for this publication is

J. Scarlett, A. Guillén i Fàbregas, A. Somekh-Baruch and A. Martinez. *Information-Theoretic Foundations of Mismatched Decoding*. Foundations and Trends® in Communications and Information Theory, vol. 17, no. 2–3, pp. 149–401, 2020.

ISBN: 978-1-68083-713-1

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## Volume 17, Issue 2–3, 2020

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Foundations and Trends® in Communications and Information Theory, 2020, Volume 17, 4 issues. ISSN paper version 1567-2190. ISSN online version 1567-2328 . Also available as a combined paper and online subscription.

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# Information-Theoretic Foundations of Mismatched Decoding

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## ABSTRACT

Shannon’s channel coding theorem characterizes the maximal rate of information that can be reliably transmitted over a communication channel when optimal encoding and decoding strategies are used. In many scenarios, however, practical considerations such as channel uncertainty and implementation constraints rule out the use of an optimal decoder. The mismatched decoding problem addresses such scenarios by considering the case that the decoder cannot be optimized, but is instead fixed as part of the problem statement. This problem is not only of direct interest in its own right, but also has close connections with other long-standing theoretical problems in information theory.

In this monograph, we survey both classical literature and recent developments on the mismatched decoding problem, with an emphasis on achievable random-coding rates for memoryless channels. We present two widely-considered

achievable rates known as the generalized mutual information (GMI) and the LM rate, and overview their derivations and properties. In addition, we survey several improved rates via multi-user coding techniques, as well as recent developments and challenges in establishing upper bounds on the mismatch capacity, and an analogous mismatched encoding problem in rate-distortion theory. Throughout the monograph, we highlight a variety of applications and connections with other prominent information theory problems.

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## Notation

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(Introduced in Section 1)

$\mathcal{X}, \mathcal{Y}$	Input and output alphabets
$m, \hat{m}$	Message and its estimate
$M$	Number of codewords
$n$	Block length
$R$	Coding rate
$W, W^n$	Channel transition law and its $n$ -letter extension
$q, q^n$	Decoding metric and its $n$ -letter extension
$\mathcal{C}$	Codebook
$\mathbf{x}^{(m)}$	$m$ -th codeword
$p_e$	Average error probability
$p_{e,\max}$	Maximal error probability
$C_M$	Mismatch capacity
$C$	Matched capacity

(Introduced in Section 2)

$H_2$	Binary entropy function
$\mathbf{X}, \mathbf{Y}$	Transmitted codeword and received sequence
$\bar{\mathbf{X}}$	Non-transmitted codeword
$P_{\mathbf{X}}$	Codeword distribution for random coding
$\hat{P}_{\mathbf{x}}$	Empirical distribution of $\mathbf{x}$
$I_{\text{GMI}}, I_{\text{LM}}$	Generalized mutual information (GMI) and LM rate
$C_{\text{GMI}}, C_{\text{LM}}$	Rates with optimized input distributions
$Q_X$	Input distribution
$\mathcal{P}$	Set of all distributions on a given alphabet
$\mathcal{P}_n$	Set of all empirical distributions on a given alphabet
$\tilde{P}_{XY}$	Auxiliary distribution in primal rates
$s, a$	Auxiliary parameters in dual rates
$b$	Auxiliary function on the output alphabet
$\mathcal{T}^n$	Type class
$Q_{X,n}$	Type approximating $Q_X$
$\bar{p}_e$	Random-coding error probability
$i_s^n$	Information density quantity used in proofs
$C_{\text{GMI}}^{(k)}, C_{\text{LM}}^{(k)}$	Multi-letter achievable rates
$C_{\text{GMI}}^{(\infty)}, C_{\text{LM}}^{(\infty)}$	Limiting multi-letter achievable rates

## (Introduced in Section 3)

$c, \Gamma$	System cost function and threshold
$a, a_l$	Auxiliary cost functions
$c^n, a^n, a_l^n$	Additive multi-letter extensions of cost functions
$\phi_c, \phi_a, \phi_l$	Means of cost functions
$\Omega_n$	Normalizing constant for cost-constrained ensemble
$\mathcal{D}_n$	Constraint set for cost-constrained ensemble
$I'_{LM}$	Fixed-cost LM rate
$r_l, \bar{r}_l$	Auxiliary parameters for cost-constrained ensemble
$L$	Number of auxiliary costs
$\mu, \sigma^2$	Noise mean and variance in additive channels

## (Introduced in Section 4)

$\Pi_X, \Pi_X^n$	Source distribution and $n$ -letter extension
$\hat{x}^{(m)}$	Codeword in rate-distortion theory
$d_0, d_1$	Encoding metric and true distortion measure
$d_0^n, d_1^n$	Additive $n$ -letter extensions of distortion functions
$D_1, D_1^*$	Distortion threshold and distortion-rate function
$Q_{\hat{X}}$	Auxiliary distribution in rate-distortion theory
$\overline{D}_1, \overline{D}_1^{(k)}$	Achievable distortion and multi-letter version
$\tilde{\mathcal{P}}$	Constraint set in achievable distortion expression
$d, D$	Distortion measure and level when $d_0 = d_1$
$R_{\text{Matched}}^*, D_{\text{Matched}}^*$	Matched rate-distortion function and distortion-rate function
$D_{\min}, D_{\prod}$	Extreme values of distortion level
$\overline{R}_{\text{iid}}, \overline{R}_{\text{cc}}$	Random coding rate-distortion functions
$\sigma^2$	Source variance

## (Introduced in Section 5)

$M_1, M_2$	Multiple-access channel codebook sizes
$R_1, R_2$	Multiple-access channel rates
$Q_1, Q_2$	Multiple-access channel input distributions
$P_{\mathbf{X}_1}, P_{\mathbf{X}_2}$	Multiple-access channel codeword distributions

## (Introduced in Section 6)

$M_0, M_1, \{M_{1u}\}$	Superposition coding codebook sizes
$R_0, R_1, \{R_{1u}\}$	Superposition coding rates
$\{n_u\}$	Refined superposition coding sub-block lengths
$Q_{UX}$	Superposition coding input distribution
$P_{UX}$	Superposition coding codeword distribution
$\mathcal{U}$	Auxiliary alphabet for superposition coding
$U$	Auxiliary codeword for superposition coding
$p_{e,\nu}, \bar{p}_{e,\nu}$	Multi-user coding error probabilities

## (Introduced in Section 7)

$E_r^{\text{iid}}, E_0^{\text{iid}}$	i.i.d. exponent and Gallager function
$E_r^{\text{cc}}, E_0^{\text{cc}}$	Constant-composition exponent and Gallager function
$E_r^{\text{cost}}, E_0^{\text{cost}}$	Cost-constrained exponent and Gallager function
$E_{\text{ex}}^{\text{cc}}, E_x^{\text{cc}}$	Constant-composition expurgated exponent and Gallager function
$\rho$	Dual error exponent parameter
$E_{\text{CK}}$	Csiszár–Körner exponent
$E_{\text{RGV}}$	Random Gilbert–Varshamov exponent
$d, \Delta$	RGV distance function and parameter

## (Introduced in Section 8)

$\epsilon_k$	Error probability with block length $k$
$\eta_k$	Minimal difference of $k$ -letter log-metric values
$q_{\max}$	Maximal metric value
$q_{\max}^k(y^k)$	Maximal metric value for fixed output sequence
$B$	Upper bound on $ \log q(x, y) $
$\mathcal{A}_k$	Output vectors with a unique metric maximizer
$\Phi_k$	Conditional probability of random codeword exceeding metric value
$\mathcal{X}_q^*$	Set of inputs with maximal metric difference
$\mathcal{M}_{\max}, \mathcal{M}_{\max,n}$	Set of maximal conditional joint distributions and type-based variant
$\overline{R}$	Single-letter mismatch capacity upper bound
$\mathcal{G}_x$	Bipartite graph associated with two channels
$\mathcal{E}_x$	Edge set associated with two channels

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