Dealing with Endogeneity in Regression Models with Dynamic Coefficients

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Dealing with Endogeneity in Regression Models with Dynamic Coefficients

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Abstract

The purpose of this monograph is to present a unified econometric framework for dealing with the issues of endogeneity in Markov-switching models and time-varying parameter models, as developed by Kim (2004, 2006, 2009), Kim and Nelson (2006), Kim et al. (2008), and Kim and Kim (2009). While Cogley and Sargent (2002), Primiceri (2005), Sims and Zha (2006), and Sims et al. (2008) consider estimation of simultaneous equations models with stochastic coefficients as a system, we deal with the LIML (limited information maximum likelihood) estimation of a single equation of interest out of a simultaneous equations model. Our main focus is on the two-step estimation procedures based on the control function approach, and we show how the problem of generated regressors can be addressed in second-step regressions.

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Contents

1

Introduction

Consider the following regression model with dynamic coefficients:

$$y_t = x_t'\beta_t + e_t, \quad e_t \sim N(0, \sigma_{e,t}^2), \tag{1.1}$$

where y_t is 1×1 and x_t is a $K \times 1$ vector of regressors. The $K \times 1$ vector of regression coefficients β_t is stochastic and time-dependent. Depending on the assumptions on the stochastic nature of β_t, we have either a Markov-switching model or a time-varying parameter model. For a time-varying parameter model, we conventionally assume that β_t is subject to a continuous shock. For example, we assume that:

$$\beta_t = \beta_{t-1} + v_t, \quad v_t \sim i.i.d.N(0, Q), \tag{1.2}$$

where the $K \times K$ matrix Q is the variance–covariance matrix of v_t. For a Markov-switching model, β_t is subject to a discrete shock. For example, we assume that β_t is dependent upon a first-order, J-state Markov-switching process S_t in the following way:

$$\beta_t = \beta_1 S_{1,t}^{\dagger} + \beta_2 S_{2,t}^{\dagger} + \ldots + \beta_J S_{J,t}^{\dagger}, \tag{1.3}$$

$$S_{j,t}^{\dagger} = \begin{cases} 1, & \text{if } S_t = j; \quad j = 1, 2, \ldots J \\ 0, & \text{otherwise,} \end{cases} \tag{1.4}$$

where the transitional dynamics of S_t are defined as:

$$Pr[S_t = j | S_{t-1} = i] = p_{ji}, \quad \sum_{j=1}^{J} p_{ji} = 1. \tag{1.5}$$

Time-varying parameter models, which date to Cooley and Prescott (1973, 1976), Rosenberg (1973), Sarris (1973), and Markov regime-switching models, originally introduced by Goldfeld and Quandt (1973) and further extended by Hamilton (1989), and have been widely used in modeling instability in economic relations.[1] With a growing body of recent empirical evidence on widespread instability in macroeconomic relations (Diebold, 1998; Stock and Watson, 1998; Perron and Qu, 2007), the importance of these time series models with dynamic coefficients has been recognized by more macroeconomists and financial economists than ever before.

However, almost all applications of these models so far have been limited to the cases of exogenous regressors or exogenous coefficients, with the following assumptions:

$$Assumption\ \#1: \quad E(e_t | x_t) = 0 \tag{1.6}$$

$$Assumption\ \#2: \quad E(e_t | \beta_t) = 0. \tag{1.7}$$

When either one of the above assumptions is violated, inferences about the model based on the conventional Kalman (1960) filter or the conventional Hamilton (1989) filter are invalid. In particular, in the case of endogenous regressors where Assumption #1 is violated, one is tempted to employ the conventional two-step procedure. That is, defining z_t to be a vector of instrumental variables, one may regress x_t on z_t to get $\hat{x}_t$, the fitted value of x_t, in the first step and then estimate Equation (1.1) by replacing x_t with $\hat{x}_t$ in the second step. However, this conventional two-step procedure is problematic when the regression coefficients are stochastic. For example, even in the case in which e_t in Equation (1.1) is independent and identically distributed (i.i.d.), the disturbance term in the second-step regression is heteroscedastic. Ignoring this heteroscedasticity would result in inefficiency in the two-step

[1] For a comprehensive review of these models, readers are referred to Kim and Nelson (1999).

estimation of the model. A more serious issue is that, unlike the case of constant coefficients, it is not easy to solve the problem of generated regressors in calculating the standard errors of the coefficient estimators.

The purpose of this monograph is to present a unified econometric framework for dealing with the issues of endogeneity in Markov-switching models and time-varying parameter models, as developed by Kim (2004, 2006, 2009), Kim and Nelson (2006), Kim et al. (2008), and Kim and Kim (2009). Note that, while Cogley and Sargent (2002), Primiceri (2005), Sims and Zha (2006), and Sims et al. (2008) consider estimation of simultaneous equations models with stochastic coefficients as a system, we focus on the LIML (limited information maximum likelihood) estimation of a single equation of interest out of a simultaneous equations model.

The control function approach, which is an econometric method used to correct for biases that arise as a consequence of selection or endogeneity, will be the main tool in dealing with the problem of endogeneity throughout this article. While the approach has been extensively applied to the sample-selection models and disequilibrium models in the microeconometrics literature, its application in the time-series econometrics literature is relatively new. The basic idea behind the control function is to model the dependence of the disturbance term on the endogenous variables in a way that allows us to construct a function such that, conditional on the function, the endogeneity problem in the regression equation of interest disappears. For example, in the case of a linear regression with constant coefficients, the two-step estimation procedure based on the control function approach proceeds as follows. In the first step, the residuals of the reduced-form equations for the endogenous regressors are estimated. Then, in the second step, the primary equation of interest is estimated with these residuals included as additional regressors.

The outline of this monograph is as follows. In Section 2, we review the basic issues associated with the control function approach, which is the main tool for dealing with endogeneity in this monograph. We investigate these issues within the framework of constant regression coefficients. In Section 3, we consider estimation of Markov-switching models with endogenous regressors, by dropping Assumption #1 in Equation (1.6) but maintaining Assumption #2 in Equation (1.7). Section 4 deals with estimation of a Markov-switching model in which Assumption #1 is maintained but Assumption #2 is dropped. In this model, while the regressors are exogenous or predetermined, the Markov-switching coefficients are correlated with regression disturbances. The issues of endogeneity within the time-varying parameter models are discussed in Section 5. In Sections 3–5, we will see how the basic Hamilton (1989) filter and the basic Kalman (1960) filter can be modified to deal with different types of endogeneity. Furthermore, we will see how the problem of generated regressors in the two-step procedure can be addressed, in light of Pagan (1984) and the results in Section 2. Finally, Section 6 provides concluding remarks.

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