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Network-Based Analysis of Rotor Angle Stability of Power Systems

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ABSTRACT

Rotor angle stability refers to the ability of synchronous machines in a power system to remain in synchronism after a disturbance. It is one of the basic requirements for secure operation of electric power systems. Traditional analysis methods for rotor angle stability are oriented to node dynamics, especially the impact of generator modeling and parameters, while power network parameters are simply treated as some coefficients in the system dynamical models. Thanks to the progress on graph theory and network science, there is an emerging trend of investigating the connections between power network structures and system dynamic behaviors. This monograph surveys the network-based results on rotor angle stability in both early and recent years, where the role of power network structure is elaborated. It reveals that rotor angle dynamics essentially link to some graph quantities (e.g., Laplacian matrix, cutset, effective resistance) defined over the underlying power network structure. New theories for angle stability are developed using advanced graph theory tools tailored for power networks. These results provide novel solutions to some important problems that have not been well addressed in the traditional node-based

studies, such as the impact of those lines with large angle differences on stability, cutset vulnerability assessment and convexification of stability constrained optimal power flow. The purpose of this monograph is to establish a network-based paradigm that sheds new light on the mechanism of angle stability under small and large disturbances.

1

Introduction

Electric power systems are a critical infrastructure in modern society, where the users (loads) get electricity supply from generators via the power transfer over the underlying power network. Rotor angle stability is an issue of fundamental importance to power system planning, operation and control. Conventionally, the studies of rotor angle stability problems are oriented to the role of node-side factors, by mainly considering the generator models. Following motivation from the subject of complex dynamical networks, this monograph introduces a network-based paradigm for angle stability analysis that focuses on the role of power network structure.

1.1 Concept of Rotor Angle Stability

Let us begin by recalling the definition and classification of power system stability. Power system stability refers to the ability of an electric power system to regain a state of operating equilibrium point after being subjected to a disturbance (Kundur *et al.*, 2004). As shown in Fig. 1.1, it can be further classified into angle stability, voltage stability and frequency stability according to the main system variables in which instability can be observed.

As the focus of this monograph, rotor angle stability refers to the ability of synchronous machines (mainly synchronous generators) to remain in synchronism after being subjected to a disturbance (Kundur *et al.*, 2004). Angle stability is a short-term issue mainly concerning generator swing dynamics. The time frame of interest in the studies of angle stability is usually 3 to 20 seconds following the disturbance (Kundur *et al.*, 2004). By the nature of disturbances, angle stability can be further classified into small-disturbance angle stability and transient stability. The term “transient stability” is equivalent to “large-disturbance angle stability”, but the former term is more commonly used in the power systems community. The rotor angle dynamics are nonlinear, e.g., see (1.1) in the following. The linearized system models are applicable to the study of small-disturbance angle stability since it relates to the system behavior following sufficiently small disturbances at an equilibrium point. In contrast, the nonlinearity must be taken into account for transient stability problems since the system states may severely deviate from the equilibrium point or the equilibrium point may be changed after large disturbances.

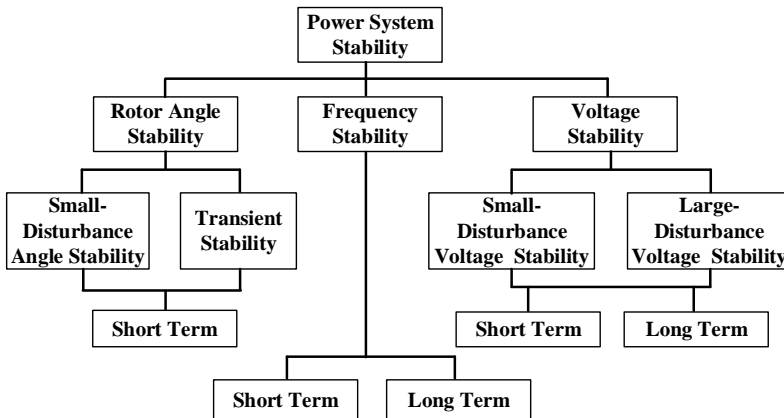


Figure 1.1: Classification of power system stability (Kundur *et al.*, 2004).

Rotor angle stability depends on whether each synchronous generator can maintain or restore the equilibrium between its electromagnetic torque and mechanical torque during the post-fault electromechanical

oscillations. At the pre-fault steady state, the system operates at an equilibrium point where all generators get their electromagnetic torques balanced with mechanical torques and the rotor speeds of all generators remain constants. When the system is disturbed, this balance is broken and generator rotors start to accelerate or decelerate according to the law of motion. The well-known swing equation captures the most fundamental characteristics of rotor dynamics (Kundur, 1994). Here we use the following version of dynamical model designed to preserve network structure (Bergen and Hill, 1981)

$$\begin{aligned} M_i \ddot{\theta}_i + D_i \dot{\theta}_i &= P_i - \sum_{j \in \mathcal{N}_i} |V_i| |V_j| B_{ij} \sin(\theta_i - \theta_j), \quad i \in \mathcal{V}_G \\ D_i \dot{\theta}_i &= P_i - \sum_{j \in \mathcal{N}_i} |V_i| |V_j| B_{ij} \sin(\theta_i - \theta_j), \quad i \in \mathcal{V}_L \end{aligned} \quad (1.1)$$

where $\mathcal{V}_G, \mathcal{V}_L$ denote the set of generator buses and load buses; $\theta_i, \dot{\theta}_i, |V_i|$ respectively denote the rotor angle, angular frequency (a representation of rotor speed) and voltage magnitude of bus i ; M_i and D_i represent the moment of inertia of a generator and frequency coefficient; P_i represents the node power injection; $j \in \mathcal{N}_i$ means bus i and bus j are directly connected by a transmission line; and B_{ij} represents the line coupling strength between bus i and bus j . A detailed explanation of these notations and derivation of (1.1) will be given in Chapter 2.

From (1.1) we observe that the rotor acceleration (or deceleration) of generator i will be restrained by the consequent increment (or decrement) in the term $|V_i| |V_j| B_{ij} \sin(\theta_i - \theta_j)$, which helps the system to regain a state of equilibrium if the disturbance is not sufficiently severe. However, this restorative effect is rather limited since $|V_i| |V_j| B_{ij} \sin(\theta_i - \theta_j)$ has a bounded value. If the disturbance pushes the system state beyond a certain limit, the system will become unstable as some generators continue to accelerate or decelerate their rotor speeds and eventually lose synchronism with other generators. Although it is not hard to have an apparent understanding of angle stability from the above discussion, the exact mechanism is highly complex due to the nonlinear system behavior.

1.2 Traditional Analysis Methods for Rotor Angle Stability

There are two mainstream methods for rotor angle stability assessment: time domain simulation and the so-called direct method. Time domain simulation applies to much higher-order models than (1.1) that contain more details of the generators and loads. It gives a comprehensive description of the system response curves during the pre-fault, fault-on and post-fault periods. Nevertheless, time domain simulation has been criticized for being computationally demanding and incapable of providing the mechanism of stability. On the other hand, the direct method, as inferred from its name, can assess stability without simulating the post-fault dynamics. The direct method is based on the energy functions (or Lyapunov functions) for power system models. Although the direct method usually considers simpler system models than time domain simulation, it is able to not only judge stability but also give an estimation of the stability region. Unlike those numerical techniques for stability region estimation such as the normal form analysis (Saha *et al.*, 1997) and reachable set computation (Jin *et al.*, 2005; Althoff and Krogh, 2014; El-Guindy *et al.*, 2017), the direct method provides an energy-based explanation to the mechanism of stability and facilitates the countermeasures for preventing instability. In the following we briefly review those aspects of the energy functions and direct methods that are more related to the theory-oriented theme of this monograph.

In general, an energy function for a power system is the sum of energies contained in system components including generators, loads, transmission lines and possibly other devices. The first energy function used for multi-machine transient stability analysis dates back to Magnusson (1947) in 1940s, which is derived from a *network reduced model* of power systems. Since the 1980s, structure preserving energy functions have emerged with the establishment of power system *structure preserving models*, e.g., the Bergen-Hill model (1.1). The early versions of energy functions consider the energies from classic second-order generators, constant-power loads and transmission lines. So far the energy functions have been developed into a big family considering more detailed description of system components, such as generator flux decay (Tsolas *et al.*, 1985; Bergen *et al.*, 1986), automatic voltage

regulators (Miyagi and Bergen, 1986), voltage dependent loads and reactive powers (Narasimhamurthi and Musavi, 1984; Hiskens and Hill, 1989). Extensive reviews of energy functions for various power system models are presented in, e.g., Pai (1981), Pai (1989) and Padiyar (2013). Normally the energy functions are constructed by directly taking the physical parameters of generators, loads and lines as the coefficients for the respective energy terms. Some recent works have attempted to tune the energy function expression by optimization techniques including sum-of-squares programming and semi-definite programming (e.g., see Anghel *et al.* (2013), Han *et al.* (2016), and Vu and Turitsyn (2016)).

With the energy function, the direct method checks stability by comparing the system energy with a threshold value called the critical energy. A power system has zero energy at the pre-fault equilibrium and growing energy during the fault-on period. If the energy at the time of fault clearance is less than the critical energy, then the state variable lies in the stability region, i.e., the post-fault system will be stable. Also the difference between the two energy values gives a measure of stability margin. Various ways for determining the critical energy have been developed mainly based on the geometry of the stability region, including the closest unstable equilibrium point (UEP) method (Prabhakara and El-Abiad, 1975; Chiang and Thorp, 1987), potential energy boundary surface (PEBS) method (Kakimoto *et al.*, 1984; Chiang *et al.*, 1988), controlling UEP method (Athay *et al.*, 1979; Chiang *et al.*, 1987; Fouad and Vittal, 1991) and boundary of stability region-based controlling UEP (BCU) method (Chiang *et al.*, 1994; Chiang and Chu, 1995). Readers can refer to Chiang (2011) for a comprehensive summary of those methods. Moreover, the extended equal area criterion (EEAC) gives another viewpoint for critical energy evaluation (Xue *et al.*, 1988; Xue *et al.*, 1989; Belhomme *et al.*, 1993). Via projection analysis of all post-fault trajectories, it simplifies a generic power system into an equivalent one-machine-infinite-bus system so that the critical energy is given by the classic concept of “deceleration area”.

Overall, most research on angle stability have been oriented to the influence of node-side factors, especially the modeling and parameters of synchronous generators. For instance, many efforts have been put into including more detailed generator dynamics into energy functions

(Padiyar, 2013) or quantifying the impact of generator parameters by sensitivity methods (e.g., see Nguyen and Pai (2003), Yuan and Fang (2009), Hou and Vittal (2013), Sharma *et al.* (2018), and Mishra *et al.* (2020)). This is probably due to the fact that rotor angle stability is conceptually linked to generator behaviors. On the other hand, the influence of power network structure in stability has been paid much less attention. Power network parameters (e.g., B_{ij} in (1.1)) are simply taken as some coefficients in the system models. Although some early works made an attempt to figure out the role of network structure (Bergen and Hill, 1981; Hill and Bergen, 1982; Chandrashekar and Hill, 1983), the lack of advanced graph theory tools prevented a further development.

1.3 Motivation of Network-Based Stability Analysis

Network-based stability analysis is motivated by revisiting power systems as dynamical networks (Hill and Chen, 2006). As background knowledge, we introduce below some basics of dynamical networks which has been a hot topic in the control systems community with much attention paid to network structural features. Consider a dynamical network with n coupled nodes (also called agents), the mathematical model of which is given by

$$\dot{x}_i = f_i(x_i) + \sum_{j \in \mathcal{N}_i} a_{ij}(x_j - x_i), \quad i = 1, 2, \dots, n \quad (1.2)$$

where x_i denotes the state of node i and $j \in \mathcal{N}_i$ means node i gets information from node j . The state of each node in the dynamical network evolves according to its local dynamics described by $f_i(x_i)$ and interaction with other nodes described by the underlying physical or communication network. Note that the node state can be a multi-dimension vector in generic dynamical network models (Strogatz, 2001). Here we use the simplified model (1.2) with scalar node states just for the convenience of illustration. System (1.2) can be rewritten into the compact form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) - \mathbf{L}_{\mathcal{G}}\mathbf{x} \quad (1.3)$$

where the vector \mathbf{x} collects all node states, the function vector $\mathbf{f}(\cdot)$ collects all functions $f_i(\cdot)$, and \mathbf{L}_G is the graph Laplacian matrix describing the underlying physical or communication network (a detailed definition of which will be given in Chapter 3).

A major concern for dynamical networks is to determine the conditions for reaching a synchronization or consensus, i.e.,

$$\lim_{t \rightarrow \infty} x_1(t) = \lim_{t \rightarrow \infty} x_2(t) = \cdots = \lim_{t \rightarrow \infty} x_n(t). \quad (1.4)$$

In case of identical nodes (i.e., all functions $f_i(\cdot)$ take the same expression), a well-known criterion (Wang and Chen, 2002; Wang and Chen, 2003) says that system (1.2) reaches a consensus if $\lambda_2(\mathbf{L}_G) \geq d$, where $\lambda_2(\mathbf{L}_G)$ is the algebraic connectivity of the corresponding graph (i.e., the second smallest eigenvalue of the Laplacian matrix) and d is a constant determined by function $f_i(\cdot)$. The synchronization conditions for dynamical networks with non-identical nodes do not take as neat form as the case with identical nodes. But some results still indicate that the algebraic connectivity has a decisive effect on reaching a consensus in dynamical networks with non-identical nodes (Zhao *et al.*, 2010; Zhao *et al.*, 2011).

In addition, consider the further simplified system below

$$\dot{\mathbf{x}} = -\mathbf{L}_G \mathbf{x} \quad (1.5)$$

which is usually called the network consensus protocol. System (1.5) reaches a consensus if and only if the Laplacian matrix \mathbf{L}_G is positive semi-definite (PSD) with only one zero eigenvalue (Olfati-Saber *et al.*, 2007). Moreover, other synchronization conditions for many variants of (1.2) or (1.5) have been established, which all show that network structural properties are crucial to the behavior of dynamical networks (e.g., see Olfati-Saber and Murray (2004), Moreau (2004), Ren and Beard (2005), Li *et al.* (2010), Yu *et al.* (2010), and Altafini (2013)).

Taking a network-based perspective, power systems are indeed a class of nonlinear dynamical networks with network structural information embedded into the dynamics. An example illustrating the modeling analogy between power systems and dynamical networks is that the small-disturbance model of (1.1) is equivalent to consensus protocol

(1.5) in an extreme case where $M_i = 0$ and $D_i = 1$. Moreover, the collective behaviors in dynamical networks, e.g., reaching a consensus, are conceptually similar to angle stability in power systems which requires that all generators get synchronized angular frequencies.

We now recap two facts from the above discussion:

- Network structure features play an important role in the synchronization/consensus of dynamical networks;
- Power systems are analogous to dynamical networks regarding the modeling and collective behaviors.

These facts indicate that the traditional node-based theory of power system stability needs to be pushed further in the direction of network-based analysis, i.e., exploiting the structural features of the underlying power networks. Thanks to the progress in dynamical networks, it is now possible to develop more powerful graph theory tools for power systems, which help to deepen the understanding of power system stability by elaborating the role of power network structure.

Following this motivation, there recently rises a trend of reinvestigating stability problems with focus on power network structure. For instance, some early works in this direction have discovered that power system model (1.1) can be interpreted as Kuramoto oscillators (a common class of dynamical network model), where the stability conditions in terms of power network parameters are derived based on the theory of dynamical networks (e.g., see Dörfler and Bullo (2011b), Dörfler and Bullo (2012), Lozano *et al.* (2012), and Dörfler *et al.* (2013)). These results have then inspired many more studies that further reveal the relationship between power system dynamics and power network structure. This monograph aims to establish a network-based paradigm for angle stability analysis by surveying the authors' results (e.g., Bergen and Hill (1981), Chandrashekar and Hill (1983), Song *et al.* (2018b), Song *et al.* (2018a), and Song *et al.* (2019b)) as well as other contemporary results from researchers all over the world.

1.4 Organization of This Monograph

In the next seven chapters we introduce in turn the power system models for angle stability analysis, new graph theories tailored for power systems and application of those tools in small-disturbance angle stability and transient stability problems. The contents of each chapter are outlined below.

Chapter 2 briefly reviews the power system models adopted for angle stability analysis and presents the structure preserving model as a basis for the following analysis.

Chapter 3 develops some graph-based matrix conditions and extends the definition of graph effective resistance to characterize the impact of negative weighted edges on the graph Laplacian spectrum.

Chapter 4 develops some new characterizations for graph cutsets and cycles.

Chapter 5 studies small-disturbance angle stability problems by applying the graph theory developed in Chapter 3. We establish network-based stability criteria that systematically uncover the mechanism of small-disturbance instability induced by the lines with large angle differences in the power network.

Chapter 6 studies transient stability problems by applying the graph theory developed in Chapter 4. We shed light on a cutset-related phenomenon in transient stability and improve the cutset index for better assessing transient stability.

Chapter 7 establishes a new type of transient stability constrained optimal power flow (TSC-OPF) model by linking transient dynamics to the extended version of effective resistance. This TSC-OPF model admits a convex relaxation form by applying the graph theory tools in Chapter 3.

Chapter 8 makes some concluding remarks and a prospect for future research directions.

For better clarity, the proofs of some important theorems will be presented in the respective chapters, while the proofs of other results will be omitted and referred to the corresponding publication.

References

- Ahmadizadeh, S., I. Shames, S. Martin, and D. Nešić. (2017). “On eigenvalues of Laplacian matrix for a class of directed signed graphs.” *Linear Algebra and its Applications*. 523: 281–306.
- Ainsworth, N. and S. Grijalva. (2013). “A structure-preserving model and sufficient condition for frequency synchronization of lossless droop inverter-based AC networks.” *IEEE Trans. Power Syst.* 28(4): 4310–4319.
- Al Maruf, A., M. Ostadijafari, A. Dubey, and S. Roy. (2019). “Small-signal stability analysis for droop-controlled inverter-based microgrids with losses and filtering.” In: *Proc. ACM Conf. Future Ener. Syst.* 355–366.
- Altafini, C. (2013). “Consensus problems on networks with antagonistic interactions.” *IEEE Trans. Autom. Control*. 58(4): 935–946.
- Althoff, M. and B. H. Krogh. (2014). “Reachability analysis of nonlinear differential-algebraic systems.” *IEEE Trans. Autom. Control*. 59(2): 371–383.
- Andreasson, M., E. Tegling, H. Sandberg, and K. H. Johansson. (2017). “Coherence in synchronizing power networks with distributed integral control.” In: *Proc. IEEE Conf. Dec. Control*. 6327–6333.
- Anghel, M., F. Milano, and A. Papachristodoulou. (2013). “Algorithmic construction of Lyapunov functions for power system stability analysis.” *IEEE Trans. Circuits Syst. I*. 60(9): 2533–2546.

- Aolaritei, L., S. Bolognani, and F. Dörfler. (2018). “Hierarchical and distributed monitoring of voltage stability in distribution networks.” *IEEE Trans. Power Syst.* 33(6): 6705–6714.
- Araposthatis, A., S. Sastry, and P. P. Varaiya. (1981). “Analysis of power-flow equation.” *Int. J. Elect. Power Energy Syst.* 3(3): 115–126.
- Athay, T., R. Podmore, and S. Virmani. (1979). “A practical method for the direct analysis of transient stability.” *IEEE Trans. Power App. Syst.* PAS-98(2): 573–584.
- Bapat, R. B. (2011). *Graphs and Matrices*. Springer.
- Barooah, P. and J. P. Hespanha. (2006). “Graph effective resistance and distributed control: Spectral properties and applications.” In: *Proc. IEEE Conf. Dec. Control.* 3479–3485.
- Bellhomme, R., H. Zhao, and M. Pavella. (1993). “Power system reduction techniques for direct transient stability methods.” *IEEE Trans. Power Syst.* 8(2): 723–729.
- Bergen, A. R., D. J. Hill, and C. L. De Marcot. (1986). “Lyapunov function for multimachine power systems with generator flux decay and voltage dependent loads.” *Int. J. Elect. Power Energy Syst.* 8(1): 2–10.
- Bergen, A. R. and D. J. Hill. (1981). “A structure preserving model for power system stability analysis.” *IEEE Trans. Power App. Syst.* PAS-100(1): 25–35.
- Bergen, A. R. and V. Vittal. (1999). *Power Systems Analysis*. Prentice Hall.
- Bernstein, A., C. Wang, E. Dall’Anese, J.-Y. Le Boudec, and C. Zhao. (2018). “Load flow in multiphase distribution networks: Existence, uniqueness, non-singularity and linear models.” *IEEE Trans. Power Syst.* 33(6): 5832–5843.
- Bonchev, D., A. T. Balaban, X. Liu, and D. J. Klein. (1994). “Molecular cyclicity and centricity of polycyclic graphs. I. Cyclicity based on resistance distances or reciprocal distances.” *International Journal of Quantum Chemistry.* 50(1): 1–20.
- Chandrashekar, K. S. and D. J. Hill. (1983). “Dynamic security dispatch: Basic formulation.” *IEEE Trans. Power App. Syst.* (7): 2145–2154.

- Chen, W., J. Liu, Y. Chen, S. Z. Khong, D. Wang, T. Başar, L. Qiu, and K. H. Johansson. (2016a). “Characterizing the positive semidefiniteness of signed Laplacians via effective resistances.” In: *Proc. IEEE Conf. Dec. Control.* 985–990.
- Chen, W., D. Wang, J. Liu, T. Başar, and L. Qiu. (2017). “On spectral properties of signed Laplacians for undirected graphs.” In: *Proc. IEEE Conf. Dec. Control.* 1999–2002.
- Chen, Y., S. Z. Khong, and T. T. Georgiou. (2016b). “On the definiteness of graph Laplacians with negative weights: Geometrical and passivity-based approaches.” In: *Proc. Amer. Control Conf.* 2488–2493.
- Chiang, H.-D. and M. E. Baran. (1990). “On the existence and uniqueness of load flow solution for radial distribution power networks.” *IEEE Trans. Circuits Syst.* 37(3): 410–416.
- Chiang, H.-D. and F. F. Wu. (1988). “Stability of nonlinear systems described by a second-order vector differential equation.” *IEEE Trans. Circuits Syst.* 35(6): 703–711.
- Chiang, H.-D. (1989). “Study of the existence of energy functions for power systems with losses.” *IEEE Trans. Circuits Syst.* 36(11): 1423–1429.
- Chiang, H.-D. (2011). *Direct Methods for Stability Analysis of Electric Power Systems: Theoretical Foundation, BCU Methodologies, and Applications.* John Wiley & Sons.
- Chiang, H.-D., C.-C. Chu, and G. Cauley. (1995). “Direct stability analysis of electric power systems using energy functions: theory, applications, and perspective.” *Proc. IEEE.* 83(11): 1497–1529.
- Chiang, H.-D. and C.-C. Chu. (1995). “Theoretical foundation of the BCU method for direct stability analysis of network-reduction power system. Models with small transfer conductances.” *IEEE Trans. Circuits Syst. I.* 42(5): 252–265.
- Chiang, H.-D. and J. S. Thorp. (1989). “The closest unstable equilibrium point method for power system dynamic security assessment.” *IEEE Trans. Circuits Syst.* 36(9): 1187–1200.
- Chiang, H.-D. and J. S. Thorp. (1987). “The closest unstable equilibrium point method for power system dynamic security assessment.” In: *Proc. IEEE Conf. Dec. Control.* Vol. 26. 72–76.

- Chiang, H.-D., F. F. Wu, and P. P. Varaiya. (1987). “Foundations of direct methods for power system transient stability analysis.” *IEEE Trans. Circuits Syst.* 34(2): 160–173.
- Chiang, H.-D., F. F. Wu, and P. P. Varaiya. (1988). “Foundations of the potential energy boundary surface method for power system transient stability analysis.” *IEEE Trans. Circuits Syst.* 35(6): 712–728.
- Chiang, H.-D., F. F. Wu, and P. P. Varaiya. (1994). “A BCU method for direct analysis of power system transient stability.” *IEEE Trans. Power Syst.* 9(3): 1194–1208.
- Chow, J. H. (2013). *Power System Coherency and Model Reduction*. Springer.
- Chua, L. O. (1980). “Dynamic nonlinear networks: State-of-the-art.” *IEEE Trans. Circuits Syst.* 27(11): 1059–1087.
- Colombino, M., D. Groß, J.-S. Brouillon, and F. Dörfler. (2019). “Global phase and magnitude synchronization of coupled oscillators with application to the control of grid-forming power inverters.” *IEEE Trans. Autom. Control.* 64(11): 4496–4511.
- Coppersmith, D., P. Doyle, P. Raghavan, and M. Snir. (1993). “Random walks on weighted graphs and applications to on-line algorithms.” *Journal of the ACM.* 40(3): 421–453.
- Cotilla-Sanchez, E., P. D. Hines, C. Barrows, S. Blumsack, and M. Patel. (2013). “Multi-attribute partitioning of power networks based on electrical distance.” *IEEE Trans. Power Syst.* 28(4): 4979–4987.
- D’Arco, S. and J. A. Suul. (2014). “Equivalence of virtual synchronous machines and frequency-droops for converter-based microgrids.” *IEEE Trans. Smart Grid.* 5(1): 394–395.
- Davy, R. J. and I. A. Hiskens. (1997). “Lyapunov functions for multi-machine power systems with dynamic loads.” *IEEE Trans. Circuits Syst. I.* 44(9): 796–812.
- Ding, T., R. Bo, F. Li, and H. Sun. (2016). “Optimal power flow with the consideration of flexible transmission line impedance.” *IEEE Trans. Power Syst.* 31(2): 1655–1656.
- Dörfler, F. and F. Bullo. (2010). “Spectral analysis of synchronization in a lossless structure-preserving power network model.” In: *IEEE SmartGridComm.* 179–184.

- Dörfler, F. and F. Bullo. (2011a). “On the critical coupling for Kuramoto oscillators.” *SIAM Journal on Applied Dynamical Systems*. 10(3): 1070–1099.
- Dörfler, F. and F. Bullo. (2011b). “Topological equivalence of a structure-preserving power network model and a non-uniform Kuramoto model of coupled oscillators.” In: *Proc. IEEE Conf. Dec. Control*. 7099–7104.
- Dörfler, F. and F. Bullo. (2012). “Synchronization and transient stability in power networks and nonuniform Kuramoto oscillators.” *SIAM Journal on Control and Optimization*. 50(3): 1616–1642.
- Dörfler, F. and F. Bullo. (2013). “Kron reduction of graphs with applications to electrical networks.” *IEEE Trans. Circuits Syst. I*. 60(1): 150–163.
- Dörfler, F., M. Chertkov, and F. Bullo. (2013). “Synchronization in complex oscillator networks and smart grids.” *Proc. Nat. Acad. Sci.* 110(6): 2005–2010.
- Dörfler, F., J. W. Simpson-Porco, and F. Bullo. (2016). “Breaking the hierarchy: Distributed control and economic optimality in microgrids.” *IEEE Trans. Control Netw. Syst.* 3(3): 241–253.
- Dörfler, F., J. W. Simpson-Porco, and F. Bullo. (2018). “Electrical networks and algebraic graph theory: Models, properties, and applications.” *Proc. IEEE*. 106(5): 977–1005.
- Doyle, P. G. and J. L. Snell. (1984). *Random Walks and Electric Networks*. Washington, DC: Math. Assoc. America.
- Essam, J. W. and M. E. Fisher. (1970). “Some basic definitions in graph theory.” *Reviews of Modern Physics*. 42(2): 271.
- Farrokhhabadi, M., C. A. Cañizares, J. W. Simpson-Porco, E. Nasr, L. Fan, P. A. Mendoza-Araya, R. Tonkoski, U. Tamrakar, N. Hatziargyriou, D. Lagos, R. W. Wies, M. Paolone, M. Liserre, L. Meegahapola, M. Kabalan, A. H. Hajimiragha, D. Peralta, M. Elizondo, K. P. Schneider, F. Tuffner, and J. T. Reilly. (2020). “Microgrid stability definitions, analysis, and examples.” *IEEE Trans. Power Syst.* 35(1): 13–29.
- Fouad, A.-A. and V. Vittal. (1991). *Power System Transient Stability Analysis Using the Transient Energy Function Method*. Pearson Education.

- Ghosh, A., S. Boyd, and A. Saberi. (2008). “Minimizing effective resistance of a graph.” *SIAM Review*. 50(1): 37–66.
- Godsil, C. D. and G. Royle. (2001). *Algebraic Graph Theory*. Springer.
- Groß, D., M. Colombino, J.-S. Brouillon, and F. Dörfler. (2019). “The effect of transmission-line dynamics on grid-forming dispatchable virtual oscillator control.” *IEEE Trans. Control Netw. Syst.* 6(3): 1148–1160.
- Grunberg, T. W. and D. F. Gayme. (2018). “Performance measures for linear oscillator networks over arbitrary graphs.” *IEEE Trans. Control Netw. Syst.* 5(1): 456–468.
- El-Guindy, A., Y. C. Chen, and M. Althoff. (2017). “Compositional transient stability analysis of power systems via the computation of reachable sets.” In: *Proc. Amer. Control Conf.* 2536–2543.
- Guo, L., C. Liang, and S. H. Low. (2017). “Monotonicity properties and spectral characterization of power redistribution in cascading failures.” In: *Annual Allerton Conference*. 918–925.
- Guo, L., C. Liang, A. Zocca, S. H. Low, and A. Wierman. (2018a). “Failure localization in power systems via tree partitions.” In: *Proc. IEEE Conf. Dec. Control*. 6832–6839.
- Guo, L. and S. H. Low. (2017). “Spectral characterization of controllability and observability for frequency regulation dynamics.” In: *Proc. IEEE Conf. Dec. Control*. 6313–6320.
- Guo, L., C. Zhao, and S. H. Low. (2018b). “Graph laplacian spectrum and primary frequency regulation.” In: *Proc. IEEE Conf. Dec. Control*. 158–165.
- Gutman, I. and W. Xiao. (2004). “Generalized inverse of the Laplacian matrix and some applications.” *Bull. Classe des Sci. Math. et Naturelles. Sci. Math.*: 15–23.
- Han, D., A. El-Guindy, and M. Althoff. (2016). “Power systems transient stability analysis via optimal rational Lyapunov functions.” In: *Proc. IEEE Power Energy Soc. Gen. Meeting*. 1–5.
- Han, T. and D. J. Hill. (2020). “ \mathcal{H}_2 -norm transmission switching to improve synchronism of low-inertia power grids.” In: *IFAC World Congress*.

- Hedman, K. W., R. P. O'Neill, E. B. Fisher, and S. S. Oren. (2009). "Optimal transmission switching with contingency analysis." *IEEE Trans. Power Syst.* 24(3): 1577–1586.
- Hill, D. J. and A. R. Bergen. (1982). "Stability analysis of multimachine power networks with linear frequency dependent loads." *IEEE Trans. Circuits Syst.* 29(12): 840–848.
- Hill, D. J. and G. Chen. (2006). "Power systems as dynamic networks." In: *Proc. IEEE Int. Symp. Circuits Syst.* 722–725.
- Hiskens, I. A. and D. J. Hill. (1989). "Energy functions, transient stability and voltage behaviour in power systems with nonlinear loads." *IEEE Trans. Power Syst.* 4(4): 1525–1533.
- Horn, R. A. and C. R. Johnson. (2012). *Matrix Analysis*. Cambridge University Press.
- Hou, G. and V. Vittal. (2013). "Determination of transient stability constrained interface real power flow limit using trajectory sensitivity approach." *IEEE Trans. Power Syst.* 28(3): 2156–2163.
- Huang, L., H. Xin, W. Dong, and F. Dörfler. (2019). "Impacts of grid structure on PLL-synchronization stability of converter-integrated power systems." *arXiv preprint arXiv:1903.05489*.
- Huang, W., D. J. Hill, and X. Zhang. (2020). "Small-disturbance voltage stability of power systems: dependence on network structure." *IEEE Trans. Power Syst.* 35(4): 2609–2618.
- Ilić, M. D. (1992). "Network theoretic conditions for existence and uniqueness of steady state solutions to electric power circuits." In: *Proc. IEEE Int. Symp. Circuits Syst.* Vol. 6. 2821–2828.
- Ishizaki, T., A. Chakraborty, and J.-I. Imura. (2018). "Graph-theoretic analysis of power systems." *Proc. IEEE.* 106(5): 931–952.
- Jin, L., H. Liu, R. Kumar, J. D. McCalley, N. Elia, and V. Ajjarapu. (2005). "Power system transient stability design using reachability based stability-region computation." In: *Proc. North Amer. Power Symp.* 338–343.
- Johansson, A., J. Wei, H. Sandberg, K. H. Johansson, and J. Chen. (2018). "Optimization of the \mathcal{H}_∞ -norm of dynamic flow networks." In: *Proc. Amer. Control Conf.* 1280–1285.

- Kakimoto, N., Y. Ohnogi, H. Matsuda, and H. Shibuya. (1984). “Transient stability analysis of large-scale power system by Lyapunov’s direct method.” *IEEE Trans. Power App. Syst.* PAS-103(1): 160–167.
- Kaye, R. and F. F. Wu. (1984). “Analysis of linearized decoupled power flow approximations for steady-state security assessment.” *IEEE Trans. Circuits Syst.* 31(7): 623–636.
- Klein, D. J. and M. Randić. (1993). “Resistance distance.” *J. Math. Chem.* 12(1): 81–95.
- Kundur, P., J. Paserba, V. Ajarapu, G. Andersson, A. Bose, C. Cañizares, N. Hatziargyriou, D. J. Hill, A. Stanković, C. Taylor, T. Van Cutsem, and V. Vittal. (2004). “Definition and classification of power system stability.” *IEEE Trans. Power Syst.* 19(3): 1387–1401.
- Kundur, P. (1994). *Power System Stability and Control*. McGraw-Hill, New York.
- Lavaei, J. and S. H. Low. (2011). “Zero duality gap in optimal power flow problem.” *IEEE Trans. Power Syst.* 27(1): 92–107.
- Lei, S., J. Wang, and Y. Hou. (2018). “Remote-controlled switch allocation enabling prompt restoration of distribution systems.” *IEEE Trans. Power Syst.* 33(3): 3129–3142.
- Li, Z., Z. Duan, G. Chen, and L. Huang. (2010). “Consensus of multi-agent systems and synchronization of complex networks: A unified viewpoint.” *IEEE Trans. Circuits Syst. I.* 57(1): 213–224.
- Lin, J., V. O. K. Li, K.-C. Leung, and A. Y. S. Lam. (2017). “Optimal power flow with power flow routers.” *IEEE Trans. Power Syst.* 32(1): 531–543.
- Low, S. H. (2014a). “Convex relaxation of optimal power flow-part I: Formulations and equivalence.” *IEEE Trans. Control Netw. Syst.* 1(1): 15–27.
- Low, S. H. (2014b). “Convex relaxation of optimal power flow-part II: Exactness.” *IEEE Trans. Control Netw. Syst.* 1(2): 177–189.
- Lozano, S., L. Buzna, and A. Díaz-Guilera. (2012). “Role of network topology in the synchronization of power systems.” *The European Physical Journal B-Condensed Matter and Complex Systems.* 85(7): 1–8.

- Madani, R., S. Sojoudi, and J. Lavaei. (2015). “Convex relaxation for optimal power flow problem: Mesh networks.” *IEEE Trans. Power Syst.* 30(1): 199–211.
- Magnusson, P. C. (1947). “The transient-energy method of calculating stability.” *AIEE Transactions.* 66(1): 747–755.
- Milano, F., F. Dörfler, G. Hug, D. J. Hill, and G. Verbič. (2018). “Foundations and challenges of low-inertia systems.” In: *Proc. Power Syst. Comput. Conf.* 1–25.
- Mishra, C., R. S. Biswas, A. Pal, and V. A. Centeno. (2020). “Critical clearing time sensitivity for inequality constrained systems.” *IEEE Trans. Power Syst.* 35(2): 1572–1583.
- Miu, K. N. and H.-D. Chiang. (2000). “Existence, uniqueness, and monotonic properties of the feasible power flow solution for radial three-phase distribution networks.” *IEEE Trans. Circuits Syst. I.* 47(10): 1502–1514.
- Miyagi, H. and A. R. Bergen. (1986). “Stability studies of multimachine power systems with the effects of automatic voltage regulators.” *IEEE Trans. Autom. Control.* 31(3): 210–215.
- Molzahn, D. K. and I. A. Hiskens. (2014). “Sparsity-exploiting moment-based relaxations of the optimal power flow problem.” *IEEE Trans. Power Syst.* 30(6): 3168–3180.
- Molzahn, D. K. and I. A. Hiskens. (2019). “A survey of relaxations and approximations of the power flow equations.” *Foundations and Trends® in Electric Energy Systems.* 4(1-2): 1–221.
- Molzahn, D. K., J. T. Holzer, B. C. Lesieutre, and C. L. DeMarco. (2013). “Implementation of a large-scale optimal power flow solver based on semidefinite programming.” *IEEE Trans. Power Syst.* 28(4): 3987–3998.
- Moreau, L. (2004). “Stability of continuous-time distributed consensus algorithms.” In: *Proc. IEEE Conf. Dec. Control.* 3998–4003.
- Narasimhamurthi, N. and M. Musavi. (1984). “A generalized energy function for transient stability analysis of power systems.” *IEEE Trans. Circuits Syst.* 31(7): 637–645.
- Nguyen, H. D., K. Dvijotham, S. Yu, and K. Turitsyn. (2019). “A framework for robust long-term voltage stability of distribution systems.” *IEEE Trans. Smart Grid.* 10(5): 4827–4837.

- Nguyen, T. B. and M. A. Pai. (2003). “Dynamic security-constrained rescheduling of power systems using trajectory sensitivities.” *IEEE Trans. Power Syst.* 18(2): 848–854.
- Olfati-Saber, R., A. Fax, and R. M. Murray. (2007). “Consensus and cooperation in networked multi-agent systems.” *Proc. IEEE.* 95(1): 215–233.
- Olfati-Saber, R. and R. M. Murray. (2004). “Consensus problems in networks of agents with switching topology and time-delays.” *IEEE Trans. Autom. Control.* 49(9): 1520–1533.
- Orfanogianni, T. and R. Bacher. (2003). “Steady-state optimization in power systems with series FACTS devices.” *IEEE Trans. Power Syst.* 18(1): 19–26.
- Padiyar, K. R. and S. Krishna. (2006). “Online detection of loss of synchronism using energy function criterion.” *IEEE Trans. Power Del.* 21(1): 46–55.
- Padiyar, K. (2013). *Structure Preserving Energy Functions in Power Systems: Theory and Applications*. CRC Press.
- Pai, M. A. (1981). *Power System Stability: Analysis by the Direct Method of Lyapunov*. North-Holland.
- Pai, M. A. (1989). *Energy Function Analysis for Power System Stability*. Springer.
- Pan, L., H. Shao, and M. Mesbahi. (2016). “Laplacian dynamics on signed networks.” In: *Proc. IEEE Conf. Dec. Control.* 891–896.
- Podmore, R. (1978). “Identification of coherent generators for dynamic equivalents.” *IEEE Trans. Power App. Syst.* (4): 1344–1354.
- Prabhakara, F. S. and A. H. El-Abiad. (1975). “A simplified determination of transient stability regions for Lyapunov methods.” *IEEE Trans. Power App. Syst.* 94(2): 672–689.
- Price, W., H.-D. Chiang, H. Clark, C. Concordia, D. Lee, J. Hsu, S. Ihara, C. King, C. Lin, and Y. Mansour. (1993). “Load representation for dynamic performance analysis.” *IEEE Trans. Power Syst.* 8(2): 472–482.
- Ren, W. and R. W. Beard. (2005). “Consensus seeking in multiagent systems under dynamically changing interaction topologies.” *IEEE Trans. Autom. Control.* 50(5): 655–661.

- Saha, S., A. Fouad, W. Kliemann, and V. Vittal. (1997). “Stability boundary approximation of a power system using the real normal form of vector fields.” *IEEE Trans. Power Syst.* 12(2): 797–802.
- Sahraei-Ardakani, M. and K. W. Hedman. (2016). “A fast LP approach for enhanced utilization of variable impedance based FACTS devices.” *IEEE Trans. Power Syst.* 31(3): 2204–2213.
- Sastry, S. and P. P. Varaiya. (1980). “Hierarchical stability and alert state steering control of interconnected power systems.” *IEEE Trans. Circuits Syst.* 27(11): 1102–1112.
- Sauer, P. W. and M. A. Pai. (1990). “Power system steady-state stability and the load-flow Jacobian.” *IEEE Trans. Power Syst.* 5(4): 1374–1383.
- Sauer, P. W. and M. A. Pai. (1997). *Power System Dynamics and Stability*. Prentice Hall.
- Schiffer, J., D. Zonetti, R. Ortega, A. Stanković, T. Sezi, and J. Raisch. (2016). “A survey on modeling of microgrids—From fundamental physics to phasors and voltage sources.” *Automatica*. 74(Dec.): 135–150.
- Seshu, S. and M. B. Reed. (1961). *Linear Graphs and Electrical Networks*. Addison-Wesley Pub. Co.
- Sharma, S., S. Pushpak, V. Chinde, and I. Dobson. (2018). “Sensitivity of transient stability critical clearing time.” *IEEE Trans. Power Syst.* 33(6): 6476–6486.
- Shubhanga, K. N. and A. M. Kulkarni. (2004). “Determination of effectiveness of transient stability controls using reduced number of trajectory sensitivity computations.” *IEEE Trans. Power Syst.* 19(1): 473–482.
- Simpson-Porco, J. W., F. Dörfler, and F. Bullo. (2013). “Synchronization and power sharing for droop-controlled inverters in islanded microgrids.” *Automatica*. 49(9): 2603–2611.
- Simpson-Porco, J. W., F. Dörfler, and F. Bullo. (2016). “Voltage collapse in complex power grids.” *Nature Comm.* 7: 10790.
- Soltan, S., D. Mazauric, and G. Zussman. (2017). “Analysis of failures in power grids.” *IEEE Trans. Control Netw. Syst.* 4(2): 288–300.

- Song, Y. (2017). “Network-Based Stability Analysis of Electric Power Systems.” *PhD thesis*. Pokfulam, Hong Kong: The University of Hong Kong.
- Song, Y., D. J. Hill, and T. Liu. (2017a). “Local stability of DC microgrids: A perspective of graph Laplacians with self-loops.” In: *Proc. IEEE Conf. Dec. Control*. IEEE. 2629–2634.
- Song, Y., D. J. Hill, and T. Liu. (2018a). “Characterization of cutsets in networks with application to transient stability analysis of power systems.” *IEEE Trans. Control Netw. Syst.* 5(3): 1261–1274.
- Song, Y., D. J. Hill, and T. Liu. (2018b). “Network-based analysis of small-disturbance angle stability of power systems.” *IEEE Trans. Control Netw. Syst.* 5(3): 901–912.
- Song, Y., D. J. Hill, and T. Liu. (2019a). “Impact of DG connection topology on the stability of inverter-based microgrids.” *IEEE Trans. Power Syst.* 34(5): 3970–3972.
- Song, Y., D. J. Hill, and T. Liu. (2019b). “On extension of effective resistance with application to graph Laplacian definiteness and power network stability.” *IEEE Trans. Circuits Syst. I.* 66(11): 4415–4428.
- Song, Y., D. J. Hill, and T. Liu. (2019c). “Static voltage stability analysis of distribution systems based on network-load admittance ratio.” *IEEE Trans. Power Syst.* 34(3): 2270–2280.
- Song, Y., D. J. Hill, T. Liu, and Y. Zheng. (2017b). “A distributed framework for stability evaluation and enhancement of inverter-based microgrids.” *IEEE Trans. Smart Grid.* 8(6): 3020–3034.
- Song, Y., Y. Zheng, T. Liu, S. Lei, and D. J. Hill. (2020). “A new formulation of distribution network reconfiguration for reducing the voltage volatility induced by distributed generation.” *IEEE Trans. Power Syst.* 35(1): 496–507.
- Strogatz, S. H. (2001). “Exploring complex networks.” *Nature.* 410(6825): 268–276.
- Tang, L. and W. Sun. (2017). “An automated transient stability constrained optimal power flow based on trajectory sensitivity analysis.” *IEEE Trans. Power Syst.* 32(1): 590–599.
- Tavora, C. J. and O. J. M. Smith. (1972). “Stability analysis of power systems.” *IEEE Trans. Power App. Syst.* PAS-91(3): 1138–1144.

- Thorp, J., M. D. Ilić, and M. Varghese. (1986). “Conditions for solution existence and localized response in the reactive power network.” *Int. J. Elect. Power Energy Syst.* 8(2): 66–74.
- Thulasiraman, K., M. Yadav, and K. Naik. (2019). “Network science meets circuit theory: Resistance distance, Kirchhoff index, and Foster’s theorems with generalizations and unification.” *IEEE Trans. Circuits Syst. I.* 66(3): 1090–1103.
- Tsolas, N., A. Arapostathis, and P. P. Varaiya. (1985). “A structure preserving energy function for power system transient stability analysis.” *IEEE Trans. Circuits Syst.* 32(10): 1041–1049.
- Van der Schaft, A. (2010). “Characterization and partial synthesis of the behavior of resistive circuits at their terminals.” *Systems & Control Letters.* 59(7): 423–428.
- Varaiya, P. P., F. F. Wu, and R.-L. Chen. (1985). “Direct methods for transient stability analysis of power systems: Recent results.” *Proc. IEEE.* 73(12): 1703–1715.
- Venikov, V. A., V. A. Stroeve, V. I. Idelchick, and V. I. Tarasov. (1975). “Estimation of electrical power system steady-state stability in load flow calculations.” *IEEE Trans. Power App. Syst.* 94(3): 1034–1041.
- Vizing, V. G. (1968). “Some unsolved problems in graph theory.” *Russian Mathematical Surveys.* 23(6): 125–141.
- Vorobev, P., S. Chevalier, and K. Turitsyn. (2019). “Decentralized stability rules for microgrids.” In: *Proc. Amer. Control Conf.* 2596–2601.
- Vu, T. L. and K. Turitsyn. (2016). “Lyapunov functions family approach to transient stability assessment.” *IEEE Trans. Power Syst.* 31(2): 1269–1277.
- Wang, C., A. Bernstein, J.-Y. Le Boudec, and M. Paolone. (2018). “Explicit conditions on existence and uniqueness of load-flow solutions in distribution networks.” *IEEE Trans. Smart Grid.* 9(2): 953–962.
- Wang, X. F. and G. Chen. (2002). “Synchronization in scale-free dynamical networks: robustness and fragility.” *IEEE Trans. Circuits Syst. I.* 49(1): 54–62.
- Wang, X. F. and G. Chen. (2003). “Complex networks: small-world, scale-free and beyond.” *IEEE Circuits Syst. Mag.* 3(1): 6–20.

- Wang, Z., B. Cui, and J. Wang. (2017). “A necessary condition for power flow insolvability in power distribution systems with distributed generators.” *IEEE Trans. Power Syst.* 32(2): 1440–1450.
- Wu, F. F. and C.-C. Liu. (1986). “Characterization of power system small disturbance stability with models incorporating voltage variation.” *IEEE Trans. Circuits Syst.* 33(4): 406–417.
- Wu, F. F. and Y.-K. Tsai. (1983). “Identification of groups of ε -coherent generators.” *IEEE Trans. Circuits Syst.* 30(4): 234–241.
- Xiang, J., Y. Wang, Y. Li, and W. Wei. (2016). “Stability and steady-state analysis of distributed cooperative droop controlled DC microgrids.” *IET Control Theory & Applications.* 10(18): 2490–2496.
- Xu, Y., J. Ma, Z. Y. Dong, and D. J. Hill. (2017). “Robust transient stability-constrained optimal power flow with uncertain dynamic loads.” *IEEE Trans. Smart Grid.* 8(4): 1911–1921.
- Xue, Y., T. Van Cutsem, and M. Ribbens-Pavella. (1989). “Extended equal area criterion justifications, generalizations, applications.” *IEEE Trans. Power Syst.* 4(1): 44–52.
- Xue, Y., T. Van Cutsem, and M. Ribbens-Pavella. (1988). “A simple direct method for fast transient stability assessment of large power systems.” *IEEE Trans. Power Syst.* 3(2): 400–412.
- Yu, W., G. Chen, and M. Cao. (2010). “Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems.” *Automatica.* 46(6): 1089–1095.
- Yu, Y., J. Fan, and F. Feng. (2006). “Relationship between the number of critical cutsets and the type of unstable equilibrium point with respect to transient angle stability.” *Proceedings of the CSEE.* 26(8): 1–6.
- Yuan, H. and Y. Xu. (2020). “Trajectory sensitivity based preventive transient stability control of power systems against wind power variation.” *Int. J. Elect. Power Energy Syst.* 117(May): 105713.
- Yuan, S. Q. and D. Z. Fang. (2009). “Robust PSS parameters design using a trajectory sensitivity approach.” *IEEE Trans. Power Syst.* 24(2): 1011–1018.
- Zelazo, D. and M. Bürger. (2014). “On the definiteness of the weighted Laplacian and its connection to effective resistance.” In: *Proc. IEEE Conf. Dec. Control.* 2895–2900.

- Zelazo, D. and M. Bürger. (2017). “On the robustness of uncertain consensus networks.” *IEEE Trans. Control Netw. Syst.* 4(2): 170–178.
- Zelazo, D. and M. Mesbahi. (2011). “Edge agreement: Graph-theoretic performance bounds and passivity analysis.” *IEEE Trans. Autom. Control.* 56(3): 544–555.
- Zhang, F. (2005). *The Schur Complement and Its Applications*. Springer.
- Zhao, J., D. J. Hill, and T. Liu. (2010). “Passivity-based output synchronization of dynamical networks with non-identical nodes.” In: *Proc. IEEE Conf. Dec. Control.* 7351–7356.
- Zhao, J., D. J. Hill, and T. Liu. (2011). “Synchronization of dynamical networks with nonidentical nodes: Criteria and control.” *IEEE Trans. Circuits Syst. I.* 58(3): 584–594.
- Zhu, L. and D. J. Hill. (2018). “Stability analysis of power systems: A network synchronization perspective.” *SIAM Journal on Control and Optimization.* 56(3): 1640–1664.
- Zhu, L. and D. J. Hill. (2020). “Synchronization of Kuramoto oscillators: A regional stability framework.” *IEEE Trans. Autom. Control.* in press. DOI: [10.1109/TAC.2020.2968977](https://doi.org/10.1109/TAC.2020.2968977).