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A Survey of Relaxations and Approximations of the Power Flow Equations

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Foundations and Trends® in Electric Energy Systems

Published, sold and distributed by:
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
United States
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Outside North America:
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PO Box 179
2600 AD Delft
The Netherlands
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The preferred citation for this publication is

D. K. Molzahn and I. A. Hiskens. *A Survey of Relaxations and Approximations of the Power Flow Equations*. Foundations and Trends® in Electric Energy Systems, vol. 4, no. 1-2, pp. 1–221, 2019.

ISBN: 978-1-68083-541-0

Foundations and Trends® in Electric Energy Systems
Volume 4, Issue 1-2, 2019

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Foundations and Trends® in Electric Energy Systems, 2019, Volume 4, 4 issues. ISSN paper version 2332-6557. ISSN online version 2332-6565. Also available as a combined paper and online subscription.

Contents

A Survey of Relaxations and Approximations of the Power Flow Equations

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ABSTRACT

The power flow equations relate the power injections and voltages in an electric power system and are therefore key to many power system optimization and control problems. Research efforts have developed a wide variety of relaxations and approximations of the power flow equations with a range of capabilities and characteristics. This monograph surveys relaxations and approximations of the power flow equations, with a particular emphasis on recently proposed formulations.

Daniel K. Molzahn and Ian A. Hiskens (2019), "A Survey of Relaxations and Approximations of the Power Flow Equations", Foundations and Trends® in Electric Energy Systems: Vol. 4, No. 1-2, pp 1–221. DOI: 10.1561/3100000012.

1

Introduction

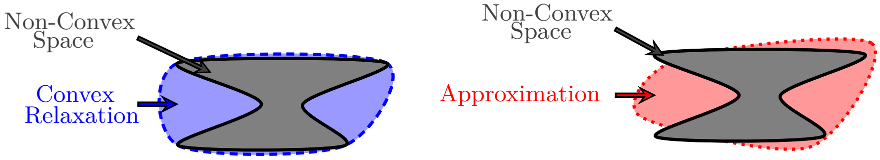
The power flow equations model the relationship between voltage phasors and power injections at nodes (buses) in an electric power system. These equations are fundamental in the analysis and operation of power systems. Accordingly, they form the key constraints in many optimization and control problems relevant to electric power systems, including optimal power flow (OPF), unit commitment, state estimation, contingency evaluation, voltage stability assessment, and dynamic stability analysis. The power flow equations are nonlinear and result in non-convex optimization problems. Moreover, at least some optimization problems containing the power flow equations (e.g., OPF problems) are generally NP-Hard [1], even for systems with radial network topologies [2], and may have multiple local solutions [3]. This inherent complexity is immediately apparent in the simple examples presented at the end of Chapter 2.

There exists a voluminous literature regarding the power flow equations. The intent of this monograph is to review various representations of the power flow equations, with a particular focus on those proposed in the last decade.

The power flow representations in this monograph are primarily presented in the context of optimization problems. However, note that while optimization plays an important role in many problems relevant to the design and operation of power systems (e.g., OPF, state estimation, unit commitment, transmission switching, expansion planning, etc. [4, 5]), various power flow representations are relevant to other important problems (stability analyses, dynamic simulations, analysis of control strategies such as volt/var control and automatic generation control, etc. [6, 7]). Moreover, while much of the literature develops power flow representations in the context of certain applications, this monograph focuses on the power flow representations themselves rather than specific problems. The reader interested in a specific problem or solution algorithm is referred to the surveys and tutorials that exist for power flow [8, 9], different formulations of optimal power flow [10–21] (and various extensions to consider, e.g., security constraints [22–25] and transient-stability constraints [26, 27]), unit commitment [28–31], state estimation [32–35], transmission switching [36], infrastructure planning [19], voltage stability analysis [37–40], cascading failure [41], distributed optimization and control methods [42–45], complex network theory [46], and more general power system stability concepts [6]. Several recent references of particular relevance are the surveys in [47] and [48] as well as the video lectures in [49], all of which review some of the topics covered in this monograph. Also note that reference implementations for several of the power flow representations presented in this monograph are provided in the software packages Matpower [50] and PowerModels.jl [51].

The power flow representations surveyed in this monograph are categorized as either *relaxations* or *approximations*. Figure 1.1 shows conceptual examples of a relaxation and an approximation of a non-convex feasible space. Relaxations enclose the non-convex feasible spaces associated with the power flow equations in a larger space. The larger space is typically chosen to be convex to enable the application of theory and algorithms developed for convex optimization problems.

Approximations use assumptions regarding certain quantities to simplify the power flow equations. Power flow approximations are capable of closely representing system behavior when the associated assumptions



(a) Convex Relaxation **(b)** Approximation

Figure 1.1: Conceptual illustrations showing a convex relaxation (blue region on the left) and an approximation (red region on the right) for the gray non-convex space.

are valid. Many power flow approximations are reasonably accurate for "typical" operating conditions.

In general, solutions to optimization problems that use power flow relaxations and approximations do not exactly satisfy the actual power flow equations. Rather, relaxations and approximations are typically employed in attempts to obtain tractable formulations which adequately represent the actual power flow physics. Optimization problems that use convex relaxations additionally provide bounds on the optimal objective value for the original non-convex problem as well as sufficient conditions for certifying problem infeasibility. Some convex relaxations also have associated sufficient conditions which guarantee their ability to provide global optima for certain limited classes of power system optimization problems. Some of these sufficient conditions can be evaluated prior to solving the relaxation based solely on the problem parameters and network topology, while other conditions are checked after solving a relaxation. In contrast, note that approximations do not provide any of the aforementioned theoretical guarantees provided by relaxations.

Solutions to relaxations and approximations may not exactly satisfy the power flow equations. This may be unacceptable for some applications, necessitating the deployment of algorithms that return a feasible power flow solution, possibly at the cost of increased computational difficulty or the lack of theoretical guarantees. A wide variety of nonlinear programming techniques have been applied to power system optimization problems. Starting from specified initializations, these techniques typically seek *local optima* for power system optimization problems, which are feasible points with objective values that are superior to all

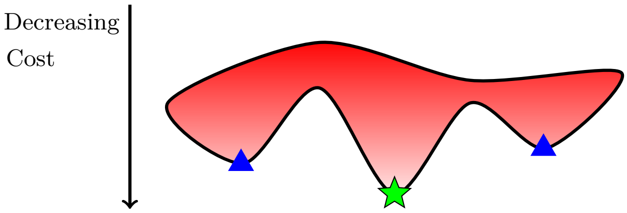


Figure 1.2: Conceptual illustration showing local optima (blue triangles) and the global optimum (green star).

nearby points but potentially inferior to the global optimum. Figure 1.2 provides a conceptual example showing the distinction between local and global optima. While surveying the power system optimization literature regarding local solution techniques is largely beyond the scope of this monograph, a brief summary of traditional nonlinear programming techniques is presented in §6. The interested reader is directed to other reviews of traditional local solution techniques, such as [13–17] for further details. Additionally, some of the power flow representations considered in this monograph form the basis of recently developed algorithms for computing local optima or "nearly globally optimal" feasible points. This monograph also reviews several such algorithms in §6.

The capabilities of various power flow relaxations and approximations are, in many ways, complementary rather than competitive with the capabilities of local solution algorithms. Local solution algorithms can benefit from the outputs resulting from power flow relaxations and approximations (e.g., using the decision variable values and the set of binding constraints to initialize certain local solution algorithms). Moreover, optimization problems may combine various power flow representations in order to balance accuracy and computational tractability. For instance, an optimization problem may have a "base case" that uses a detailed model of the power flow physics and multiple "scenarios" that use simplified power flow representations for the sake of computational tractability. As another example, an algorithm could decompose the solution of a complicated mixed-integer nonlinear program into two steps: first solve a mixed-integer problem with a simplified power flow model to select values for the discrete variables, and then apply a local solution algorithm to the continuous optimization problem that results from fixing the discrete variables and employing a higher-fidelity power flow model.

The theoretical guarantees provided by relaxations also complement the capabilities of local solution algorithms. Infeasibility of a relaxation certifies that the original optimization problem is infeasible, but feasibility of a relaxation is not sufficient to guarantee feasibility of the original problem. Conversely, a local solution algorithm can show that a problem is feasible, but failure of a local algorithm to converge to a feasible point does not guarantee that the original problem is infeasible. Thus, relaxations and local solution algorithms have complementary capabilities with respect to the question of problem feasibility. Furthermore, many global solution algorithms compute an *optimality gap* by comparing the objective value bound from a relaxation with the achievable objective value from a feasible point obtained via a local solution algorithm. In order to provably obtain a global optimum, these algorithms then use a variety of techniques to shrink the optimality gap. Also note that the objective value bounds can be directly useful, for instance, in algorithms that aim to achieve robustness with respect to a set of possible uncertainty realizations, compute bounds on voltage stability margins, etc. The references at the end of §7.2 provide examples of these and other synergistic uses of various power flow representations.

The remainder of this monograph is organized as follows. Chapter 2 describes the power flow equations. Chapter 3 overviews the optimization tools which form the basis for the power flow representations. Chapters 4 and 5 review the literature of power flow relaxations and approximations, respectively. Chapter 6 overviews various techniques for obtaining a feasible point, focusing on recent developments. Chapter 7 concludes the monograph and discusses open research topics.

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