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ABSTRACT

Online auctions are one of the most fundamental facets of the modern economy and power an industry generating hundreds of billions of dollars a year in revenue. Auction theory has historically focused on the question of designing the best way to sell a single item to potential buyers, with the concurrent objectives of maximizing revenue generated or welfare created. Theoretical results in this area have typically relied on some prior Bayesian knowledge agents were assumed to have on each other. This assumption is no longer satisfied in new markets such as online advertising: similar items are sold repeatedly, and agents are unaware of each other or might try to manipulate each other. On the other hand, statistical learning theory now provides tools to supplement those missing pieces of information given enough data, as agents can learn from their environment to improve their strategies.

This monograph covers recent advances in learning in repeated auctions, starting from the traditional economic

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study of optimal one-shot auctions with a Bayesian prior. We then focus on the question of learning optimal mechanisms from a dataset of bidders' past values. The sample complexity as well as the computational efficiency of different methods will be studied. We will also investigate online variants where gathering data has a cost to be accounted for, either by sellers or buyers ("earning while learning"). Later in the monograph, we will further assume that bidders are also adaptive to the mechanism as they interact repeatedly with the same seller. We will show how strategic agents can actually manipulate repeated auctions, to their own advantage. A particularly interesting example is that of reserve price improvements for strategic buyers in second price auctions.

All the questions discussed in this monograph are grounded in real-world applications and many of the ideas and algorithms we describe are used every day to power the Internet economy.

1

Introduction: Scope and Motivation

The main purpose of auction theory is to construct a set of rules that will be used by a seller to sell one or several items to a group of potential buyers, who will send messages (or *bids*) to the seller – usually indicating how much they value the item or how much they are willing to pay to acquire it. In almost all cases, it is sufficient to define only two rules. First, the *allocation rule* describes which buyer wins the auction (if a unique non-divisible item is sold), depending on the different messages received; if the item is divisible, the allocation rule describes how the item is shared between winners. Second, the *payment rule* indicates to buyers how much they are going to pay to the seller, again based on the different messages. Those rules are known publicly before the auction starts, and they influence the behavior, or strategy, of the different buyers.

When choosing an allocation and a payment rule, the seller might have several constraints to respect: **1)** maximizing the revenue she is getting from the auction (*revenue maximization*); **2)** ensuring the participation of buyers in the auction and making sure they have an incentive to participate (*individual rationality*); **3)** ensuring that given the rules of the auction, it is in the best interest of buyers to reveal how much they truly value an item (*incentive compatibility*) as it may make revenue maximization easier. On the other side of the game, the buyers strategically adjust the bids sent to the seller depending on auction rules in order to maximize their own utility.

Historically, auctions have often been designed so that buyers have an incentive to bid in a way that reflects how much they truly value the items that are for sale. This constraint still leaves plenty of choices for auction design, and a large part of the literature has focused on designing auctions that maximize the seller's revenue, assuming buyers are rational. However, with the advent of the Internet and the automation of auctions, the landscape of possible applications has changed drastically, necessitating more complex settings to accurately study the incentives and behaviors at play. More recently, the auction literature has aimed at understanding how the design of an auction platform impacts seller's revenue, and the global welfare and behavior of buyers and sellers in contexts where sellers (and sometimes buyers) participate in a very large number of auctions each day. These setups reflect situations appearing in modern online marketplaces.

1.1 Bayesian mechanism design

Auction theory has focused for a long time on the simplest case: there is a single, non-divisible item to be sold to a set of predefined buyers in a one-shot auction. The chosen mechanism indicates which buyer (if any) gets the item and at which price. The seminal works of Vickrey (1961), Myerson (1981) and Riley and Samuelson (1981) emphasize the importance of the information structure of an auction system. It consists of the information owned privately by the buyers and the information that the seller has on each buyer. This information owned privately by the buyers is the value they give to the item, i.e, the highest price they are willing to pay to get the item. The uncertainties upon these different values lie at the gist of the seller's optimization problem: otherwise, she would just have to sell the item to the buyer with the highest value, at this price or infinitesimally less.

To handle this deficit of information about buyers, it is standard to take a "Bayesian" viewpoint and assume that the seller has some probabilistic prior on the values given to the item by each bidder. This prior distribution is usually called the value distribution and it encompasses the seller's uncertainty on a specific bidder's values. There are of course several possibilities for how this value distribution is constructed. For instance, in wine or art auctions, it often comes from expert knowledge about an admissible price for a good wine bottle or for an important piece of art.

1.2 Learning theory and auction design

It is now possible for Internet platforms to run billions of auctions a day and store most of the historical data coming from them. This digitization of auction mechanisms was the first step into gathering data to optimize selling mechanisms. Auctions are now used in most Internet platforms to organize interactions between the different stakeholders. eBay was one of the first big online platforms to use ascending auction to sell objects on the platform. Google and most search engines companies started to use auctions to sell ad opportunities on their front page. For instance, they let advertisers bid on some keywords to get sponsored links above the first results for a certain user query. Nowadays, Facebook and LinkedIn are also using them to determine which ad to display, Amazon and most e-commerce marketplaces decide which products are going to be sponsored (and/or advertised) through an auction mechanism and auctions are also used to sell carbon permits by the European Union or to run large electricity markets.

To exploit this new source of available information (i.e., enormous datasets of past bids), practitioners used advanced statistical learning algorithms in connection with the classical Bayesian theory. Indeed, beyond the AI hype, machine learning algorithms are now widely applied in the industry for numerous applications: the value distribution is no longer coming from some given and fixed prior, but learned (hopefully accurately and efficiently) on historical— bidding—data. The first large-scale field experiment in production showed how engineers at Yahoo could handle their huge datasets to learn an optimal reserve price per key word (Ostrovsky and Schwarz, 2011). This results in data-driven mechanisms whose design use techniques coming from a large variety of fields, including statistics, machine learning, game theory and Economics. Similarly, bidders on these online platforms also gather data and use new statistical learning techniques to improve their bidding strategies against automated mechanisms. This flood of data and the associated paradigm shift it constitutes opens many new interesting practical problems, new theoretical questions and new interesting games to study.

1.2.1 Repeated auctions only from a seller's standpoint

The first natural repeated game setting consists of understanding how the seller can learn a revenue-maximizing auction mechanism from a dataset of bids or values. In the example of eBay marketplace, the seller (eBay) observes numerous auctions a day for similar items. Hence, from its point of view, the mechanism is repeated and she can aim at optimizing some long-term revenue. On the contrary, buyers are individuals that participate in a few, if not a single, auctions at best. Then, from their point of the view, the mechanism still looks like a one-shot auction and they are bound to implement myopic short-term strategies, optimizing point-wisely their utility (by opposition to long-term and effectively in expectation). Let us consider the simplifying assumption where bidder values on the platform are sampled from a certain unknown distribution, that encompasses the variability in their readiness to pay a certain price. Assuming the bidders actually bid their true value (for instance, if the mechanism chosen is fixed and "incentive-compatible", i.e., bidding one's value is optimal for buyers), the seller has then access at the end of the day to a dataset of buyer values.

Inspired by the computational learning formalism, Elkind (2007), Balcan *et al.* (2008), and Cole and Roughgarden (2014) initiated a line of research aiming at finding approximations of the revenue-maximizing auction, if possible, efficiently, with approximation guarantees depending on the size of the dataset gathered (a.k.a., the sample complexity). This setting is called the *batch learning setting.* A variant considers the case where the flow of buyers is continuously coming on the platform and the seller can update continuously her mechanism. This is *the online* *learning setting* introduced in Cesa-Bianchi *et al.* (2014). In all these problems, it is crucial that the samples gathered in the dataset have the same distribution as the samples that will be gathered and treated in the future.

1.2.2 Repeated auctions from seller and bidder standpoint

The crucial assumption of myopic/short-sighted/impatient bidders facing a patient seller is unfortunately not necessarily satisfied, depending on the setting. In modern-day practice, typically large online ad platforms, such as Google DoubleClick or AppNexus, are selling ad opportunities for large publishers such as some of the biggest online newspapers. The main difference with the aforementioned eBay example is that only a few companies are actually bidding in these auctions. They are furthermore doing so repeatedly and participating in a massive number of auctions.

Indeed, most companies willing to display ads actually rely on third-parties, *demand-side platforms* (DSP), that are buying and displaying ads for them (because of technical constraints, even sending bids in real-time might actually be quite complex). These aggregated bidders are repeatedly interacting with the (same) seller, billions of times a day. Consequently, this type of buyer can also optimize for long-term utility and need not be myopic. Thus, even if the seller is using one-shot incentive compatible auctions - for instance to gather data in order to later design and switch to a revenue maximizing mechanism -, the bidder might have an interest in not bidding "truthfully", as classical theory would suggest is optimal for them. Indeed, if buyers do not bid their values, this will modify the distribution of "values" observed by the seller. Subsequently, the mechanism chosen to optimize their revenue will be different from what it would have been had the bidders been naïve, to the advantage of the buyers (Tang and Zeng, 2018; Nedelec *et al.*, 2019b).

Intuitively, this is possible because the information asymmetry that arose in the eBay example between the seller and the bidders – one optimizing over the long-term, the other over the short-term – is almost reversed. If the seller must commit to a specific mechanism or a family of mechanisms, for instance for contractual reasons, and buyers have this information, they can strategically leverage it by, for example, changing their bidding behavior. In the end, the respective utilities of the seller and buyers will somehow depend on the underlying amount of asymmetry between them. Several works have started studying various intermediate settings, for example, when bidders are (almost) identical (Kanoria and Nazerzadeh, 2014), or are patient, but not as patient as the seller (Amin *et al.*, 2013), etc.

1.3 Organization of the monograph

In this monograph, our overarching objective is to provide a widely accessible introduction to the fascinating topics of classical and modern auction theory while bringing to the fore the statistical and machine learning lenses to the topic. We will very clearly state the differences between the different information-asymmetry settings we will review, and point to cutting edge theoretical and practical solutions adapted to them. We will also show how new statistical tools can be used to tackle some important and well-known problems from Economics. Furthermore, those questions open many new interesting problems in Economics since algorithms are replacing classical sellers and buyers. We believe that modern auction theory offers a nice framework to understand what data and Computer Science can bring to modern Economics.

In Section 2, we survey the main results of the Bayesian auction literature, initiated with the seminal works of Vickrey and Myerson. Those results form some of the backbone of classical auction theory and are widely used in Internet practice. We will recall what is the revenue-maximizing auction once the seller has a prior on bidder's valuations and introduce some approximations of the revenue-maximizing auction when the seller must use simpler auctions. In Section 3, we focus on the setting derived from the eBay use case and tackle both the batch learning setting and the online learning setting. We recall some key concepts of statistical learning theory, derive the sample complexity of some of the learning algorithms used to compute a revenue-maximizing auction and show their computational complexity. In Section 4, we focus on the less studied but crucially important setting where bidders can be strategic regarding the mechanism itself since they have multiple interactions with the seller. We review some of the main methods that have been devised to keep bidders from being strategic in that context, show their limitations and introduce some very new results and approaches developed for bidders to take advantage of the seller's learning process.

This monograph only assumes basic familiarity with standard notions of Machine Learning, Statistics and Data Science and is written with a reader having this background in mind. We hope our monograph will be useful to engineers and researchers looking for an introduction to the beautiful and fast developing topics of modern auction theory and applications.

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