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**The Economics and  
Mathematics of  
Aggregation: Formal  
Models of Efficient  
Group Behavior**

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# The Economics and Mathematics of Aggregation: Formal Models of Efficient Group Behavior

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## The Economics and Mathematics of Aggregation: Formal Models of Efficient Group Behavior

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### Abstract

The goal of this article is to provide a general characterization of group behavior in a market environment. A crucial feature of our approach is that we do not restrict the form of individual preferences or the nature of individual consumptions; we allow for public as well as private consumption, for intragroup production, and for any type of consumption externalities across group members. Our only assumption is that the group always reaches Pareto efficient decisions.

We analyze two main issues. One is testability: what restrictions (if any) on the aggregate demand function characterize the efficient behavior of the group? The second question relates to identifiability; we investigate the conditions under which it is possible to recover the underlying structure — namely, individual preferences, the decision process and the resulting intragroup transfers — from the group's aggregate behavior.

Our approach applies to large (markets) or small (households) groups, with both private and public consumptions, with and without restrictions on trade, with monetary or real endowments. In particular,

our approach generalizes the classical analysis of the aggregate demand of a market economy, as pioneered by Gerard Debreu, Ralph Manted and Hugo Sonnenchein; we devote a section of our work to this specific but important case.

We show that in all these contexts, aggregation of individual behaviors involves a common mathematical structure, whereby the aggregate demand of the group, considered as a vector field, can be decomposed into a sum of gradients. The proper way to understand this structure, and ultimately to find necessary and sufficient condition for such a decomposition to be possible, is to use tools which were developed about 100 years ago, mainly by the French mathematician Elie Cartan, and which are known now-a-days as exterior differential calculus (EDC). The last section of this article is devoted to an exposition of EDC and contains the proofs of the results in the preceding ones.

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# 1

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## Introduction

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The study and characterization of economic behavior in a market context is one of the goals of microeconomic theory. Most existing results concentrate on two extreme cases. One is individual behavior. It has been known for at least one century that individual demand, as derived from the maximization of a single utility function under budget constraint, satisfies specific and stringent properties (homogeneity, adding up, Slutsky symmetry and negativeness). The alternative case concerns the aggregate behavior of a large number of agents. The main conclusion of the so-called Debreu–Mantel–Sonnenschein (henceforth DMS) literature<sup>1</sup> is that, if the number of agents exceeds the number of commodities, aggregate (excess) demand does not exhibit specific properties, except for the obvious ones (continuity, homogeneity, Walras Law). In other words, the standard assumptions of microeconomic theory generate considerable structure at the individual level, but this structure is essentially lost by (large) aggregation.

However, many interesting economic situations lie somewhere in-between these two polar situations. These are cases where the group

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<sup>1</sup>See Shafer and Sonnenschein (1982) for a general survey.

## 2 Introduction

under consideration includes more than one individual, but is not large enough for the aggregation results of DMS type to apply. For instance, standard demand theory uses data on households or families, most of which gather several individuals. The behavior of even large firms is routinely analyzed as stemming from the interaction of a small number of agents (management and unions, manager and shareholders, top manager and division heads, etc.), who bargain under some global financial constraint. The same remark obviously applies to committees, clubs, villages and other local organizations, who have also attracted much interest. In short, many economic decisions are made by small, multi-person groups.

The goal of this book is to provide a general characterization of group behavior in a market environment. The general problem we consider can be stated as follows. Consider a group consisting of  $S$  members. The group has limited resources; specifically, its global consumption vector  $\xi$  must satisfy a standard market budget constraint of the form  $p^T \xi = y$  (where  $p$  is a vector of prices and  $y$  is total group income). Any demand vector belonging to the global budget set thus defined can be consumed by the members. Some of the goods can be privately consumed, while others may be publicly used. Empirically, the intragroup allocation of resources or consumptions is not observable; the group is perceived as a ‘black box’, and only its *aggregate* behavior, summarized by the demand function  $\xi(p, y)$ , is recorded. A crucial feature of our approach is that we do *not* restrict the form of individual preferences (except for the standard convexity assumptions) or the nature of individual consumptions. That is, we allow for public as well as private consumption, for intragroup production, and for any type of consumption externalities across group members. What restrictions (if any) on the aggregate demand function characterize the efficient behavior of the group? And when is it possible to recover the underlying structure — namely, individual preferences, the decision process and the resulting intragroup transfers — from the group’s aggregate behavior? These are the main questions addressed in what follows.

Our work has a direct relationship with club theory; our models encompass situations in which groups share public consumptions (‘local public goods’), or more generally ‘confer externalities on each other’

(Scotchmer, 2002, p. 1999). The literature on clubs, however, is mostly concerned with group formation, on the one hand, and the existence of an equilibrium on the other hand — which in turn raises the question of the correct equilibrium concept. In this book, we consider the group as exogenously given, and we assume that the decision process, whatever its specific features, leads to efficient allocations (as we shall discuss below in some detail). Our main interest is in the empirical consequences of these assumptions: do they generate testable predictions on the group's (aggregate) behavior, and to what extent is it possible to recover the group's fundamentals (preferences, decision process) from the sole observation of this behavior (more on this below). In that sense, the two approaches are largely complementary.<sup>2</sup>

Finally, we use, throughout the book, a 'differentiable' perspective; i.e., we consider demand *functions*, which are moreover assumed to be 'smooth' (typically differentiable). An alternative viewpoint relies on a 'revealed preferences' approach, which only assumes the availability of finite sets of price and demand vectors. Although this viewpoint is not presented here, it is largely complementary to our analysis. The interested reader may profitably refer to the work of Cherchye et al. (2007).

*Decision process: the collective viewpoint.* Any analysis of group behavior faces a basic problem, namely the representation of the decision process adopted by the group. Various paths can be followed at this point. One is to construct a very detailed model, based on specific assumptions — say, a non-cooperative bargaining framework in which agents can make offers in a given order and according to detailed rules, while being characterized by specific fallback options. An obvious drawback of this approach is that, since the predicted outcome depends on

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<sup>2</sup>In addition, many technical details differ between the two approaches. For instance, the club literature typically consider individual budget constraints; while this assumption is not incompatible with our setting, our context a priori only requires a budget constraint to be satisfied at the group level (the particular case of individual budget constraints is however considered in Section 5). In clubs, agents typically optimize individually and cannot observe other agents' preferences or endowments, whereas we assume Pareto efficiency. Finally, we do not need specific assumptions on preferences (e.g., transferable utility, or "essential" private goods). For a general presentation of the club literature, see Scotchmer (2002).

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the particular rules adopted, empirical applications require a precise knowledge of these rules. We can certainly think of situations in which the rules governing the decision process are indeed publicly known; one may think, for instance, of the allocation of a commodity by an auction, or of the vote of a committee. Most of the time, however, the rules are unobservable; they may actually fail to exist, at least in the formal and explicit sense required by the theory.

In this book, we adopt an alternative, axiomatic perspective. Specifically, we follow the ‘collective’ approach, initially introduced by Chiappori (1988a,b, 1992) and Browning and Chiappori (1998), and simply assume that the group always reach Pareto efficient decisions. We view efficiency as a natural assumption in many contexts, and as a natural benchmark in all cases. The ‘collective’ point of view is indeed becoming a standard tool in household economics. Other models, in particular in the literature on firm behavior, are based on cooperative game theory in a symmetric information context, where efficiency is paramount (see for instance the ‘insider–outsider’ literature, and more generally the models involving bargaining between management and workers or unions). The analysis of intra-group risk sharing, starting with Townsend’s seminal paper (1994), provides other interesting examples. Finally, even in the presence of asymmetric information, first best efficiency is a natural benchmark. For instance, a large part of the empirical literature on contract theory tests models involving asymmetric information against the null of symmetric information and first best efficiency (see Chiappori and Salanie 2000 for a recent survey). However, it is important to note that we place no restriction on the form of the decision process beyond efficiency.

*Distribution factors.* In many situations, the group’s decision depends not only on prices, but also on factors that can affect the influence of various members on the decision process. Think, for instance, of the decision process as a bargaining game; since bargaining under symmetric information typically generates efficient outcomes, such a context is indeed a particular case of the collective model. Typically, the outcome of such a game depends on the members’ respective bargaining positions. It follows that *any factor of the group environment that may influence the members’ respective bargaining strengths will*

*potentially affect the outcome.* Such effects are of course paramount, and their relevance is not restricted to bargaining in any particular sense. In general, group behavior depends not only on preferences and budget constraint, but also on the members' respective 'power'; any variable that changes the powers may have an impact on observed collective behavior.

Variables that affect the members' decision powers within the group are usually called *distribution factors*. In many cases, such variables are readily observable. To take a very basic example, think of the group as a small open economy with private consumption only (in the DMS tradition). Any efficient outcome is an equilibrium; and the particular equilibrium that will prevail depends typically on individual incomes (or endowments). Then initial incomes are distribution factors for the group under consideration. Other examples are provided by the literature on household behavior. In their study of household labor supply, Chiappori et al. (2002) use the state of the marriage market, as proxied by the sex ratio by age, race and state, and the legislation on divorce as particular distribution factors affecting the intrahousehold decision process, hence its outcome (labor supplies in that case). They find, indeed, that any improvement in women's position (e.g., more favorable divorce laws, or excess 'supply' of males on the marriage market) significantly decreases (resp. increases) female (resp. male) labor supply. In a similar context, Rubalcava and Thomas (2000) refer to the generosity of single parent benefits and reach identical conclusions. Thomas et al. (1997), using an Indonesian survey, show that the distribution of wealth by gender at marriage — another candidate distribution factor — has a significant impact on children health in those areas where wealth remains under the contributor's control.<sup>3</sup> Duflo (2003) has derived related conclusions from a careful analysis of a reform of the South African social pension program that extended the benefits to a large, previously not covered black population. She finds that the recipient's gender — still a typical distribution factor — matters for the consequences of the transfers on children's health. Finally, Attanasio and Lechène (2009), studying the conditional cash transfer program 'Progresa' in Mexico,

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<sup>3</sup>See also Galasso (1999) for a similar investigation.

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which is given to women, show that the consequences over consumption of an increase in income due to Progresá is quite different from any other source of income. Again, they conclude that the gender of the recipient makes all the difference.

Practically, the observation of distribution factors crucially enhances the analytical performance of our models; they allow for more robust tests and more general identification procedures. Distribution factors will therefore play an important role in what follows.

*Testability and identifiability.* The collective model provides a very flexible tool for analyzing the decisions made by the group. It is compatible with a host of decision processes, from decentralized bargaining to centrally coordinated mechanisms; with a number of structural frameworks, involving private or public consumption, externalities and intragroup production; and with any allocation of powers within the collectivity. While this flexibility is a major advantage of the approach, one may fear it is excessive, in the sense that it deprives the model from any empirical content; i.e., it is not the case that *any* behavior is compatible with (some version of) this general setting? We shall see that, on the contrary, the model, general as it is, still generates *testable implications* on the group's behavior, at least when the size of the group is small enough; remember that from DML, no testable implication can be found for 'large' groups. In a Popperian view, testability is a standard requirement of a scientific approach, hence the importance of this result.

Another requirement is related to what is usually called 'identifiability' of the collective framework. As for any theoretical model, the collective approach relates unobservable, structural parameters (individual preferences, intragroup decision process) with observed, 'reduced-form' behavior — here, an aggregate demand function. Ultimately, our interest is mostly directed at the underlying structure, either because we want to understand the characteristics of the decision process or because we are concerned with welfare issues. An important question, therefore, is whether it is possible to recover that structure from the type of data that is available — i.e., demand behavior.

Testability on the one hand, identifiability on the other hand are the two main problems that this book will analyze.

*Identifiability and identification.* From a methodological perspective, it may be useful to define more precisely what is meant by ‘recovering the underlying structure’. The structure, in our case, is defined by the (strictly convex) preferences of individuals in the group and the decision process. Because of the efficiency assumption, for any particular cardinalization of individual utilities the decision process is fully summarized by the Pareto weights corresponding to the outcome at stake. The structure, thus, consists in a set of individual preferences (with a particular cardinalization) and Pareto weights (with some normalization — e.g., the sum of Pareto weights is taken to be one).

This structure is not observable; what can be recorded is the group’s aggregate demand function  $\xi(p, y)$ . In practice, the ‘observation’ of  $\xi(p, y)$  is a complex process, that entails specific difficulties. For instance, one never observes a (continuous) function, but only a finite number of values on the function’s graph. These values are measured with some errors, which raises problems of statistical inference. In some cases, the data are cross-sectional, in the sense that different groups are observed in different situations; specific assumptions have to be made on the nature and the form of (observed and unobserved) heterogeneity between the groups. Even when the same group is observed in different contexts (say, from panel data), other assumptions are needed on the dynamics of the situation, e.g., on the way past behavior influences present choices. All these issues, which lay at the core of what is usually called the *identification* problem, are outside the scope of this paper.

Our interest, here, is in what has been called the *identifiability* problem, which can be defined as follows: when it is the case that the (hypothetically) perfect knowledge of a smooth *demand function*  $\xi(p, y)$  uniquely defines the underlying structure within a given class? Formally, for any given structure, the maximization of the (Pareto) weighted sum of utilities generates a unique demand function. This defines a mapping from the set of structures to the set of demand functions. Identifiability obtains if this mapping is *injective*, in the sense that two different structures can never generate the same demand function. In other words, non-identifiability does not result from the econometrician’s inability to exactly recover the form of demand functions — say, because only noisy estimates of the parameters can be

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obtained, or even because the functional form itself (and the stochastic structure added to it) is inadequate — but from deeper structural reasons. The identification problem has, at least to some extent, econometric or statistical answers. For instance, confidence intervals can be computed for the parameters (and become negligible when the sample size grows); the relevance of the functional form can be checked using specification tests; etc. The non-identifiability problem has a different nature: even if a *perfect* fit to *ideal* data was feasible, it would still be impossible to recover the underlying structure from observed behavior.<sup>4</sup>

In the case of *individual* behavior, as analyzed by standard consumer theory, identifiability is an old but crucial result. Indeed, it has been known for several decades that under minimal smoothness conditions,<sup>5</sup> an individual demand function uniquely identifies the underlying preferences. Usual as this property may have become, it remains one of the strongest results in microeconomic theory. It implies, for instance, that assessments about individual well-being can unambiguously be made from the sole observation of demand behavior — a fact that opens the way to all of applied welfare economics. The present work can be seen as an attempt at generalizing this classical identifiability property to efficient groups of arbitrary sizes.

Identifiability is a necessary condition for identification. If different structures are observationally equivalent, there is no hope that observed behavior will help to distinguish among them — only ad hoc functional form restrictions can do that. Since observationally equivalent models may have very different welfare implications, non-identifiability severely limits our ability to formulate reliable normative judgments: any normative recommendation based on a particular structural model is unreliable, since it is ultimately based on the purely arbitrary choice of one underlying structural model among many.

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<sup>4</sup>The distinction between identification and identifiability can be traced back to Koopmans's (1949) seminal paper (we thank Martin Browning for suggesting this reference). A difference is that Koopmans defines a 'structure' as 'a combination of a specific set of structural equations and a specific distribution function of the latent variables' — a 'model' being defined as a 'set of structures'. Koopmans clearly distinguishes two types of identification problems, namely those linked with 'statistical inference' and those due to 'identifiability'.

<sup>5</sup>Essentially, preferences must be lipschitzian (see Mas Colell, 1977).

Note, finally, that identifiability is only a necessary first step for identification (in the standard, econometric sense). Whether an *identifiable* model is econometrically *identified* depends on the stochastic structure representing the various statistical issues (measurement errors, unobserved heterogeneity, ...) discussed above. After all, the abundant empirical literature on consumer behavior, while dealing with a model that is always identifiable, has convinced us that identification crucially depends on the nature of available data.

*Parametric versus non-parametric identifiability.* The identifiability problem may be approached from a parametric or a non-parametric perspective. In the parametric approach, a particular functional form is chosen for the structural model, and a reduced form for the demand function is derived. In such a context, uniqueness or identifiability are *conditional on the functional form*; i.e., they obtain (at best) within a specific and narrow set of candidate functions, namely those compatible with the functional form chosen at the outset. Throughout this paper, our approach, on the contrary, is explicitly *non-parametric*. That is, we try to find conditions that guarantee uniqueness within the general class of smooth, strictly convex preferences and differentiable Pareto weights. One can readily provide examples in which identifiability obtains in a parametric sense, but not in the non-parametric setting (it is then functional form dependent).<sup>6</sup>

In practice, parametric models are often convenient. In particular, we do *not* suggest that parametric estimations should not be used, or even that it should be resorted to with some reluctance. Postulating a specific functional form is a standard, well established and often extremely fruitful methodology. We do however submit that the status of the conclusions drawn from parametric estimations crucially depend on whether or not the underlying model is *non-parametrically identifiable*. If it is, then the reliability of the parametric estimates (and, consequently, of the conclusions drawn from it) is directly related to the quality of the empirical fit, as assessed by standard econometric tests. If the econometrician can convince himself (and the scientific community) that the model provides a pretty faithful representation of

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<sup>6</sup>See for instance Blundell et al. 2006.

the real phenomenon, then the same level of trust could in principle be put into the conclusions derived from it. The case is much weaker in the absence of non-parametric identifiability. A good empirical fit is no longer sufficient: by definition, many different structural models, with potentially divergent normative implications, have exactly the same fit (since they generate the same reduced forms), hence are exactly as well supported by the data as the initial one.

Of course, this discussion should not be interpreted too strictly. In the end, identifying assumptions are (almost) always needed. The absence of non-parametric identifiability, thus, should not necessarily be viewed as a major weakness. We believe, however, that it justifies a more cautious interpretation of the estimates. More importantly, we submit, as a basic, methodological rule, that an explicit analysis of non-parametric identifiability is a necessary first step in any consistent empirical strategy — if only to suggest the most adequate identifying assumptions. Applying this approach to collective models is indeed an important outcome of our approach.

*Structure of the book and main results.* We may now briefly describe the general structure of the book. Section 2 describes the basic framework under consideration; it describes the main concepts and assumptions, and states the questions that will be addressed in the remainder. Section 3, which discusses the first problem, namely the characterization of aggregate demand, which establish several results. First, whenever the number of commodities is strictly larger than the number of group members, the model generates strong testable restrictions on group demand. These restrictions take the form of partial differential equations and inequations (the ‘ $\text{SNR}(H - 1)$ ’ conditions) that directly generalize the standard Slutsky conditions of consumer theory. Moreover, these conditions are also sufficient, at least for local integration: for any ‘smooth’ aggregate demand  $X$  satisfying  $\text{SNR}(H - 1)$  and for any regular point  $p$ , it is possible to construct a group of  $H$  members such that an optimal consumption plan for the group aggregates into  $X$  on an open neighborhood of  $p$ . Finally, the conditions remain sufficient for local integration even under the additional restrictions that commodities are either all privately or all

publicly consumed. A by-product of this result is that the private or public nature of intragroup consumption is not testable from aggregate data on group demand.

Section 4 addresses the second problem, namely identification. We first show that in its most general formulation, the model is not identifiable: any given aggregate demand satisfying  $\text{SNR}(H - 1)$  can be derived either from a continuum of models with private consumption only, or from a continuum of models with public consumption only, or actually from a continuum of other models. However, a simple exclusion assumption is in general sufficient to guarantee full, non-parametric identifiability of the welfare-relevant structure. Specifically, the *collective indirect utility* of each member is defined as the utility level that member ultimately reaches for given prices, household income and possibly distribution factors, taking into account the allocation of resources prevailing within the household. If each agent of the group is excluded from consumption of (at least) one commodity, then, in general, the *collective indirect utility* of each member can be recovered (up to some increasing transform), irrespective of the total number of commodities. The result obtains only ‘in general’ because one can characterize pathological situations in which it does not hold; technically, the demand function must then satisfy a set of partial differential equations (in that sense, these cases are ‘non-generic’). Ironically, the most striking example of such a pathology is the widely used unitary setting, in which the group is described as if there was a unique decision maker. While analytically convenient, the unitary representation entails a huge cost, since it precludes the (non-parametric) identification of individual consumption and welfare. One of the main conclusions of this section, therefore, is that in a general sense, *non-unitary models are indispensable for addressing issues related to intrahousehold behavior*.

Sections 5 and 6 are devoted to a particular but widely studied case, that of an exchange economy. The restriction, here, is twofold: each commodity is privately consumed (without externality), so that the group can be analyzed as an exchange economy (efficiency is then a consequence of the welfare theorems); and the intragroup distribution of resources is characterized either by constant individual endowments

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(the excess demand case, in Shafer and Sonnenschein's 1982 terminology) or constant nominal incomes (the market demand case). Section 5 deals with the case of complete markets. We first consider the so-called 'Sonnenschein problem', which can be stated as follows: in a 'large' economy (i.e., one with more agents than commodities), which properties, if any, characterize aggregate demand? We provide a new and shorter proof for the case of aggregate excess demand, initially solved by Debreu (1974) and Mantel (1974, 1976); we then describe the proof of the (more difficult) market demand case, solved by Chiappori and Ekeland (1999a), before considering in last subsection the case of a 'small' economy. Section 6 extends the previous results to incomplete markets, and generally to constraints on trade.

Throughout this book, the techniques of exterior differential calculus are repeatedly used; they are absolutely crucial to many of the proofs provided. Since these tools may not be familiar to most mathematical economists, we provide a detailed presentation in an Appendix. The Appendix is self-contained, and can be read independently of this book; we believe that its usefulness goes beyond aggregation theory.

## References

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- Aloqeili, M. (2004), ‘The generalized Slutsky relations’. *Journal of Mathematical Economics* **40**, 71–91.
- Andreu, J. (1982), ‘Rationalization of market demand on finite domains’. *Journal of Economic Theory* **28**(1), 201–204.
- Attanasio, O. and V. Lechène (2009), ‘Efficient responses to targeted transfers’. working paper, UCL.
- Browning, M., F. Bourguignon, P. A. Chiappori, and V. Lechene (1994), ‘Incomes and outcomes: A structural model of intra-household allocation’. *Journal of Political Economy* **102**, 1067–1096.
- Browning, M. and P. A. Chiappori (1998), ‘Efficient intra-household allocations: A general characterization and empirical tests’. *Econometrica* **66**, 1241–1278.
- Browning, M., P. A. Chiappori, and A. Lewbel (2005), *Estimating Consumption Economies of Scale, Adult Equivalence Scales, and Household Bargaining Power*. Mimeo, Boston College.
- Browning, M., P. A. Chiappori, and Y. Weiss, *Household Economics*. Cambridge University Press, forthcoming.
- Bryant, R., S. Chern, R. Gardner, H. Goldschmidt, and P. Griffiths (1991), *Exterior Differential Systems*. MSRI Publications (18), Springer-Verlag.

- Cherchye, L., B. De Rock, and F. Vermeulen (2007), 'The collective model of household consumption: A nonparametric characterization'. *Econometrica* **75**(2), 553–574.
- Chiappori, P. A. (1988a), 'Nash-bargained household decisions: A comment'. *International Economic Review* **29**, 791–796.
- Chiappori, P. A. (1988b), 'Rational household labor supply'. *Econometrica* **56**, 63–89.
- Chiappori, P. A. (1992), 'Collective labor supply and welfare'. *Journal of Political Economy* **100**, 437–467.
- Chiappori, P. A. and O. Donni (2006), *Learning From a Piece of Pie: The Empirical Content of Nash Bargaining*. Mimeo, Columbia University.
- Chiappori, P. A. and I. Ekeland (1997), 'A convex darbox theorem'. *Annali della Scuola Normale Superiore di Pisa* **4.25**, 287–297.
- Chiappori, P. A. and I. Ekeland (1999a), 'Aggregation and market demand : An exterior differential calculus viewpoint'. *Econometrica* **67**(6), 1435–1458.
- Chiappori, P. A. and I. Ekeland (1999b), 'Disaggregation of excess demand functions in incomplete markets'. *Journal of Mathematical Economics* **31**, 111–129.
- Chiappori, P. A. and I. Ekeland (1999c), 'Disaggregation of excess demand functions in incomplete markets'. *Journal of Mathematical Economics* **31**, 111–129.
- Chiappori, P. A. and I. Ekeland (2000), 'Corrigendum to "Disaggregation of excess demand functions in incomplete markets"'. *Journal of Mathematical Economics* **33**, 531–532.
- Chiappori, P. A. and I. Ekeland (2004), 'Individual excess demand'. *Journal of Mathematical Economics* **40**, 41–57.
- Chiappori, P.-A., B. Fortin, and G. Lacroix (2002), 'Marriage market, divorce legislation and household labor supply'. *Journal of Political Economy* **110**(1), 37–72.
- Chiappori, P. A. and B. Salanié (2000), 'Empirical applications of contract theory: A survey of some recent work'. invited lecture, World Congress of the Econometric Society; forthcoming in *Advances in Economics*, M. Dewatripont and L. Hansen, eds.

- Debreu, G. (1974), 'Excess demand functions'. *Journal of Mathematical Economics* **1**, 15–23.
- Diamond, P. and M. Yaari (1972), 'Implications of the theory of rationing for consumers under uncertainty'. *American Economic Review* **62**, 333–343.
- Diewert, W. E. (1977), 'Generalized Slutsky conditions for aggregate consumer demand functions'. *Journal of Economic Theory* **15**, 353–362.
- Donni, O. (2003), 'Collective household labor supply: Non-participation and income taxation'. *Journal of Public Economics* **87**, 1179–1198.
- Duffie, D. (1992), 'The nature of incomplete security markets'. In: J.-J. Laffont (ed.): *Advances in economic theory (6th World Congress)*. Econometric Society Monographs, Cambridge University Press, pp. 214–262.
- Duflo (2003), 'Grandmothers and granddaughters: Old age pension and intra-household allocation in South Africa'. *World Bank Economic Review* **17**(1), 1–25.
- Ekeland, I. and N. Djitte (2006), 'An inverse problem in the economic theory of demand'. *Annales de l'Institut Henri Poincaré "Analyse non linéaire"* **23**, 269–281.
- Ekeland, I. and L. Nirenberg (2002), 'The convex Darboux theorem'. *Methods Appl. Anal.* **9**, 329–344.
- Epstein, L. G. (1981), 'Generalized duality and integrability'. *Econometrica* **49**, 655–678.
- Fong, Y. and J. Zhang (2001), 'The identifiability of unobservable independent and spousal leisure'. *Journal of Political Economy* **109**(1), 191–202.
- Fortin, B. and G. Lacroix (1997), 'A test of neoclassical and collective models of household labor supply'. *Economic Journal* **107**, 933–955.
- Galasso, E. (1999), *Intrahousehold Allocation and Child Labor in Indonesia*. Mimeo, BC.
- Geanakoplos, J. and H. Polemarchakis (1980), 'On the disaggregation of excess demand functions'. *Econometrica* pp. 315–331.
- Kapan, T. (2009), 'Essays on household bargaining'. PhD Dissertation, Columbia University.

- Laisney, F. (2004), 'Welfare analysis of fiscal and social security reforms in Europe: Does the representation of family decision process matter?'. *Review of Economics of the Household*, (forthcoming).
- MacGill, M. and M. Quinzii (1997), *Incomplete Markets*. Vol 1, MIT Press.
- Mantel, R. (1974), 'On the characterization of aggregate excess demand'. *Journal of Economic Theory* **7**, 348–353.
- Mantel, R. (1976), 'Homothetic preferences and community excess demand functions'. *Journal of Economic Theory* **12**, 197–201.
- Mantel, R. (1977), 'Implications of economic theory for community excess demand functions'. Cowles Foundation Discussion Paper no. 451, Yale University.
- Mas Colell, A. (1977), 'The recoverability of consumer's preferences from market demand behavior'. *Econometrica* **45**(6), 1409–1430.
- Rubalcava, L. and D. Thomas (2000), *Family Bargaining and Welfare*. Mimeo RAND, UCLA.
- Scotchmer, S. (2002), 'Local public goods and clubs'. In: A. J. Auerbach and M. Feldstein (eds.): *Handbook of Public Economics, vol. 4*. Elsevier.
- Shafer, W. and H. Sonnenschein (1982), 'Market demand and excess demand functions'. In: K. Arrow and M. Intriligator (eds.): *Handbook of Mathematical Economics*. vol 2, chapter 14, North-Holland.
- Sonnenschein, H. (1973), 'Do Walras' identity and continuity characterize the class of community excess demand functions'. *Journal of Economic Theory* **6**(4), 345–354.
- Thomas, D., D. Contreras, and E. Frankenberg (1997), *Child Health and the Distribution of Household Resources at Marriage*. Mimeo RAND, UCLA.
- Tobin, J. (1952), 'A survey of the theory of rationing'. *Econometrica* **20**, 521–553.
- Townsend, R. (1994), 'Risk and insurance in village India'. *Econometrica* **62**, 539–591.
- Vermeulen, F. (2005), 'And the winner is... An empirical evaluation of two competing approaches to household labour supply'. *Empirical Economics* **30**(3), 711–734.