Marketing Dynamics: A Primer on Estimation and Control

Prasad A. Naik
University of California, Davis, USA
panaik@ucdavis.edu
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Prasad A. Naik
University of California, Davis, USA
panaik@ucdavis.edu
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Abstract

This primer provides a gentle introduction to the estimation and control of dynamic marketing models. It introduces dynamic models in discrete- and continuous-time, scalar and multivariate settings, with observed outcomes and unobserved states, as well as random and/or time-varying parameters. It exemplifies how various dynamic models can be cast into the unifying state space framework, the benefit of which is to use one common algorithm to estimate all dynamic models.

The primer then focuses on the estimation part, which answers questions such as: how much is the sales elasticity of advertising? How much sales lift can managers expect for a certain level of price promotion? What is the best sales forecast for the next quarter? The estimation relies on two principles: Kalman filtering and the likelihood principle. The Kalman filter recursively infers the means and covariances of an unobserved state vector as the observed outcomes arrive over time. This evolution of moments is then embedded in the likelihood function to obtain parameter estimates and their statistical significance.

Next, the primer elucidates the control part, which answers questions such as: how much should managers spend on advertising over time and across regions? What is the best promotional timing and depth? How should managers optimally respond to competing brands’ actions and resulting outcomes? The control part relies on the maximum principle and the optimality principle. Pontryagin’s maximum principle allows managers to determine the optimal course of action (for example, the optimal levels and timing of advertising spends or price promotions) to attain a specified goal, such as profit maximization. Bellman’s optimality principle, on the other hand, offers insights into optimal course correction when implementing the best plan as the state of a system varies dynamically and/or stochastically. Finally, the
primer presents three examples on the application of optimal control, differential games, and stochastic control theory to marketing problems, and illustrates how to discover novel insights into managerial decision-making.
In 1696, Johann Bernoulli posed a challenge to his contemporary “sharpest mathematical minds of the globe” with the following problem: If in a vertical plane two points A and B are given, then what is the trajectory of an object C starting from A to arrive at B in the shortest possible time falling under its own weight? He added that “this problem ... is not ... purely speculative and without practical use ... Rather it even appears ... that it is very useful also for other branches of science than mechanics.” This problem is called Brachystochrone problem because, in Greek, βραχιστζ = shortest and χρονoζ = time. Leibniz described this problem as “splendid” and furnished the solution (a cycloid) in a letter to Bernoulli, while Newton presented his solution to the Royal Society anonymously\footnote{Johann Bernoulli ascribes the anonymous solution to Newton because he noted that “you can tell the lion by its claws” (ex ugue leonem). The solution is the cycloid curve for the point C whose coordinates \((x(t), y(t))\) evolve, starting from the point A at \((0,0)\) at \(t = 0\), according to \(x(\theta) = \alpha(\theta - \sin(\theta))\) and \(y(\theta) = \alpha(\cos(\theta) - 1)\), where \(\theta(t) = t\sqrt{g/\alpha}, g = 9.8 \text{ m/sec}^2\), and the parameters \((\alpha, T)\) are determined by the terminal condition \((x(\theta(T)), y(\theta(T))) = B\).} For the quoted text and a definitive account of the intellectual history, see Sussmann and Willems [1997].
This event gave birth to the mathematical branch, later known as the calculus of variation, due to Leonhard Euler, who was Bernoulli’s student and who discovered what is now called the Euler’s equation to solve such dynamic optimization problems. Intense mathematical research over the subsequent three hundred years culminated into the modern control theory. Consistent with Bernoulli’s prognosis, this theory found use in launching man on Moon, landing Curiosity on Mars, deploying unmanned drones or designing driverless cars, high precision manufacturing using robots, providing navigation guidance turn by turn to users on roads, seas, and air, besides numerous applications in “other branches of science” including Operations Research (for example, Berstekas [2005]), Economics (for example, Stokey and Lucas [1989], Aghion and Howitt [1998], Ljungqvist and Sargent [2004], Weber [2011], Kamien and Schwartz [2012]), Management Science (for example, Sethi and Thompson [2000], Dockner et al. [2000]), and Marketing (for example, Erickson [2003], Jørgensen and Zaccour [2004]).

In Marketing, the point A represents the current state of the company’s brand sales or consumer’s utility. The point B marks the desired state the decision-maker wants to arrive at. The object C is the decision-maker (for example, CEO, brand managers, consumers), and its use of own weight denotes the set of actions (for example, price, advertising, brand choices) available for transitioning the system from state A to state B. The shortest time specifies the decision-maker’s objectives (for example, maximize the stream of future profit or utility). In Section 2 I clarify the terms state, system, transition, actions (or controls), and objectives, but note here that this simple abstraction is “splendid” because it not only unifies diverse problems across many applications, but also offers a systematic approach for solving them.

The purpose of this primer is to impart this systematic approach for solving dynamic marketing problems. To pursue this pedagogical focus, this article does not aim to review dynamic models in the extant marketing literature, for which readers are referred to [Bowman and Gatignon 2010], [Neslin and van Heerde 2009], [Shankar 2008], [Hanssens et al. 2003], and [Leeflang et al. 2000].

Solving dynamic problems involves two distinct topics: parameter estimation and optimal control. The former refers to describing
the system of relations among current states, past states, and actions via econometric time-series models (for example, Pauwels [2004], Steenkamp et al. [2005]). The latter refers to managing the system by determining the optimal course of actions to execute over the future planning horizon (for example, Kumar et al. [2008], Esteban-Bravo et al. [2014]). The foundation for parameter estimation stands on the Kalman Filter and the Likelihood Principle, whereas that for optimal control stands on the Pontryagin’s Maximum Principle and Bellman’s Principle of Optimality.

This monograph elucidates these four principles and related concepts, focusing on how to estimate dynamic models in Sections 3 and 4, and how to solve the control problems in Sections 5 and 6. But first, I clarify the terms, present the diversity of dynamic models, unify them via the canonical state space model, and highlight the value of unification.


References


References


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