Power Control in Wireless Cellular Networks
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Power Control in Wireless Cellular Networks

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Abstract

Transmit power in wireless cellular networks is a key degree of freedom in the management of interference, energy, and connectivity. Power control in both the uplink and downlink of a cellular network has been extensively studied, especially over the last 15 years, and some of the results have enabled the continuous evolution and significant impact of the digital cellular technology.

This survey provides a comprehensive discussion of the models, algorithms, analysis, and methodologies in this vast and growing literature. It starts with a taxonomy of the wide range of power control problem formulations, and progresses from the basic formulation to more sophisticated ones. When transmit power is the only set of optimization variables, algorithms for fixed SIR are presented first, before turning to their robust versions and joint SIR and power optimization. This is followed by opportunistic and non-cooperative power control. Then joint control of power together with beamforming pattern, base station assignment, spectrum allocation, and transmit schedule is surveyed one-by-one.
Throughout the survey, we highlight the use of mathematical language and tools in the study of power control, including optimization theory, control theory, game theory, and linear algebra. Practical implementations of some of the algorithms in operational networks are discussed in the concluding section. As illustrated by the open problems presented at the end of most chapters, in the area of power control in cellular networks, there are still many under-explored directions and unresolved issues that remain theoretically challenging and practically important.
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Introduction

1.1 Overview

Transmit powers represent a key degree of freedom in the design of wireless networks. In both cellular and ad hoc networks, power control helps with several functionalities:

- *Interference management*: Due to the broadcast nature of wireless communication, signals interfere with each other. This problem is particularly acute in interference-limited systems, such as CDMA systems where perfect orthogonality among users is difficult to maintain. Power control helps ensure efficient spectral reuse and desirable user experience.

- *Energy management*: Due to the limited battery power in mobile stations, handheld devices, or any “nodes” operating on small energy budget, energy conservation is important for the lifetime of the nodes and even the network. Power control helps minimize a key component of the overall energy expenditure.

- *Connectivity management*: Due to uncertainty and time-variation of wireless channels, even when there is neither
signal interference nor energy limitation, the receiver needs to be able to maintain a minimum level of received signal so that it can stay connected with the transmitter and estimate the channel state. Power control helps maintain logical connectivity for a given signal processing scheme.

To define a scope that allows an in-depth treatment within 150 pages, we will focus on power control in cellular networks in this survey, emphasizing primarily its use in interference management while occasionally touching upon energy and connectivity management. Most of the treatment is devoted to uplink transmission from mobile station (MS) to base station (BS), although extensions to downlink transmission from a BS to MSs are sometimes discussed as well. In many formulations uplink problems are more difficult to solve, although there are exceptions like joint power control and beamforming, and in other formulations uplink and downlink problems present interesting duality relationships. Uplink power control is also often more important in systems engineering of cellular networks.\footnote{First, BS power consumption is of less importance in comparison to MS power consumption. Second, the downlink intra-cell interference is much smaller in comparison to uplink intra-cell interference, because maintaining orthogonality of resource allocation (e.g., code allocation in CDMA, tone allocation in OFDM, or frequency and time slot allocation in GSM) to MSs within a cell on the downlink is easily accomplished by the BS. Third, BS locations are fixed and inter-cell interference is less bursty.}

Within the functionality of interference management, there are several types of problem statements, including optimizing Quality of Service (QoS) metrics such as utility functions based on throughput and delay, achieving network capacity in the information-theoretic sense with technology-agnostic converse theorems, or maintaining network stability in queueing-theoretic sense when there are dynamic arrival and departure of users. This survey focuses on the first type of problems, which is already rich enough that a detailed taxonomy of problem formulations is warranted and will be provided later in this section.

Given the specialization stated above and the range of power control problems in wireline systems like DSL, it is clear that this survey only covers part of the broad set of problems in interference management. Within this scope, there is already a wide and growing range
of results that are mathematically interesting and practically important. After surveying the key formulations, their relationships with each other, and the key properties of convexity and decomposability in this opening section, we organize the core materials in eight sections. Sections 2–4 present the basic formulations, starting with the simplest case of power control with fixed equilibrium SIR targets in Section 2 and progressing to the case of controlling transient behaviors and admission in Section 3, and that of jointly controlling power and SIR assignment in Section 4. Sections 5 and 6 then present extensions to opportunistic and non-cooperative power control, respectively.

Power control is often conducted jointly with other resource allocation when spatial, spectral, and temporal degrees of freedom are offered. In Sections 7–9 we discuss joint power control and beamforming, base station assignment, frequency allocation, and scheduling, for both fixed SIR and variable SIR cases. Each of Sections 2–9 starts with an overall introduction and concludes with a discussion of open problems. The mathematical techniques of optimization theory, control theory, game theory, and linear algebra will also be highlighted across these eight sections. Practical impacts of the theory of power control in the wireless industry have been substantial over the years, and some of these engineering implications in operational networks are summarized in Section 10.

Power control in wireless networks has been systematically studied since the 1970s. Over the last 15 years, thanks to the tremendous growth of cellular networks and its transformative impacts on society, extensive research on cellular network power control has produced a wide and deep set of results in terms of modeling, analysis, and design. We have tried to include as many contributions in the bibliography as possible, to survey the key results and methodologies in a balanced manner, and to strike a tradeoff between a detailed treatment of each problem and a comprehensive coverage of all major issues. While these lofty goals may not have been attained to perfection, we hope this survey will serve both as a partial summary of the state-of-the-art and a sketchy illustration of the exciting open problems in the area of power control in cellular networks.
1.2 Notation

The following notation are used throughout this survey. Vectors are denoted in bold small letter, e.g., \( \mathbf{z} \), with their \( i \)th component denoted by \( z_i \). Matrices are denoted by bold capitalized letters, e.g., \( \mathbf{Z} \), with \( Z_{ij} \) denoting the \( \{i,j\} \)th component. Vector division \( \mathbf{x}/\mathbf{y} \) and multiplication \( \mathbf{x}\mathbf{y} \) are considered component-wise, and vector inequalities denoted by \( \succeq \) and \( \preceq \) are component-wise inequalities. We use \( \mathbf{D}(\mathbf{x}) \) to denote a diagonal matrix whose diagonal elements are the corresponding components from vector \( \mathbf{x} \). A summary of key notation is provided in Table 1.1 at the end of this section.

1.3 Taxonomy of Problem Formulations

1.3.1 Basic System Model

Consider a general multi-cell network where \( N \) MSs establish links to \( K \) BSs, as illustrated in Figure 1.1. We assume that each MS is served by one of the \( K \) BSs, thereby establishing \( N \) logical links. Let \( \sigma_i \) denote the serving BS for link \( i \).

Let \( C_i \) denote the set of links whose transmit power appear as interference to link \( i \) for a given receiver design. This definition allows us to consider both orthogonal and non-orthogonal uplinks. In a non-orthogonal uplink, such as CDMA, transmitted power from all links appear as interference, so we set \( C_i = \{ j | j \neq i \} \). For an orthogonal uplink, such as OFDM, links terminating on the same BS are orthogonal and do not contribute interference to one another. In this case, we set \( C_i = \{ j | \sigma_j \neq \sigma_i \} \).

Let \( h_{kj} \) denote the amplitude gain from MS \( j \) to BS \( k \). Define the \( N \times N \) power-gain matrix \( \mathbf{G} \) by

\[
G_{ij} = ||h_{\sigma_i,j}||^2,
\]

which represents the power gain from MS on link \( j \) to the receiving BS on link \( i \). Correspondingly define a normalized gain matrix \( \mathbf{F} \) where

\[
F_{ij} = \begin{cases} 
G_{ij}/G_{jj} & \text{if } j \in C_i, \\
0 & \text{if } j \notin C_i.
\end{cases}
\]

Let \( \mathbf{D}_h = \text{diag}(G_{11}, \ldots, G_{NN}) \) be the diagonal matrix containing direct link channel gains, which depend on \( h \).
### 1.3 Taxonomy of Problem Formulations

#### Table 1.1 Summary of key notation.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{C}, \mathcal{D}, \mathcal{X}, \mathcal{Y}, \mathcal{K}$</td>
<td>Sets in $\mathbb{R}^N$ ($\mathcal{X}$ denotes the closure of $\mathcal{X}$)</td>
</tr>
<tr>
<td>$g(\cdot), \phi(\cdot)$</td>
<td>Scalar-valued functions</td>
</tr>
<tr>
<td>$c, d$</td>
<td>Scalar constants</td>
</tr>
<tr>
<td>$\mathcal{L}(\cdot)$</td>
<td>Lagrangian</td>
</tr>
<tr>
<td>$\lambda_i, \mu_i, \nu_i$</td>
<td>Lagrangian multipliers (or prices)</td>
</tr>
<tr>
<td>$x^*$</td>
<td>Optimizer or stationary point of a problem</td>
</tr>
<tr>
<td>$x[t]$</td>
<td>Variable at the $t$th iteration</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Step size for an iterative algorithm</td>
</tr>
<tr>
<td>$N$ (indexed by $i$ or $j$)</td>
<td>Number of links (MS)</td>
</tr>
<tr>
<td>$K$ (indexed by $k$)</td>
<td>Number of base-stations (BS)</td>
</tr>
<tr>
<td>$M$ (indexed by $m$)</td>
<td>Number of BS antennas</td>
</tr>
<tr>
<td>$C_i$</td>
<td>A set of links interfering with link $i$</td>
</tr>
<tr>
<td>$h_{ki}$ or $h_{ki}$</td>
<td>Complex channel amplitude from MS $i$ to BS $k$</td>
</tr>
<tr>
<td>$G$</td>
<td>Absolute link gain matrix, $G_{ij} =</td>
</tr>
<tr>
<td>$F$</td>
<td>Normalized link gain matrix, $F_{ij} = G_{ij}/G_{ii}$ if $j \in C_i$ and $F_{ij} = 0$ O.W.</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Direct link gain matrix $D_h = \text{diag}{G_{11}, \ldots, G_{NN}}$</td>
</tr>
<tr>
<td>$D(\cdot)$</td>
<td>Diagonal matrix operator</td>
</tr>
<tr>
<td>$\gamma_i$ (in vector $\gamma$)</td>
<td>SIR value of link $i$</td>
</tr>
<tr>
<td>$n_i = E[z_i]$ (in vector $n$)</td>
<td>Thermal noise for link $i$</td>
</tr>
<tr>
<td>$v_i = \gamma_i n_i/G_{ii}$</td>
<td>Product of normalized noise with SIR target for link $i$</td>
</tr>
<tr>
<td>$\eta_i = n_i/G_{ii}$ (in vector $\eta$)</td>
<td>Normalized noise for link $i$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Rise-Over-Thermal</td>
</tr>
<tr>
<td>$p_i$ (in vector $p$)</td>
<td>Transmission power of MS $i$</td>
</tr>
<tr>
<td>$p_i^m$ (in vector $p^m$)</td>
<td>Transmit power constraint for link $i$</td>
</tr>
<tr>
<td>$q_i$ (in vector $q$)</td>
<td>Interference plus noise power for MS $i$</td>
</tr>
<tr>
<td>$q_i^m$ (in vector $q^m$)</td>
<td>Interference constraint for link $i$</td>
</tr>
<tr>
<td>$\Gamma, B$</td>
<td>A set of feasible SIRs</td>
</tr>
<tr>
<td>SIR$_i(p)$</td>
<td>SIR function for MS $i$</td>
</tr>
<tr>
<td>$\rho(\cdot)$</td>
<td>Spectral radius operator</td>
</tr>
<tr>
<td>$\mathbf{I}(p)$</td>
<td>Standard interference function</td>
</tr>
<tr>
<td>$R_I$</td>
<td>Feasibility index of a standard interference function</td>
</tr>
<tr>
<td>$\lambda_F$</td>
<td>Lyapunov exponent associated with matrix $F$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Lyapunov function of power control algorithm</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Robustness parameter</td>
</tr>
<tr>
<td>$K_{1+\epsilon}$</td>
<td>Invariant cone parameterized by robustness parameter</td>
</tr>
<tr>
<td>$\tilde{p}_i = \log p_i$</td>
<td>Log transformation of power</td>
</tr>
<tr>
<td>$\xi_i$</td>
<td>Energy consumption budget (percentage over total power)</td>
</tr>
<tr>
<td>$U_\alpha(\cdot)$ (with parameter $\alpha$)</td>
<td>Utility function (for $\alpha$ fairness)</td>
</tr>
<tr>
<td>$s_i$</td>
<td>Spillage factor</td>
</tr>
<tr>
<td>$\ell_i$</td>
<td>Load factor</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Convex combination weight</td>
</tr>
<tr>
<td>$S$ (indexed by $s$)</td>
<td>Number of states</td>
</tr>
<tr>
<td>$\pi_s$</td>
<td>Probability that the system is in state $s$</td>
</tr>
<tr>
<td>$p_{s,i}$</td>
<td>Transmit power of user $i$ in state $s$</td>
</tr>
<tr>
<td>$G_{s,ik}$</td>
<td>Path gain from base station to user $i$ in state $s$</td>
</tr>
</tbody>
</table>

(Countinued)
Table 1.1 (Continued).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{s,i}$</td>
<td>Performance measure of user $i$ in state $s$</td>
</tr>
<tr>
<td>$a_{i,j}$</td>
<td>Fixed weight on each $g_{s,j}^i(p_{s,j})$ to reflect priority</td>
</tr>
<tr>
<td>$v_i$</td>
<td>Minimum fraction of the total transmit power by user $i$</td>
</tr>
<tr>
<td>$\bar{v}_i$</td>
<td>Minimum fraction of the expected total system utility by user $i$</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>Stochastic gradient of Lagrangian</td>
</tr>
<tr>
<td>$P_T$</td>
<td>Total transmit power in downlink transmission</td>
</tr>
<tr>
<td>$\xi_i$</td>
<td>Signal-interference product of user $i$</td>
</tr>
<tr>
<td>$\psi_i$</td>
<td>Non-orthogonality factor in CDMA spreading code</td>
</tr>
<tr>
<td>$E_b/I_0$</td>
<td>The bit energy to interference density ratio</td>
</tr>
<tr>
<td>$L$ (indexed by $l$)</td>
<td>Number of (orthogonal) carriers</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Multiplexing gain for MS $i$ in a CDMA network</td>
</tr>
<tr>
<td>$D_t$ and $D_f$</td>
<td>Number of total information bits and bits in a packet</td>
</tr>
<tr>
<td>BER($\gamma_i$)</td>
<td>Bit error rate function</td>
</tr>
<tr>
<td>$f(\cdot)$</td>
<td>Packet success rate function</td>
</tr>
<tr>
<td>$A_i$</td>
<td>A set of feasible power allocation policies for MS $i$</td>
</tr>
<tr>
<td>$w_i$ (in matrix $W$)</td>
<td>Uplink beamforming vector for MS $i$</td>
</tr>
<tr>
<td>$\hat{w}_i$</td>
<td>Downlink beamforming vector for MS $i$</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Information symbol of unit power for MS $i$</td>
</tr>
<tr>
<td>$G_{ex}$</td>
<td>Extended coupling matrix for beamforming</td>
</tr>
<tr>
<td>$1$</td>
<td>A vector whose components are 1’s</td>
</tr>
<tr>
<td>$\hat{p}_i$</td>
<td>Downlink transmission power of MS $i$</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>The BS that serves MS $i$</td>
</tr>
<tr>
<td>$S_i$</td>
<td>A set of allowable BSs that MS $i$ can connect to</td>
</tr>
<tr>
<td>$b_i, B^m$</td>
<td>Bandwidth for MS $i$ and total bandwidth allowed</td>
</tr>
<tr>
<td>$r_i(\gamma_i)$</td>
<td>Rate function of link $i$</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>Instantaneous rate region</td>
</tr>
<tr>
<td>$\mathcal{X}$</td>
<td>The stable arrival rate region</td>
</tr>
<tr>
<td>Conv($\mathcal{R}$)</td>
<td>Convex hull of $\mathcal{R}$</td>
</tr>
<tr>
<td>$T$</td>
<td>Length of time resource</td>
</tr>
</tbody>
</table>

Let $p_j$ be the transmit signal power on link $j$. Since $h_{a,j}$ is the path gain from MS on link $j$ to its serving BS, the receiver on link $j$ receives the signal at the power of $p_j\|h_{a,j}\|^2 = G_{ij}p_j$. If $j \in C_i$, this transmission will appear as interference to link $i$ with a power of $\|h_{a,j}\|^2p_j = G_{ij}p_j$. The total interference and noise at the BS serving MS $i$ is given by

$$q_i = \sum_{j \in C_i} G_{ij}p_j + n_i = \sum_{j=1}^M F_{ij}G_{jj}p_j + n_i,$$

where $n_i \geq 0$ is the power of noise other than interference from other links. In matrix notation, (1.3) can be written as

$$q = FDh p + n.$$

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Let $\gamma_i$ be the SIR achieved by link $i$. With the above notation, $\gamma_i = G_{ii}p_i/q_i$, or equivalently,

$$D_{h}p = D(\gamma)q,$$

(1.5)

where $D(\gamma) = \text{diag}(\gamma_1, \ldots, \gamma_M)$.\(^2\) Combining (1.4) and (1.5), we get the following basic equations relating the key quantities:

$$q = FD(\gamma)q + n,$$

(1.6)

and

$$D_{h}p = D(\gamma)FD_{h}p + D(\gamma)n.$$

(1.7)

An important factor that determines the total uplink capacity in commercial networks today is the interference limit $q^\text{in}$, often stated

---

\(^2\) The difference between the $D(\gamma)$ and $D_{h}$ notation is that the diagonal entries of $D(\gamma)$ is exactly $\gamma$ while those of $D_{h}$ are functions of $h$. 
in the form $q^m = \kappa n$ for some constant $\kappa \geq 1$. With this definition, the interference, $q_i$, at each BS $i$, is not allowed to be larger than a factor $\kappa$ greater than the thermal noise $n_i$. The factor, $\kappa$, is called the Interference over Thermal (IOT), and typically quoted in dB,

$$\text{IOT} = 10\log_{10}(\kappa).$$

A related measure is the Raise over Thermal (ROT), the ratio of the interference and the signal power to the thermal noise. The IOT limits bound the interference to the cell and the power required for new MSs to access the network. Typical IOT values in commercial networks range from 3 to 10 dB.

1.3.2 Optimization Variables

Whether a power control problem is formulated as cooperative or non-cooperative, over a period of time or for a target equilibrium, it often involves an optimization formulation. An optimization can be described by four tuples: optimization variables, objective function, constraint set, and constant parameters.

Obviously transmit power vector $p$ is an optimization variable in all formulations in this survey. In Sections 7–9, beamforming pattern, BS assignment, bandwidth allocation, and time schedules also become variables. In addition to these primary variables, there are also secondary variables that are functions of them. An important example is that, in Section 4, SIR vector $\gamma$ also becomes a variable.

1.3.3 Objectives

There are two types of terms in the objective function: QoS-based utility and resource cost. Cost function for resource usage is relatively simple. It is often an increasing, convex function $V$ of the underlying resource, e.g., linear function of transmit power.

Utility functions require more discussion. The most general utility function assumes the form of $U(\beta)$, where $\beta$ is a vector of metrics. Often it is assumed to be additive across MSs indexed by $i$: $U(\beta) = \sum_i U_i(\beta)$, and locally dependent: $U(\beta) = \sum_i U_i(\beta_i)$.
1.3 Taxonomy of Problem Formulations

Metric $\beta_i$ may be the achieved throughput or goodput (and $U$ would be an increasing function), or delay, jittering, or distortion (and $U$ would be a decreasing function). These metrics are in turn functions of transmit power and other optimization variables in a given power control problem formulation.

For example, one QoS metric of interest is throughput, which is a function of SIR, which is in turn a function of transmit powers. There are several expressions of this metric. One is in terms of the capacity formula:

$$\beta_i(\gamma_i) = d\log(1 + c\gamma_i), \quad (1.8)$$

where $c$ and $d$ are constants that depend on modulation scheme, symbol period, and target Bit Error Rate (BER). In high-SIR regime, the above expression can be approximated by log function: $\beta_i(\gamma_i) = d\log c\gamma_i$. In low-SIR regime, it can be approximated by linear function: $\beta_i(\gamma_i) = dc\gamma_i$. It turns out both approximations help with formulating a convex optimization problem as discussed later. When other degrees of freedom such as schedules and beamforming patterns are involved, the expression becomes more complicated. Another expression for throughput is through the packet success rate function $f$ that maps SIR to the probability of successfully decoding a packet:

$$\beta_i(\gamma_i) = R_i f(\gamma_i), \quad (1.9)$$

where $R_i$ is the transmission rate.

There are also other QoS metrics such as delay that depend on SIR, and they will be introduced as the sections progress.

Back to the utility function $U_i$ itself, it is often modeled as a monotonic, smooth, and concave function, but in more general form as required by some applications, it may not be smooth or concave. It can capture any of the following: happiness of users, elasticity of traffic, efficiency of resource allocation, and even the notion of fairness. Consider a family of utility functions parameterized by $\alpha \geq 0$ [122]:

$$U_i(\beta_i) = \begin{cases} \log(\beta_i) & \text{if } \alpha = 1, \\ \frac{1}{(1 - \alpha)^{-1}\beta_i^{1-\alpha}} & \text{if } \alpha \neq 1. \end{cases} \quad (1.10)$$

Maximizing such an “$\alpha$-fair utility function” leads to an optimizer that satisfies the definition of $\alpha$-fairness in economics literature. For exam-
ple, proportional fairness is attained for $\alpha = 1$ and maxmin fairness for $\alpha \to \infty$. It is also often believed that larger $\alpha$ means more fairness.

A utility function that will help provide convexity of problem formulation while approximating linear utility function is the following pseudo-linear utility function:

$$U_i(\beta_i) = \log(\exp(\beta_i/c) - 1),$$

(1.11)

where $c > 0$ is a constant. This utility function is fairer than the linear utility function but approximates the linear utility at high QoS values. In particular, we have

$$\log(\exp(\beta_i/c) - 1) \to \beta_i/c \quad \text{as} \quad \beta_i \to \infty,$$

$$\log(\exp(\beta_i/c) - 1) \to -\infty \quad \text{as} \quad \beta_i \to 0.$$ 

Sometimes, QoS-based utility and resource cost are combined into a single objective function for each user, either additively as in utility minus power, or multiplicatively as in throughput over power.

### 1.3.4 Constraints

There are three major types of constraints in power control problems. First is the set of constraints reflecting technological and regulatory limitations, e.g., total transmit power, maximum transmit power for each user, and IOT or ROT. These are usually simple constraints mathematically.

Second are constraints based on inelastic, individual users’ requirements, e.g., two MSs’ received SIR at a BS need to be the same, or one MS’s rate cannot be smaller than a threshold. It is not always possible to meet these constraints simultaneously. In these cases, the power control problem is infeasible.

The third type of constraints, called feasibility constraints, is most complicated. In the information-theoretic sense, it would be the capacity region of an interference channel, which remains unknown. In the queueing-theoretic sense, it would be the stochastic stability region. In this survey, we focus instead on constraints that are defined with respect to QoS feasibility region, which is closely related to SIR feasibility region.
An SIR vector $\gamma \succeq 0$ is called feasible if there exists an interference vector, $q \succeq 0$, and power vector $p \succeq 0$, satisfying (1.6) and (1.7), respectively. It is reasonable to assume that the network of BSs and MSs represented by the channel matrix $F$ in Section 1.3.1 is connected, implying that $F$ is a primitive matrix. Let $\rho(\cdot)$ denote the spectral radius function of such a positive, primitive matrix. The following lemma from [198] is one of the fundamental results that characterizes SIR feasibility based on spectral radius of system matrices $F$ and $D(\gamma)$:

**Lemma 1.1.** An SIR vector $\gamma \succeq 0$ is feasible if and only if $\rho(FD(\gamma)) < 1$, when $n \neq 0$, and $\rho(FD(\gamma)) = 1$, when $n = 0$.

Further discussions on SIR and QoS feasibility regions will be provided in Sections 2 and 4.

### 1.3.5 Problem Formulations

We are ready to provide a quick preview of some representative problem formulations in the rest of the survey. Given the vast landscape of power control problems covered, a “problem tree” in Figure 1.2 serves as a high-level guide to the relationships among these problems. Meanings of each level of branching off are as follows:

- **Level 1**: Optimization vs. game theory approach.
- **Level 2**: Deterministic optimization within each time-slot vs. opportunistic approach.
- **Level 3**: Variable SIR vs. fixed SIR approach.
- **Level 4 and below**: Joint power control and a subset of the following: beamforming, BS assignment, bandwidth allocation, and scheduling.

Next, we present one or two representative problems in each of the nodes in the problem tree. Symbols are defined in the Table of Notation. The meanings, justifications, solutions, and implications of these problems are not discussed here, since they will be extensively studied.
in the following 8 sections. This preview puts the following sections in the appropriate corners of the problem landscape.

Problem (O), distributed power control, discussed in Section 2

\[
\begin{align*}
\text{minimize} & \quad \sum_i p_i \\
\text{subject to} & \quad \text{SIR}_i(p) \geq \gamma_i, \ \forall i \\
\text{variables} & \quad p.
\end{align*}
\] (1.12)

Problem (M), robust distributed power control, discussed in Section 3

\[
\begin{align*}
\text{minimize} & \quad \sum_i p_i + \phi(\epsilon) \\
\text{subject to} & \quad \text{SIR}_i(p) \geq \gamma_i(1 + \epsilon), \ \forall i \\
\text{variables} & \quad p, \ \epsilon.
\end{align*}
\] (1.13)

Problem (N), power control for optimal SIR assignment, discussed in Section 4

\[
\begin{align*}
\text{maximize} & \quad \sum_i U_i(\gamma_i) \\
\text{subject to} & \quad p(\gamma) \leq p^m \\
\text{variables} & \quad \gamma, \ p.
\end{align*}
\] (1.14)
1.3 Taxonomy of Problem Formulations

Problem (E), opportunistic power control, discussed in Section 5:

\[
\begin{align*}
\text{maximize} & \quad \sum_s \pi_s \sum_i U_{s,i}(p_{s,i}) \\
\text{subject to} & \quad \sum_s \pi_s g_{s,i}(p_s) \geq c_i, \quad \forall i \\
& \quad \sum_i p_{s,i} \leq P_T, \quad \forall s \\
\text{variables} & \quad p_s, \quad \forall s.
\end{align*}
\] (1.15)

Problem (C), Non-cooperative power control, discussed in Section 6:

\[
\begin{align*}
\text{maximize} & \quad U_i(\gamma_i) - V_i(p_i) \\
\text{subject to} & \quad \text{SIR}_i(p_i, \sigma_i, p_{-i}) \geq \gamma_i, \quad \forall i \\
& \quad p \preceq p^m \\
& \quad \sigma_i \in S_i, \quad \forall i \\
\text{variables} & \quad p, \quad \gamma, \quad \sigma.
\end{align*}
\] (1.16)

Problem (K), Joint PC and beamforming power minimization, discussed in Section 7:

\[
\begin{align*}
\text{minimize} & \quad \sum_i p_i \\
\text{subject to} & \quad \text{SIR}_i(W, p) \geq \gamma_i, \quad \forall i \\
\text{variables} & \quad p, \quad W.
\end{align*}
\] (1.17)

Problem (J), Joint PC and beamforming for utility maximization, discussed in Section 7:

\[
\begin{align*}
\text{maximize} & \quad \sum_i U_i(\gamma_i) \\
\text{subject to} & \quad \text{SIR}_i(W, p) \geq \gamma_i, \quad \forall i \\
& \quad p \preceq p^m \\
\text{variables} & \quad p, \quad \gamma, \quad W.
\end{align*}
\] (1.18)

Problem (L), Joint PC and BS assignment, discussed in Section 8:

\[
\begin{align*}
\text{minimize} & \quad \sum_i p_i \\
\text{subject to} & \quad \text{SIR}_i(p, \sigma) \geq \gamma_i, \quad \forall i \\
& \quad \sigma_i \in S_i, \quad \forall i \\
\text{variables} & \quad p, \quad \sigma.
\end{align*}
\] (1.19)
Problem (H), Joint PC and scheduling in frequency domain, discussed in Section 9:

maximize $\sum_i U_i(r_i)$

subject to $r_i = \sum_{l=1}^{L} b_l \log(1 + c_\gamma^l), \forall i$

$\gamma^l \in \Gamma^l, \forall l$

$\sum_l p_i^l \leq p_i^m, \forall i$

variables $p_i^l, \gamma^l, \forall l.$

(1.20)

Problem (I), Joint PC and scheduling in time domain, discussed in Section 9:

maximize $\sum_i U_i(r_i)$

subject to $r \in X = \text{Conv}(\mathcal{R}(\Gamma))$

$\gamma \in \Gamma$

variables $r, \gamma.$

(1.21)

Finally, the above list of representative formulations are compared in Table 1.2. The columns represent the fields describing the problem, and each row corresponds to one node in the tree of problems.

1.4 Convexity and Decomposability Structures

1.4.1 Convexity

Convexity has long been recognized as the watershed between easy and hard optimization problems. Convex optimization refers to minimization of a convex objective function over a convex constraint set. For convex optimization, a local optimum is also a global optimum (and unique if the objective function is strictly convex), duality gap is zero under mild conditions, and a rich understanding of its theoretical and numerical properties is available. For example, solving a convex optimization is highly efficient in theory and in practice, as long as the constraint set is represented efficiently, e.g., by a set of upper bound inequality constraints on other convex functions. Zero duality gap further enables distributed solutions through dual decomposition.
## 1.4 Convexity and Decomposability Structures

Table 1.2: Table of representative problem formulations.

<table>
<thead>
<tr>
<th>Chapters</th>
<th>Constraints</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>√</td>
<td>w_i, t</td>
</tr>
<tr>
<td>E</td>
<td>√</td>
<td>w_i</td>
</tr>
<tr>
<td>F</td>
<td>4,7,9</td>
<td>√</td>
</tr>
<tr>
<td>G</td>
<td>2,3,7,9</td>
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<td>9</td>
<td>√</td>
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<td>7</td>
<td>√</td>
</tr>
<tr>
<td>N</td>
<td>4</td>
<td>√</td>
</tr>
<tr>
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<td>√</td>
</tr>
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<td>√</td>
</tr>
<tr>
<td>O</td>
<td>2</td>
<td>√</td>
</tr>
</tbody>
</table>

**Objective:** $\sum U_i(\gamma_i) - \sum V_i(p_i)$  
**Constraint:** $E[\sum U_i(\gamma_i)]$  
**Variables:** $w_i, t, r_i, \gamma_i, p_i, \sigma_i, \epsilon_i$
To check if a power control problem is convex optimization, we need to check both its objective and constraints. We want the objective function being maximized (e.g., utility function of rate) to be concave, and the one being minimized (e.g., cost function of power consumption) to be convex in the optimization variables. As will be discussed in many sections later, concavity of utility function may not always hold. In non-cooperative power control formulations, quasi-concavity property of selfish utility functions plays a similarly important role for proving the existence of Nash equilibrium. We also want the constraint set to be convex, and in an efficient representation. Sometimes, a log change of variables turns an apparently non-convex problem into a convex one as in the Geometric Programming approach that has been shown to solve a wide range of constrained power control problems in high-SIR regime and a smaller set of problems in general-SIR regime [40]. More sufficient conditions for such convexity will be discussed in later sections.

Sometimes discrete optimization variables need to be introduced, thus turning the problem into a non-convex one. Three important examples include BS assignment among a finite set of BS choices, scheduling an MS to transmit or not, and a discrete set of available power levels.

1.4.2 Decomposability

While convexity is the key to global optimality and efficient computation, decomposability is the key to distributed solutions of an optimization problem. Unlike convex optimization, however, there is no definition of a decomposable problem. Rather, decomposability comes in different degrees. If a problem can be decomposed into subproblems whose coordination does not involve communication overhead, its solution algorithm can be distributed without any message passing. In other instances, subproblems being solved by different network elements (e.g., MS and BS) need to be coordinated by passing messages among these elements. Counting such communication overhead is not always easy either, it often depends on how far and how often are messages passed and how many bits each message contains. In general, message passing across multiple BS is difficult, whereas between a BS and the MS in its
cell is more feasible. Frequency and length of these control messages will be further discussed in Section 10.

There are various decomposition techniques from optimization theory, such as dual decomposition, primal decomposition, and penalty function method. However, one of the key constraints in power control problems, the SIR feasibility constraint, turns out to be coupled in a way that is not readily decomposed by these techniques. In Section 4, we will show how a reparametrization of this set reveals decomposability structures and leads to a distributed algorithm.

In contrast to the global optimization formulations, distributed algorithms are, by definition, already provided in non-cooperative game formulations of power control, as surveyed in Section 6. The challenge then becomes showing that such distributed interactions among selfish network elements will lead to a desirable equilibrium, e.g., they also maximize the global utility function for the whole network. The following two approaches are complementary: modeling through global optimization and searching for decomposition method, or modeling through selfish local optimization and characterizing the loss of social welfare optimality.
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