Sensing and Filtering: A Fresh Perspective Based on Preimages and Information Spaces

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Sensing and Filtering: A Fresh Perspective Based on Preimages and Information Spaces

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Abstract

This monograph presents an unusual perspective on sensing uncertainty and filtering with the intention of *understanding* what information is minimally needed to achieve a specified task. Information itself is modeled using *information space* concepts, which originated from dynamic game theory (rather than *information theory*, which was developed mainly for communication). The guiding principle in this monograph is avoid sensing, representing, and encoding more than is necessary. The concepts and tools are motivated by many tasks of current interest, such as tracking, monitoring, navigation, pursuit-evasion, exploration, and mapping. First, an overview of sensors that appear in numerous systems is presented. Following this, the notion of a virtual sensor is explained, which provides a mathematical way to model numerous sensors while abstracting away their particular physical implementation. Dozens of useful models are given, each as a mapping from the physical world to the set of possible sensor outputs. Preimages with respect to this mapping represent a fundamental source of uncertainty: These are

equivalence classes of physical states that would produce the same sensor output. Pursuing this idea further, the powerful notion of a *sensor lattice* is introduced, in which *all* possible virtual sensors can be rigorously compared. The next part introduces filters that aggregate information from multiple sensor readings. The integration of information over space and time is considered. In the spatial setting, classical triangulation methods are expressed in terms of preimages. In the temporal setting, an information-space framework is introduced that encompasses familiar Kalman and Bayesian filters, but also introduces a novel family called *combinatorial filters*. Finally, the planning problem is presented in terms of filters and information spaces. The monograph concludes with some discussion about connections to many related research fields and numerous open problems and future research directions.

Contents

1

Introduction

Think about the devices we build that intermingle sensors, actuators, and computers. Whether they be robot systems, autonomous vehicles, sensor networks, or embedded systems, they are completely blind to the world until we equip them with sensors. All of their accomplishments rest on their ability to sift through sensor data and make appropriate decisions. This monograph therefore takes a completely *sensor-centric* view for designing these systems.

It is tempting (and common) to introduce the most complete and accurate sensors possible to eliminate uncertainties and learn a detailed, complex model of the surrounding world. In contrast, this monograph heads in the opposite direction by starting with sensing first and then *understanding* what information is minimally needed to solve specific tasks. If we can accomplish our mission without knowing certain details about the world, then the overall system may be more simple and robust.

This can be partly understood by considering computational constraints. One way or another, we want computers to process and interpret the data obtained from sensors. The computers might range from limited embedded systems to the most powerful computer systems. The source of their data is quite different from classical uses of computers, in which data are constructed by humans, possibly with the help of software. When data are obtained from sensors, there is a direct *sensor mapping* from the physical world onto a set of sensor readings. Even though sensors have been connected to computers for decades, there has been a tendency to immediately digitize the sensor data and treat it like any other data. With the proliferation of cheap sensors these days, it is tempting to easily gather hordes of sensor data and google them for the right answer. This may be difficult to accomplish, however, without carefully understanding the sensor mapping. A large part of this monograph is therefore devoted to providing numerous definitions and examples of practical sensor mappings.

When studying sensors, one of the first things to notice is that most sensors leave a huge amount of ambiguity with regard to the state of the physical world. Example: How much can we infer about the world when someone triggers an infrared sensor to turn on a bathroom sink? In many fields, there is a common temptation to place enough powerful sensors so that as much as possible about the physical world can be reconstructed. The idea is to give a crisp, complete model that tends to make computers happy. In this monograph, however, we argue that it is important to start with the particular task and then determine its *information requirements*: What critical pieces of information about the world do we need to maintain, while leaving everything else ambiguous? The idea is to "handle" uncertainty by avoiding big models whenever possible. This is hard to accomplish if we design a general purpose robot with no clear intention in mind; however, most devices appearing in practice have specific, well-defined tasks to perform.

Depending on your background, there might be surprises in this monograph:

1. **Discrete vs. continuous: Not very important:** Even though computation is discrete and the physical world is usually modeled with continuous spaces, the distinction is not too important here. The field of hybrid systems is devoted to the interplay between continuous models, usually expressed with differential equations, and discrete computation models.

The point in this monograph, however, is to study sensor mappings. These may be from continuous to continuous spaces, continuous to discrete, or even discrete to discrete (if the physical world is modeled discretely).

2. **Information spaces, not information theory:** As an elegant and useful mathematical framework for characterizing information transmitted through a noisy channel, Shannon's information theory is extremely powerful. The concepts are fundamental to many fields; however, *information spaces* were formulated since the 1940s in the context of game theory and control theory for systems that are unable to determine their state. Thus, this monograph talks more about how to accomplish tasks in spite of huge amounts of ambiguity in state, rather than measuring information content, using entropy-based constructs. There may indeed be interesting connections between the two subjects, but they are not well understood and are therefore not covered here.
3. **Perfectly accurate and reliable sensors yield huge amounts of uncertainty:** Uncertainty in sensing systems is usually handled by formulating statistical models of disturbance. For example, a global positioning system (GPS) may output location coordinates, but a Gaussian noise model might be used to account for the true position. It is important, however, to study the often neglected source of uncertainty due simply to the sensor mapping. Consider the sensor pad at the entrance to a parking garage or drive-through restaurant. It provides one bit of information, usually quite reliably and accurately. It performs its task well, in spite of enormous uncertainty about the world: What kind of car drove over it? Where precisely did the car drive? How fast was it going? We are comfortable allowing this uncertainty to remain uncertain. We want to study these situations broadly. This is complementary to the topic of noisy sensors, and both issues can and should be addressed simultaneously. This monograph, however, focuses mainly on the underrepresented topic of uncertainty that arises from the sensor mapping.

Based on the discussion above, it is clear that sensing and computation are closely intertwined. For robotic devices, *actuation* additionally comes into play. This means that commands are issued by the computer, causing the device to move in the physical world. Therefore, many problems of interest mix all three: sensing, actuation, and computation. Alternative names for sensing are *perception* or even *learning*, but each carries distinct connotations. A broader name for actuation is *control*, which may or not refer to forcing changes in the physical world. Based on this three-way mixture and its increasing relevance, we are forced more than ever to develop new mathematical abstractions and models that reduce complexity and meet performance goals.

Figure 1.1 shows a conceptual distinction between classical computation and the three-way mixture considered in this paper. In Figure 1.1(a), the Turing machine model is shown, in which a state machine interacts with a boundless binary tape. This and other computation models represent useful, powerful abstractions for ignoring the physical world. Figure 1.1(b) emphasizes the interaction between the physical world and a computer. Imagine discarding the Turing tape and interacting directly with a wild, unknown, chaotic world through sensing and actuation.

A natural question arises: What is the "state" of this system? In the case of the Turing machine the full state is given by: the finite machine state, head position, and the binary string written on the tape. For Figure 1.1(b), this becomes replaced by two kinds of states: internal and external. The internal state corresponds to the state inside of the computation box. Some or all of the internal state will be called an *information state* (or *I-state*), to be defined later. The external state

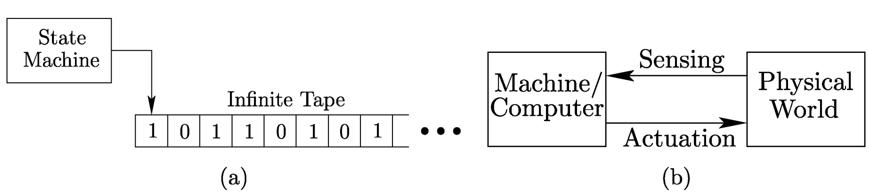

Fig. 1.1 (a) For classical computation, the full state is given by the finite machine state, the head position, and the binary string written on the tape. (b) In this monograph, there is both an internal computational state and an external physical state.

corresponds to the state of the physical world. The internal state is closer to the use of state in computer science, whereas the external state is closer to its use in control theory. The internal vs. external distinction is more important than discrete vs. continuous; either kind of state may be continuous or discrete.

These internal states will be defined to live in an *information space* (or *I-space*), which is where filtering and planning problems naturally live when sensing is involved. In this monograph, we will define and interpret these spaces in many settings. A continuing mission is to make these spaces as small as possible while being able to efficiently compute over them and to understand their connection to the external states.

Here are some key themes to take from this monograph:

- Start from the task and try to *understand* what information is actually *required* to be extracted from the physical world.
- Since sensors leave substantial uncertainty about the physical world, they are best understood as inducing partitions of the external state space into indistinguishable classes of physical states.
- We can design *combinatorial filters* that are structurally similar to Bayesian or Kalman filters, but involve no probabilistic models. These are often dramatically simpler in complexity. They are also perfectly compatible with probabilistic reasoning: Stochastic models can be introduced over them.
- There is no problem defining enormous physical state spaces, provided that we do not directly compute over them. However, state estimation or recovery of a particular state in a giant state space should be avoided if possible.
- Virtual sensor models provide a powerful intermediate abstraction that can be implemented by many alternative physical sensing systems.

The remainder of this monograph is divided into four main parts:

1. **Physical sensors:** Before going into mathematical models, a broad overview of real sensors will be given along with discussions about what we would like to sense.

2. **Virtual sensors:** This part introduces mathematical models of sensors that are abstracted away from the particular physical implementation. Using a definition of the physical state space, a sensor is defined as a mapping from physical states to data that can be measured.
3. **Filtering:** Information accumulates from multiple sensor readings over time or space and needs to be efficiently combined. *Spatial filters* generalize ancient triangulation methods and combine information over space. For *temporal filters*, we find and attempt to "live" in the smallest I-space possible, given the task. The concepts provide a generalization of Kalman and Bayesian filters. The new family includes reduced complexity filters, called *combinatorial filters*, that avoid physical state estimation.
4. **Discussion:** In the final part, the transition to *planning* is briefly considered. A *plan* specifies actuation primitives (or actions) that are conditioned on the I-states maintained in a filter and manipulate the world to achieve tasks. Related research and future research challenges are then presented to end the monograph.

Filtering and planning can be distinguished by being *passive* and *active*, respectively. A filtering problem might require making inferences, such as counting the number of people in a building or determining the intent of a set of autonomous vehicles. A planning problem usually disturbs the environment, for example by causing a robot to move a box across the floor.

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