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A Survey on the Low-Dimensional-Model-based Electromagnetic Imaging

Lianlin Li¹, Martin Hurtado², Feng Xu³, Bing Chen Zhang⁴, Tian Jin⁵, Tie Jun Cui⁶, Marija Nikolic Stevanovic⁷ and Arye Nehorai⁸

1 Peking University; lianlin.li@pku.edu.cn
2 National University of La Plata; martin.hurtado@ing.unlp.edu.ar
3 Fudan University; fengxu@fudan.edu.cn
4 Institute of Electronics, Chinese Academy of Sciences; bczhang@mail.ie.ac.cn
5 National University of Defense Technology; tianjin@nudt.edu.cn
6 Southeast University; tjcui@seu.edu.cn
7 University of Belgrade; mnikolic@etf.rs
8 Washington University in St. Louis; nehorai@ese.wustl.edu

ABSTRACT

The low-dimensional-model-based electromagnetic imaging is an emerging member of the big family of computational imaging, by which the low-dimensional models of underlying signals are incorporated into both data acquisition systems and reconstruction algorithms for electromagnetic imaging, in order to improve the imaging performance and break the bottleneck of existing electromagnetic imaging methodologies. Over the past decade, we have witnessed profound impacts of the low-dimensional models on electromagnetic imaging. However, the low-dimensional-model-based electromagnetic imaging remains at its early stage, and many
important issues relevant to practical applications need to be carefully investigated. Especially, we are in the big-data era of booming electromagnetic sensing, by which massive data are being collected for retrieving very detailed information of probed objects. This survey gives a comprehensive overview on the low-dimensional models of structure signals, along with its relevant theories and low-complexity algorithms of signal recovery. Afterwards, we review the recent advancements of low-dimensional-model-based electromagnetic imaging in various applied areas. We hope this survey could bridge the gap between the model-based signal processing and the electromagnetic imaging, advance the development of low-dimensional-model-based electromagnetic imaging, and serve as a basic reference in the future research of the electromagnetic imaging across various frequency ranges.
Electromagnetic imaging has been a powerful technique in various civil and military applications across medical imaging, geophysics, space exploration, resources and energy survey, etc., where the operational frequency ranges from the very low frequency (like tens of Hertz) through microwave, millimeter wave, and Therahertz, up to optical frequencies [47, 127, 23, 192, 96, 150, 70]. The electromagnetic imaging problem or the electromagnetic inverse scattering problem consists of determining the unknown features (including geometrical and physical parameters) of an object from processing measured electromagnetic data [127, 192]. Essentially, it is strongly nonlinear and ill-posed due to the complicated interaction between the electromagnetic wavefield and the imaging scene [127, 192]. In principle, this problem could be addressed by employing nonlinear iterative optimization methods, but these iterative methods are computationally prohibitive even for the moderate-scale problem [127]. In practice, one resorts to the linearized approximate solution to the rigorous inverse scattering problem, for example, Born-approximation [127, 23]. Nonetheless, the resulting inverse problem remains notoriously ill-posed since the measurements available are inadequate typically compared to the unknowns to be
retrieved. Especially, there are increasing continuously demands on the imaging resolution of detailed information of probed object nowadays, which broaden the gap between the unknowns of interest and the measurements available further. Moreover, the measurements are noisy and suffer from unknown ambiguous parameters, which makes the electromagnetic imaging problem more challenging.

Put formally, the electromagnetic imaging can be formulated as $y = A_\theta(x) + n$, where the quantity $y$ indicates the vectorized measurements corrupted with additive noise $n$, $x$ denotes the unknown (e.g., the reflectivity of imaged scene) to be retrieved, $A_\theta$ is a mapping operator with the subscript $\theta$ highlighting possible unknown ambiguous parameters [47, 127, 23, 192, 96, 150]. As argued above, the nonlinear inverse scattering, i.e. $A_\theta$ being nonlinear, is limited to the small-scale problem due to its very expensive computational cost. For this reason, we are restricted ourselves into the case of $A_\theta$ being linear. Furthermore, without the loss of generality, we assume no ambiguous parameters involved, implying that the subscript $\theta$ vanishes. Consequently, $A_\theta$ becomes $A$. Then, the electromagnetic imaging problem consists of retrieving the unknowns $x$ from the noisy measurements $y$. In probabilistic framework, the estimation of $x$ amounts to evaluating the posterior probability of $x$ conditional on $y$ [157, 87, 166]:

$$
\Pr(x|y) = \frac{1}{Z} \Pr(y|x) \Pr(x)
$$

(1.1)

where $Z = \int \Pr(y|x) \Pr(x) \, dx$ is the normalized factor (or the partition function), $\Pr(y|x)$ is the likelihood function of $x$, and $\Pr(x)$ is the prior knowledge on $x$. Once obtaining the posteriori probability $\Pr(x|y)$, we can numerically or analytically calculate desirable statistical quantities. We are particularly interested in the maximum a posteriori (MAP) mode denoted by $x_{\text{MAP}}$ [157, 87, 166]:

$$
x_{\text{MAP}} = \arg \max_x \log \Pr(x|y)
$$

$$
= \arg \max_x \left[ \log \Pr(y|x) + \log \Pr(x) \right]
$$

(1.2)

$$
= \arg \min_x \left[ \frac{1}{2} \| y - A(x) \|_2^2 - \log \Pr(x) \right]
$$

(1.3)

As pointed out previously, Eq. 1.3 is ill-posed due to the inadequate
measurements compared to the unknowns to be retrieved, especially when the finer details of probed scene are desirable. Since the measurements are incomplete, an infinite number of solutions, however, being non-meaningful, match measurements. Therefore, one crucial task of electromagnetic imaging is to select the most meaningful solution out of the potential solutions. In terms of Bayesian analysis, the model function $\text{Pr}(x)$ provides the speculative knowledge on the underlying signal $x$, which is, if correct, helpful in suppressing remarkably the solution uncertainty by complementing the incomplete measurements. In this sense, one feasible approach to the above task is the exploration of the correct model $\text{Pr}(x)$ in the design of imaging systems and imaging algorithms.

Most of real-world signals have low-dimensional models, known as being of the structured sparsity or structured compressibility [16, 62, 46, 10]. Here, we mean by structure that a transformed domain or manifold, being either deterministic or probabilistic, exists such that over which the transformed coefficients are sparse or compressible. By sparsity, we mean that a signal of length $n$ has $k \ll n$ nonzero elements; in contrast, we mean by compressibility that a signal of length $n$ can be approximated with certain accuracy by a signal with only $k \ll n$ nonzero coefficients [16]. Low-dimensional signal models affect significantly the data acquisition, analysis and later processing [63, 172, 59], which has profoundly broken bottlenecks set by the well-known Nyquist-Shannon theory founded by Kotelnikov, Nyquist, Shannon, Whittaker et al. A celebrated theory known as compressive sensing, founded by Candès, Tao, Romberg, Donoho et al., states that the sparse or compressible signal can be accurately and efficiently retrieved from its low-dimensional projections [32, 31, 30, 29, 48]. Afterwards, many more realistic and richer low-dimensional signal models along with the guarantee of theories and algorithms have been discovered and investigated, which affect the data acquisition, analysis and processing significantly [59, 15, 80, 81, 64]. For instance, for tree- and block-structured signals, Baraniuk et al. established the theory of model-based compressive sensing (CS) along with reconstruction algorithms [80].

Low-dimensional-model based signal processing (model-based SP,
for short) differs from the Nyquist-Shannon theory based methods (conventional SP, for short) in several important aspects, as summarized in Table 1.1. As opposed to the conventional SP, which only uses the information provided by the measurement, model-based SP uses both the measurement and the prior knowledge, enabling us to break the limits in classical signal processing. For example, the number of measurements required by the Nyquist-Shannon theory can be dramatically reduced [80, 81, 64, 20], the Rayleigh resolution limit can be readily beat [35, 27], and so on. In addition, these two frameworks differ in the manner in which they deal with signal recovery. For conventional SP, the signal recovery is accomplished through simple sinc interpolation with marginal computational cost [46]. In contrast, the model-based signal recovery is achieved by implementing nonlinear iterative algorithms, which needs apparently expensive computational resources [80, 81, 64].

Table 1.1: Comparisons between two frameworks of conventional and low-dimensional-model-based signal processing.

<table>
<thead>
<tr>
<th></th>
<th>Conventional SP</th>
<th>Model-based SP</th>
</tr>
</thead>
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<tr>
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<tr>
<td>Math. Model ( y = Ax )</td>
<td>( \dim(y) \geq \dim(x) )</td>
<td>( \dim(y) \ll \dim(x) )</td>
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<tr>
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</tr>
</tbody>
</table>

Over past years, we have witnessed the notable impacts of low-dimensional-model-based signal processing, more strictly, the sparse-model-based signal processing, on the electromagnetic imaging [30, 114, 113, 131, 85, 101, 151, 9, 144, 21, 176, 86, 149, 147, 125, 17, 37, 38,
Imaging techniques, that exploit low-dimensional-model of the underlying scene, are becoming more and more popular thanks to their ability to mitigate the theoretical and practical difficulties arising in the associated inverse problem, while properly complying with several common applicative requirements (e.g., reduced computational costs, high spatial resolution, and robustness to the noise). Such an increased interest is proved by a vast of publications in several areas (e.g., electromagnetic inverse scattering, radar, microwave imaging, and array synthesis), and special sessions in relevant international conferences, as well as special issues in leading-edge journals. Invoked by the concept of compressive sensing, Hunt et al. [85] and Li et al. [101] invented the single-pixel images for microwave imaging of sparse scenes, demonstrating the important potential of sparse-signal model of imaged scene in developing the apparatus of low-complexity and low-cost data acquisition. By now, the most affected issue related to electromagnetic imaging is the development of low-order model-based imaging algorithms. For instance, there are a large amount of efforts have been made to melt the sparsity-promoted regularization with the iterative algorithms of electromagnetic inverse scattering, being different in the selection of sparsifying transformations, leading to the feature-enhanced reconstruction. As argued above, the non-linear electromagnetic inverse scattering is limited to the small-scale problem due to very expensive computational complexity. For this reason, we leave the discussion about them out of this survey, and refer to [151, 9, 144, 21, 176] for detailed discussions. For the linearized electromagnetic imaging (Born-based tomography [86, 149, 147, 125] and signal-based radar imaging [17, 37, 38, 84, 180, 99, 195, 185]), the sparse-model (or compressible-model) of imaged scene have been exploited, and a large amount of low-order model-based imaging algorithms have been developed, demonstrating that the usefulness of sparse-model in enhancing the image quality and reducing the number of measurements. It is really appealing to incorporate low-dimensional models of underlying signals into the electromagnetic imaging, in order to reduce the number of measurements, improve the imaging resolution, enhance the capability of object recognition and classification, and so on. For this reason, we refer to this methodology of electromagnetic
imaging as the low-dimensional-model-based electromagnetic imaging (model-based electromagnetic imaging, for short). Here, we would like to give a formal description about it:

**Definition 1:** The low-dimensional-model-based electromagnetic imaging is the object-oriented and feature-enhanced electromagnetic imaging methodology by incorporating the knowledge of the structured models of underlying signals into the data acquisition system and the reconstruction algorithm, in order to reducing the number of measurements, improve the imaging resolution, enhance the capability of object recognition and classification, and so on.

Although the model-based signal processing by itself has arrived at relatively mature level with a solid body of theories [59, 32, 31, 30, 29, 48, 15, 80, 81, 64, 35, 27, 76, 34, 28, 54, 49, 50, 51, 52, 53] and algorithms [55, 159, 41, 94, 198, 33, 112, 115, 120, 183, 160, 168, 72, 161, 91], its interactions with electromagnetic imaging remains challenging and many important issues are deserved to be studied in-depth. The model-based electromagnetic imaging focuses on four major aspects as following.

First, it is desirable the development of the next-generation imaging system with low-cost, low-complexity and high efficiency by the way of the optimal design of the waveform, the programmable or reconfigurable antenna, the configuration of sparse sensor array, etc. For instance, the establishment of novel compressive radar is appealing for ultra-wideband (UWB) radar imaging [84, 180], since it is really too costly, or even physically impossible, to build devices capable of acquiring samples at the necessary rate in the context of classical signal processing.

Second, it is desirable to establish an easy-implementation of imaging formulations, which account for the real interaction between the electromagnetic wavefield and the probed scene. The interaction between the electromagnetic wavefield and the probed scene is nonlinear in essence; however, most of mathematical formulations of electromagnetic imaging considered so far are linear, failing to capture fully the undergoing physical mechanism in some practical cases [164, 26, 42].
Nikolic et al. attempted to link the electromagnetic mechanism and the sparse-signal model by using the equivalent electromagnetic currents, and demonstrated that with this model, the sparseness of imaged scene supports the reconstruction of non-convex shape of 2D PEC targets [123]. However, such methodology is limited to the single-frequency imaging configuration, since the equivalent currents vary with the operational frequencies.

Third, it is desirable to discover more realistic and richer low-dimensional models of the underlying electromagnetic information. By now, the low-dimensional models utilized in the area of electromagnetic imaging are nearly simple sparse or compressible models; more realistic and richer models are not fully investigated.

Fourth, we are in the deluge of massive electromagnetic data coming from the continuously increasing demands on retrieving very detailed information of objects nowadays. Therefore, it is crucial to develop efficient reconstruction algorithms for treating massive measurements and high-dimensional variables. From the aspect of computational complexity, it is also important to develop algorithmic frameworks trading off the imaging accuracy with the computational cost.


References


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