Hybrid Dynamical Systems: An Introduction to Control and Verification

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Contents

Abstract

Hybrid dynamical systems are a class of complex systems that involve interacting discrete-event and continuous-variable dynamics. They are important in applications in embedded systems, cyber-physical systems, robotics, manufacturing systems, traffic management, biomolecular networks, and have recently been at the center of intense research activity in the control theory, computer-aided verification, and artificial intelligence communities. This paper provides a tutorial introduction to this multidisciplinary research area. A number of fundamental topics, such as modeling, abstraction, verification, supervisory control, stability analysis, stabilization, and optimal control of hybrid systems are introduced and discussed. Additionally, more advanced topics are briefly discussed at the end of each chapter with references given for further reading.

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1

Introduction

Hybrid dynamical systems contain heterogeneous dynamics that interact with each other and determine their behaviors over time. By heterogeneity, we mean systems containing two different kinds of dynamics: Time-driven continuous variable dynamics, usually described by differential or difference equations; and event-driven discrete variable dynamics, the evolutions of which depend on if-then-else type of rules, usually described by automata or Petri nets. These two kinds of dynamics interact with each other and generate complex dynamical behaviors, such as switching once the value of a continuous variable passes through a threshold, or state jumping upon certain discrete event occurring to mention but a few.

As an example, consider a typical room temperature control system in winter. Assume that the set point of the thermostat is 70 degrees Fahrenheit. The furnace will turn on if the room temperature is below the set point, and turn off otherwise. The room temperature control system is actually a typical hybrid system as the furnace, along with the heat flow characteristics of the room, form the continuous variable dynamics, whereas the on-off thermostat can be modeled as a discrete event system with two states "ON" and "OFF". In addition, the transition between these two discrete states is triggered by the temperature in the room, while the evolution of the temperature depends on whether the furnace is on or off, i.e., discrete state of the thermostat. Hence, the temperature control system contains interacting discrete and continuous dynamics, and can be modeled and studied as a hybrid system.

Hybrid systems actually arise in a great variety of applications, such as manufacturing systems [Pepyne and Cassandras, 2000], air traffic management [Tomlin et al., 1998], automotive engine control [Balluchi et al., 2000], chemical processes [Engell et al., 2000], to mention but a few. Hybrid systems also arise from the hierarchical organization of complex systems, and from the interaction of discrete planning algorithms and continuous control algorithms in autonomous, intelligent systems. Hybrid systems have a central role in networked embedded control systems that interact with the physical world, and as such play a key role in the understanding of the evolution of systems that contain an information and networking core and interact tightly with the physical world and human operators; such systems are also referred to as Cyber-Physical Systems (CPS) [Lee, 2008, Baheti and Gill, 2011]. Studies in hybrid systems could provide a unified modeling framework for CPS, and systematic methods for performance analysis, verification, and design.

Besides their enormous practical importance, hybrid systems also represent a fascinating and highly challenging area of study that encompasses a variety of theoretical research questions. Actually, the introduction of switching and state jumps in hybrid systems non-trivially extend the dynamical behaviors that can be modeled by hybrid system models compared with traditional modeling frameworks, such as ordinary differential equations and automata. Hence, hybrid system models are of interest in themselves, and have been successfully used to model a large variety of complex systems, such as gene-regulatory networks [De Jong, 2002], communication networks [Hespanha, 2004b] and robotic systems [Egerstedt, 2000]. However, the price associated with the increased modeling power is the difficulty in analyzing properties of the evolution or solution of a hybrid system model, such as the existence and uniqueness of a solution, and the continuity of trajectories with respect to initial conditions. These difficulties have motivated significant and intense research activities targeting formal analysis and synthesis of hybrid systems. On the other hand, the introduction of switching logic into controllers may help to achieve performance that exceeds any fixed classical linear or nonlinear smooth controller; for example, there are some nonlinear systems that cannot be stabilized by any smooth feedback control law, but can be asymptotically stabilized by a hybrid controller [Hespanha et al., 1999]. Moreover, to meet challenging high-performance design requirements that reflect multiple objectives such as response speed, accuracy, optimality, robustness, and disturbance attenuation, a multi-modal (hybrid) control architecture may be the proper choice. When the requirements are represented by time and event-based behaviors or when the plant to be controlled has tight interactions of continuous variable and discrete event dynamics, one needs to employ hybrid control methods [Antsaklis and Nerode, 1998, Antsaklis, 2000].

The history of hybrid system research can be traced back at least to the 1960s to the study of engineering systems that contained relays and/or hysteresis, see e.g., [Witsenhausen, 1966]. However, hybrid systems began to seriously attract the attentions of researchers in the early 1990s, mainly because of the widespread development and implementation of digital micro controllers and embedded devices. The last two decades have seen considerable research activities in modeling, analysis and synthesis of hybrid systems. The investigation of hybrid systems is a fascinating discipline bridging control engineering, mathematics and computer science.

Computer scientists tend to look at hybrid systems primarily as discrete (computer) programs interacting with the physical environment. They extend their computational models, such as finite state machine, automata and petri nets, from discrete systems to hybrid systems by embedding the continuous variable dynamics into these discrete models, see e.g., [Alur et al., 1993, 1995]. Typically, these approaches are able to deal with complex discrete dynamics and emphasize analysis results (verification) and simulation methodologies. Such approaches typically ask whether certain properties, such as safety, liveness and fairness that are formulated in temporal logic formulas, hold true or not for a given hybrid system model. This is called the verification of hybrid systems, and one of the main verification methods is symbolic model checking, which is based on the computation of reachable sets for hybrid systems. Consequently, a good deal of research effort has focused on developing sophisticated techniques drawn from optimal control, game theory, and computational geometry to calculate or approximate the reachable sets for various classes of hybrid systems, see e.g., [Chutinan and Krogh, 2003, Tomlin et al., 2003].

On the other hand, researchers from the areas of dynamical systems and control theory have approached hybrid systems as a collection of differential/difference equations with discontinuous or multi-valued right-hand sides. Representative modeling frameworks in this category include piecewise affine/linear systems [Sontag, 1981, Johansson, 2003a] and switched systems [Liberzon, 2003, Lin and Antsaklis, 2009]. They extend the models and methodologies for traditional continuous variable systems, such as ordinary differential/difference equations, by including discrete variables so as to describe the jumping or switching phenomena. Typically, these approaches are able to deal with complex continuous variable dynamics and focus mainly on stability, controllability, robustness and synthesis issues.

The methods for hybrid systems are distributed across a wide spectrum, ranging from methods known in the discrete (cyber-) domain at one end, to traditional approaches for the continuous physical systems at the other. Rooted at opposite ends, both computer scientists and control theorists have made significant contributions to the field of hybrid systems by extending traditional methods from the purely discrete or continuous domain to deal with hybrid systems. However, in general, there has been little work on integrating methods from these two domains. This is possibly because the formal methods pursued in computer science traditionally lie in the realm of discrete mathematics, while control theory approaches lie mainly in the realm of continuous mathematics. A noticeable trend in the recent hybrid system literature emphasizes the synthesis of hybrid controllers for continuous or hybrid dynamical systems to satisfy complicated temporal logic specifications. This is known as symbolic control or hybrid supervisory control, which can be seen as a crosstalk between these two schools of thoughts.

This tutorial paper seeks to balance the emphasis on methods from both computer science and control theory, and hopefully gives the readers a relatively complete picture of the whole field of hybrid dynamical systems. For such a purpose, the rest of paper is organized as follows. First, several modeling frameworks for hybrid systems, namely hybrid automata, switched systems and piecewise affine systems, are introduced in Chapter 2. Chapter 3 briefly reviews the results on stability, stabilization and optimal control of hybrid systems. Then, Chapter 4 investigates the verification problems for hybrid systems with a particular focus on model checking approaches. Finally, Chapter 5 reviews the developments of hybrid supervisory control, also known as symbolic control, which can be seen as an effort to combine the results and approaches from both systems and control theory and computer sciences.

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