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Analysis and Synthesis of Reset Control Systems

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Contents

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ABSTRACT

This survey monograph overviews a large core of research results produced by the authors in the past decade about reset controllers for linear and nonlinear plants. The corresponding feedback laws generalize classical dynamic controllers because of the interplay of mixed continuous/discrete dynamics. The obtained closed-loop system falls then within the category of hybrid dynamical systems, with the specific feature that the hybrid nature arises from the nature of the controller, rather than the nature of the plant, which is purely continuous-time. Due to this fact, the presented results focus on performance and stability notions that prioritize continuous-time evolution as compared to the discrete-time one. Dwell-time logics (namely, conditions preventing consecutive jumps that are too close to each other) are indeed enforced on solutions, to ensure that the continuous evolution of solutions is complete (no Zeno solutions occur).

After presenting a historical motivation and an overview of the results on this topic in Part I, several results on stability

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and performance analysis and on control design for general linear continuous-time plants are developed in Part II. These results are developed by exploiting the well-established formalism for nonlinear hybrid dynamical systems introduced by Andy Teel and co-authors around 2004. With this formalism, by ensuring sufficient regularity of the reset controller dynamics, we ensure robustness of stability with respect to small disturbances and uncertainties together with suitable continuity of solutions, generally regarded as well-posedness of the hybrid closed loop. Throughout Part II, we provide several simulation studies showing that reset control strategies may allow to attain better performance with respect to the optimal ones obtained by classical continuous-time controllers.

Finally, in Part III we focus on planar systems, that is reset closed loops involving a one-dimensional linear plant and a one-dimensional reset controller. For this simple interconnection interesting stability conditions can be drawn and relevant extensions addressing the reference tracking problem are introduced, illustrating them on a few relevant case studies emerging in the automotive field.

Part I

Background

1

Introduction

1.1 Historical overview

Reset controllers were proposed for the first time more than 50 years ago by Clegg (1958), with the aim at providing more flexibility in linear controller designs and at potentially removing fundamental performance limitations of linear controllers (see, e.g., the motivating example in Section 2.2). The first systematic designs for reset controllers were reported in the 1970s by Krishnan and Horowitz (1974) and Horowitz and Rosenbaum (1975) and there has been a renewed interest in this class of systems in the late 1990s with Beker *et al.* (1999b), Beker *et al.* (1999a), Beker *et al.* (2001b), Beker *et al.* (2004), Chait and Hollot (2002), Chen *et al.* (2000a), Chen *et al.* (2000b), Chen *et al.* (2001), Haddad *et al.* (2000), Hollot *et al.* (1997), Hollot *et al.* (2001), Hu *et al.* (1997), and Zheng *et al.* (2000).

More specifically, a reset controller, according to its historical definition, is a linear controller whose output is reset to zero whenever its input and output satisfy an appropriate algebraic relationship (even though generalizations not necessarily resetting to zero will be also considered in this survey). For instance, in Beker *et al.* (2004) and the references cited therein, a class of reset controllers was considered where the output of the controller is reset to zero whenever its input is equal to zero. The Clegg integrator introduced in Clegg (1958) acts like a linear integrator whenever its input and output have the same sign and it resets its output to zero otherwise (see Section 1.2). Consequently, its describing function has the same magnitude plot as the linear integrator but it has 51.9° less phase lag. This feature of the Clegg integrator was used for the first time in Krishnan and Horowitz (1974) to provide a systematic procedure for controller design exploiting this device. Subsequently, a new reset device called the *First Order Reset Element (FORE)* was introduced in Horowitz and Rosenbaum (1975), essentially generalizing the Clegg integrator's base linear dynamics by also allowing for a nonzero real pole. Horowitz and Rosenbaum (1975) also proposed a design procedure consisting of two steps. First, a non-reset part of the controller was designed to achieve all design specifications except for the overshoot. Then, in the second step, the pole of the FORE was selected to reduce the overshoot. It was illustrated through examples and simulations that the controller in the first step of the procedure could indeed be designed with lower phase margin, which provided more design flexibility. A nice account of these results and their relation to more recent developments in reset control are given in Chait and Hollot (2002).

The first example that clearly illustrated the advantages of reset over linear controllers was presented in Beker *et al.* (2001a) where a reset controller was designed to achieve design specifications that are impossible to achieve by any linear controller (see also Feuer *et al.*, 1997). Indeed, for linear plants including an integral action, if the desired rise time is sufficiently small, then the output must overshoot with any linear controller. However, a reset controller is designed in Beker *et al.* (2001a) that overcomes this fundamental performance limitation of linear controllers. To date, this appears to be the only real situation where reset designs have been shown to outperform the best possible classical design, nevertheless, practical experience reveals that desirable closed-loop behavior is obtained when suitably embedding resets in otherwise continuous-time control devices. Examples of such experiences can be found in the experimental applications reported in Zheng *et al.* (2007), Fernandez *et al.* (2008), Wu *et al.* (2007), Li *et al.*

(2009), Bakkeheim *et al.* (2008), Guo *et al.* (2009), Panni *et al.* (2014), Cordioli *et al.* (2015), Zheng *et al.* (2000), Carrasco and Baños (2012), Li *et al.* (2011), Vidal and Baños (2010), and Villaverde *et al.* (2011). The difficulty in proving rigorous statements with reset systems was due to the lack of suitable stability and performance analysis tools for systems whose solutions may experience instantaneous jumps (this is the case for an integrator reset to zero).

From a theoretical viewpoint, first attempts to rigorously analyze stability of reset systems with Clegg integrators can be found in Hu *et al.* (1997) and Hollot *et al.* (1997). In particular, an integral quadratic constraint was proposed in Hollot *et al.* (1997) to analyze stability of a class of reset systems. However, the proposed condition was conservative as it was independent of reset times. BIBO stability analysis of reset systems consisting of a second-order plant and a FORE was conducted in Chen *et al.* (2001) (see also Chen *et al.*, 2000b). The proofs are based on an explicit characterization of reset times which are proved to be equidistant under mild conditions. Using this fact, the authors prove asymptotic and BIBO stability of the reset system via the discrete-time model of the system that describes the system at reset times only. However, the same approach could not be used to analyze higher-order reset systems. Stability analysis of general reset systems can be found in Beker *et al.* (2004) (see also Hollot *et al.*, 2001; Chen *et al.*, 2000a)) where Lyapunov-based conditions for asymptotic stability were presented and computable conditions for quadratic stability based on linear matrix inequalities (LMIs) were given. Moreover, in Beker *et al.* (2004), BIBO stability of general reset systems was obtained as a consequence of quadratic stability and an internal model principle was proved for reference tracking and disturbance rejection.

In the last decade, perhaps triggered by the inspiring work of Beker *et al.* (2004), a significant renewed interest in Lyapunov-based analysis and synthesis for reset systems has been witnessed by the scientific community. This survey monograph reports on a research strand that started around 2005, motivated by the results in Beker *et al.* (2004), wherein some recent stability and performance analysis tools for hybrid dynamical systems have been brought to bear into the framework of reset control systems. While many alternative and relevant approaches have been developed during the last decade (a selection of them is briefly overviewed in the following Section 1.5), we specifically concentrate here on a research strand that emerged from the hybrid Lyapunov theory proposed in Goebel *et al.* (2012). Some important differences between what is reported here and alternative approaches are discussed in the next sections, which explain the spirit of our approach to reset control. To this end, we need to somewhat come back to the very origins of reset control and take a close look at the analog circuit proposed by J.C. Clegg in 1958, which is discussed in the next section.

1.2 The Clegg integrator circuit

In 1958, J.C. Clegg published a paper (Clegg, 1958) where he proposed a modification to the existing analog control schemes to reduce the phase lag induced by a linear integrator. The relevance of Clegg's work was mostly targeted to analog control, because digital control systems were still non-existent in the late 1950s, nevertheless, follow-up works considered digital versions of the scheme proposed by Clegg. Let us here consider the analog device proposed by Clegg and derive the corresponding equations.

In the ideal case of using infinite gain operational amplifiers, it is well known that a linear integrator can be implemented using a resistor on the input path and a capacitor on the feedback path of the circuit, as represented in Figure 1.1. The corresponding input/output relation of the linear integrator can be written in the time domain as $\dot{x}_c = -\frac{1}{RC}e$, where we use x_c for the integrator output, to resemble the fact that the integrator state is the state of a feedback controller from the tracking error e. In this figure, R is the resistance on the input branch of the

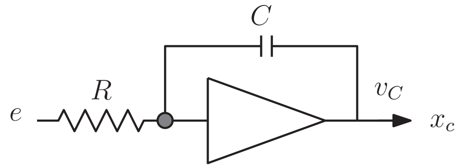

Figure 1.1: A linear analog integrator.

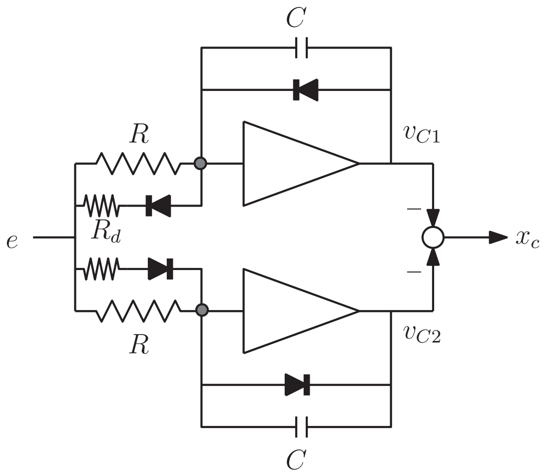

Figure 1.2: The "Clegg integrator".

amplifier, C is the capacity in the feedback branch, and v_C is the voltage across the capacitor.

The modification proposed by Clegg corresponds to the scheme of Figure 1.2 (which is reported here from Clegg (1958) with a sign inversion at the output, for convenience of exposition), where we use the same notation as in Figure 1.1, with possibly different voltages v_{C1} and v_{C2} and extra diodes and resistors. We describe the Clegg integrator dynamics assuming that $R_d \ll R$ and disregarding the forward-bias voltage drop across the diodes, leading to the following ideal relationships between voltage and current across capacitors, resistors, and diodes:

$$\begin{aligned} i_C(t) &= C\frac{dv_C(t)}{dt} \\ i_R(t) &= v_R(t)/R \\ i_D(t) &= \begin{cases} +\infty, & \text{if } v_D(t) > 0, \\ 0 & \text{if } v_D(t) \leq 0. \end{cases} \end{aligned}$$

First note that by the infinite gain assumption of the operational amplifier, the input voltages (marked by gray dots on the figure) are always zero. Then, the two capacitors' voltages satisfy $v_{C1}(t) \leq 0$ and $v_{C2}(t) \geq 0$ for all times (otherwise the infinite current flowing in the diodes would instantaneously discharge the capacitor). Moreover, when $e(t) < 0$, regardless of the preceding voltage stored in the upper capacitor, the current flowing in the two diodes and through the upper R_d will (almost) instantaneously impose $v_{C1}(t) = 0$. However, when $e(t) \geq 0$, the upper circuit will correspond to the linear integrator because the diodes will both be open (being subjected to a non-positive voltage). Similarly for the lower circuit, if $e(t) > 0$, we will have $v_{C2}(t) = 0$ and if $u(t) \leq 0$ the circuit will integrate. Since, as commented above, $v_{C1}(t) \leq 0$ and $v_{C2}(t) \geq 0$ for all times, given $x_c(t) := -v_{C1}(t) - v_{C2}(t)$, the integrating and reset conditions for both circuits can be written as the following hybrid dynamics

$$\begin{cases} \dot{x}_c = \dfrac{1}{RC} e, & \text{is allowed when } x_c e \geq 0, \\ x_c^+ = 0, & \text{is allowed when } x_c e \leq 0, \end{cases} \tag{1.1}$$

where we insist on the fact that the situation $e(t) = 0$ leads to an undetermined behavior of the circuit mostly dependent on the effect of unmodeled noise and uncertainties. In this model, we use the shorthand notation $\dot{x}$ for $\frac{d}{dt}x(t,j)$ and x^+ for $x(t, j+1)$ which will be formally defined later on in the survey. The strategy that we prefer to adopt for handling such undetermined cases is to allow for multiple solutions to the dynamics (1.1), thereby considering in our model all the possible scenarios. See the discussion given later in Remark 1.1.

One way to understand the hybrid model (1.1) for the Clegg integrator is to call its first equation the "flow" equation and its second equation the "jump" equation. The two conditions at the right-hand side become then the "flow" condition and the "jump" condition. At any time, a solution to the hybrid system (1.1) may then flow or jump depending on whether its value at that time belongs to the so-called "jump set" (namely, the set of states for which the jump condition is true) or it belongs to the "flow set". In case both conditions are true, then the solution will be free to choose whether flowing or jumping, thereby establishing a peculiar non-uniqueness feature.

Further insight into Equation (1.1) can be gained by observing that e and x_c can never have opposite signs. Indeed, if $e \geq 0$, then $v_{C2}^+ = 0$, and since $v_{C1} \leq 0$ for all times, $ex_c \geq 0$. Similarly for the case $e \leq 0$.

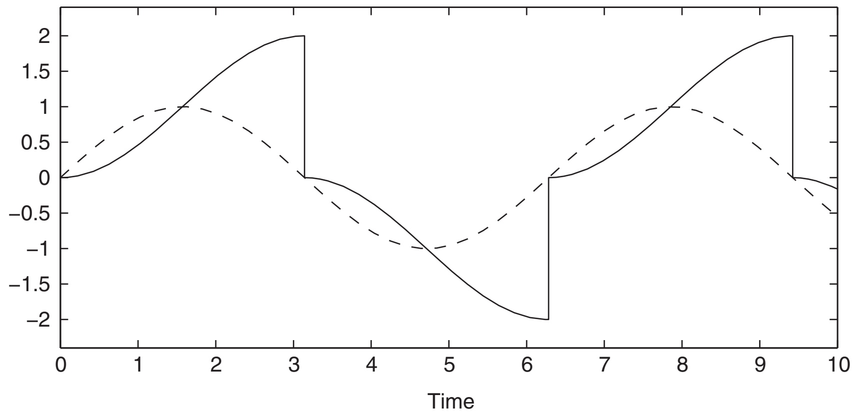

Figure 1.3: Response of the Clegg integrator (solid) to a sine input (dashed).

On the other hand, whenever $e \neq 0$, there will always be one circuit integrating (the upper one if $e > 0$ and the lower one if $e < 0$) and the other circuit will be forced to be at zero. For illustrative purposes, Figure 1.3 represents the Clegg integrator state (solid curve) when the input e is selected as a sine wave with unit frequency (dashed curve). Note that x_c and e always have the same sign, which is ensured by the fact that around times $t_1 = 3.2$ and $t_2 = 6.2$ the solution jumps. In particular, the upper condition of Equation (1.1) would not be satisfied if the solution did not jump, indeed violation of this condition forbids flowing.

One way to interpret the dynamics (1.1) is to regard it as a linear filter with a pole at the origin embedded with a special resetting rule dependent on the value of the input and output of the filter at each time. This interpretation is the starting point for the FORE generalization discussed in the next section. The interest of Clegg integrator will become clear in Section 2.2 when using Clegg integrator for a linear plant, and in Chapter 3 when designing hybrid loops for nonlinear control systems. As it will be explained in these parts (and in many other parts of this survey), the performance of Clegg integrators, and more generally of reset control is better to what could be done with classical linear or nonlinear controllers. The extra flexibility given by the jump rule is exploited to improve the performance of the closed-loop system in the presence of reset controllers.

1.3 Modeling issues with reset control

The model (1.1) derived in the previous section for the hybrid dynamics can be easily generalized to the following dynamics, where the eigenvalue associated with the continuous dynamics is not necessarily at zero:

$$\begin{cases} \dot{x}_c = a_c x_c + b_c e, & \text{is allowed when } x_c e \geq 0, \\ x_c^+ = 0, & \text{is allowed when } x_c e \leq 0, \end{cases} \tag{1.2}$$

the model of the Clegg integrator corresponding to $a_c = 0$ and $b_c = \frac{1}{RC}$. The generalized dynamical system (1.2) was introduced in Horowitz and Rosenbaum (1975) and therein called First Order Reset Element (FORE). In Horowitz and Rosenbaum (1975) and follow-up works, this generalization was meant for stable filters (namely, $a_c \leq 0$), but it will be emphasized in this survey that this is not a necessary assumption and indeed unstable selections ($a_c > 0$) lead sometimes to desirable aggressive control actions.

It should be acknowledged that models (1.1) and (1.2) do not correspond to the models originally developed from the 1960s. Indeed, while Clegg's discussion in Clegg (1958) well referred to the dynamical behavior of his analog circuit as a circuit resetting to zero whenever input and output had opposite signs, his qualitative description incorporated the following observation:

> "Whenever the input voltage e passes through zero from either direction, the output voltage is quickly dropped to zero."

This sentence propagated into the follow-up work of Horowitz and co-authors, who never really wrote down equations but only described in words this behavior specifying resetting the controller state to zero at zero-crossings of the input. Much later in the 1990s, Hollot and co-authors (see, e.g., Chait and Hollot, 2002; Beker *et al.*, 2004, and references therein), and then also Baños and co-authors (see, e.g., Baños and Barreiro, 2011, and references therein) reported the following dynamical description of the Clegg mechanism:

$$\begin{cases} \dot{x}_c = a_c x_c + b_c e, & \text{if } e \neq 0, \\ x_c^+ = 0, & \text{if } e = 0. \end{cases} \tag{1.3}$$

The modified hybrid dynamics (1.3) was then used as the baseline hybrid reset control mechanism implemented in the control logic of modern digital control systems. Somehow the nice and intrinsic robustness properties of the analog circuit proposed by Clegg got lost along the route towards digitalization of modern feedback control. In particular, dynamics (1.3) no longer describes the behavior of the circuit in Figure 1.2 for $a_c = 0$ because solutions to Equation (1.3) starting from $x_c \neq 0$ and $e x_c < 0$ do not lead to an instantaneous reset to zero of the controller state x_c. In particular, dynamics (1.3) is associated with resetting in a so-called "thin set", as opposed to dynamics (1.2) (and the behavior of Clegg's circuit) where resets are enforced in half of the input–output space of the controller. This difference is well highlighted in Figure 1.4 where the e axis has been reversed in anticipation for negative error feedback interconnection of the FORE with a linear plant. In Figure 1.4 the sets enabling continuous flow of solutions are denoted by $\mathcal{F}$ (the "flow set") and the sets enabling discrete jump of solutions are denoted by $\mathcal{J}$ (the "jump set"). Figure 1.4 also shows the possible different evolutions of

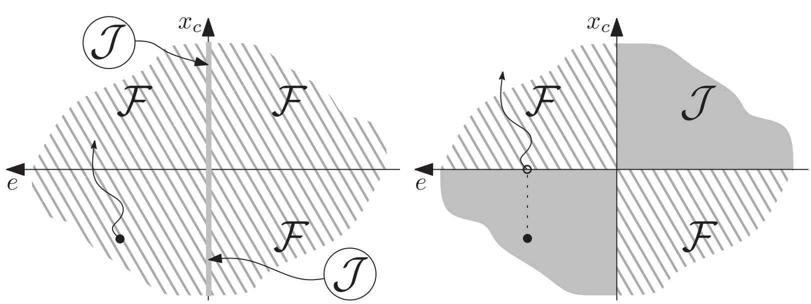

Figure 1.4: The jump (gray) and flow (striped) sets for the model (1.3) (left), and original model (1.2) (right) proposed by Clegg.

two solutions starting from the same initial conditions, for the two dynamics.

1.4 Using thin jump sets: Existence of solutions and robustness

The model (1.2) for the First Order Reset Element (FORE) was first introduced in Nešić *et al.* (2005) and Zaccarian *et al.* (2005), where the hybrid dynamical systems formalism of Goebel *et al.* (2009) and Goebel *et al.* (2012) has been employed for the first time for representing the peculiar evolutions of reset control systems whose state (notably, the controller state) may be integrated following a differential equation during the continuous-time flow phase, or may be reset to zero following a discrete update law at the jump times.

The useful features of model (1.2) have been first characterized in Nešić *et al.* (2005) and Zaccarian *et al.* (2005), and are worth summarizing here, with specific reference to the typical scenario of a Clegg integrator (or a more general FORE) interconnected in error feedback with a linear continuous-time plant $\mathcal{P}$, as represented in Figure 1.5. When focusing on stabilization only (that is, $r = 0$), the general dynamics arising from using this model may be well represented by using the notation in Goebel *et al.* (2009) and Goebel *et al.* (2012), and corresponds to the following closed loop involving the overall state $x := \left[\begin{smallmatrix} x_p \\ x_c \end{smallmatrix}\right]$, with x_p being the state of the plant $\mathcal{P}$:

$$\begin{cases} \dot{x} \;= Ax + Bd & \text{is allowed when } x \in \mathcal{F} \\ x^+ = Gx & \text{is allowed when } x \in \mathcal{J}, \end{cases} \tag{1.4}$$

where A, G and B are suitable constant matrices and the jump and flow sets correspond to the following symmetric cones, defined on the

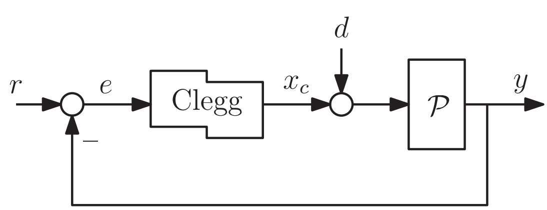

Figure 1.5: A Clegg integrator in error feedback interconnection with a linear plant.

basis of an output equation $y = C_y x$, and already shown at the right of Figure 1.4 (remember that we are looking at a negative feedback interconnection that motivates reversing the horizontal axis)

$$\begin{aligned} \mathcal{F} &:= \left\{ (x_p, x_c) : \begin{bmatrix} x_c \\ y \end{bmatrix}^\top \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y \end{bmatrix} \le 0 \right\}, \\ \mathcal{J} &:= \left\{ (x_p, x_c) : \begin{bmatrix} x_c \\ y \end{bmatrix}^\top \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y \end{bmatrix} \ge 0 \right\}. \end{aligned} \tag{1.5}$$

The overall dynamics (1.4), (1.5) falls into the larger class of homogeneous hybrid systems (see, e.g., Goebel and Teel, 2010; Tuna and Teel, 2006), and will be the modeling framework adopted in this survey. We emphasize that the jump and flow sets defined in Equation (1.5) are closed. This condition is necessary for the theoretical developments in Goebel *et al.* (2012, Ch. 7) to apply. Those results (which have also been used in Nešić *et al.* (2005) and Zaccarian *et al.* (2005) and later works) allow us to establish existence of solutions from any initial conditions (therefore, some type of well posedness of the hybrid dynamics) in addition to suitable robustness properties of asymptotic stability of the origin for the error dynamics of the closed loop represented in Figure 1.5.

Remark 1.1. (Non-uniqueness of solutions) Note that asking that the sets $\mathcal{F}$ and $\mathcal{J}$ be closed implies that there are some regions of the state space belonging to both sets. Therefore, solutions may jump or flow in these regions, so that the solutions to the arising reset linear systems may be nonunique. Nonuniqueness becomes a necessary notion when wanting to establish robust results for the reset system, as a matter of fact, when the feedback system is affected by (arbitrarily small) noise, the state could be pushed in several different directions and different solutions may correspond to different noise selections. These and other robustness issues are addressed and solved in the hybrid framework that we adopt here and in the stability results that we will rely on in this survey.

We emphasize now that the desirable existence and robustness properties highlighted above for the adopted modeling framework are not guaranteed, in general, whenever relying on the alternative model (1.3). When using that model, and using the notation of some recent papers, the closed loop in Figure 1.5 may be represented by the equations

$$\begin{cases} \dot{x} &= Ax + Bd, \quad \text{if } x \notin \mathcal{M} \\ x^+ &= Gx, \quad\quad\quad \text{if } x \in \mathcal{M}, \end{cases} \tag{1.6}$$

where $\mathcal{M} := \{x : C_y x = 0, \text{ and } (I - G)x \neq 0\}$ (recall that we defined $y = C_y x$).

Model (1.6), which is based on Equation (1.3), is actually used in a large number of results that can be found in the literature (see Bupp *et al.* (2000), Chait and Hollot (2002), Beker *et al.* (2004), Baños and Barreiro (2011), Barreiro *et al.* (2014), and Ghaffari *et al.* (2014) just to cite a few) but is associated to some subtle issues related to existence of solutions. In particular, the following observation was already made in Nešić *et al.* (2005) regarding (Beker *et al.*, 2004, Theorem 1), which establishes asymptotic stability of the origin under suitable Lyapunov conditions. Consider however the reset system (1.6) with $d = 0$ and $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$, $G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $C_y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$. Then it is not clear how to define solutions for an initial condition satisfying $C_y x_0 = 0$, $(I - G)x_0 = 0$. Indeed, in that case $x_0 \notin \mathcal{M}$ and the reset is not possible at the initial time, which means that the dynamics can only be governed by the flow equation (1.6) for small $t \geq 0$. Moreover, integrating the differential equation (1.6) from the same initial condition yields $C_y x(t) = 0$ for all t and $(I - G)x(t) = \begin{bmatrix} 0 & 0 & x_3(t) \end{bmatrix}^\top$, which is initially zero but is nonzero for all small t (thus $x(t) \in \mathcal{M}$ for $t > 0$ and thus flowing from the initial condition is not possible). Note that the conditions of Beker *et al.* (2004, Theorem 1) hold for this example by simply selecting $V(x) = |x|^2$, which yields $\dot{V} = -2V$ and $\Delta V \leq 0$. However, the established stability conditions hold for a system that does not guarantee existence of solutions from some initial conditions.

Due to the reasons above, and due to the lack of guarantee of robustness of asymptotic stability, we will restrict our attention to the formalism in dynamics (1.4), (1.5) and will establish robust properties via the Lyapunov tools of Goebel *et al.* (2012).

Remark 1.2. It should be further emphasized that in the model (1.6) resets are only possible on the hyperplane $C_y x = 0$ (as long as some flow has occurred since the last reset), whereas in our model (1.4), (1.5) resets are enforced on a sector $\mathcal{J}$. As a consequence, solutions to model (1.6) flow also in regions of the state space where our model does not allow flowing solutions. The consequence of this fact is that when wanting to use Lyapunov tools to prove asymptotic or exponential stability of the origin, using Equation (1.6) it is necessary to impose a "decrease along flows" condition in almost all the state space, which then implies, by continuity, that this condition holds everywhere. Instead, with our model, and using the Lyapunov tools of Goebel *et al.* (2012), we only need to impose the "decrease along flows" condition in half of the state space (well understood from Figure 1.4) and this leads to less conservative conditions. In particular, even for just a Clegg integrator connected to an integrator plant $\mathcal{P} = \frac{1}{s}$ (this is the system considered in Beker *et al.* (2001a), corresponding to $A = \left[\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right]$) the model (1.6) cannot lead to a strict Lyapunov function proving asymptotic stability of the origin. We will show in the next chapters that desirable stability properties can be shown with model (1.4), (1.5) even in cases when the continuous-time linear dynamics associated with matrix A is exponentially unstable, because the stabilization is obtained by way of the resetting mechanism. This peculiar feature of stabilizing a plant by way of exponentially diverging inputs, that are eventually reset to some value leads to desirable and aggressive control actions, is well illustrated by the simulations and experimental studies reported in the last part of this survey.

1.5 An overview of recent reset systems results

In the previous section we clarified that this survey monograph is focused on robust reset systems, arising from the use of model (1.1) and the arising closed-loop representation (1.4), (1.5).

This modeling framework has been used in a range of recent papers to address various analysis and design questions for reset systems. Those papers constitute the basis for the results in this survey and are shortly described below. The first paper where the modeling framework has been used is Zaccarian *et al.* (2005), where the main observations of this chapter were first pointed out, together with Nešić *et al.* (2005), where Lyapunov like conditions for $\mathcal{L}_2$ stability and exponential stability of reset systems were proposed (later improved and revised in Nešić *et al.* (2008b)). Among other things, the conditions proposed in these papers involved locally Lipschitz Lyapunov functions as opposed to the continuously differentiable ones considered in Beker *et al.* (2004). This allowed us to consider piecewise quadratic Lyapunov functions in verifying exponential or $\mathcal{L}_2$ stability of reset systems (see Zaccarian *et al.* (2011), which is a revised and improved version of Zaccarian *et al.* (2005)). In later years, further developments of this field provided some explicit Lyapunov constructions for a class of planar reset systems, reported in Zaccarian *et al.* (2006), while the properties of reset set-point stabilizers and necessary and sufficient conditions for exponential and $\mathcal{L}_2$ stability have been reported first in Zaccarian *et al.* (2007) and Nešić *et al.* (2008a), and then revised and better illustrated, together with several properties of homogeneous hybrid systems, in Nešić *et al.* (2011). This modeling framework has also been used to provide LMI-based approaches for the $\mathcal{H}_2$ performance analysis and $\mathcal{L}_2$ performance analysis of reset control systems, respectively, in Witvoet *et al.* (2007) and Aangenent *et al.* (2008) (see also Aangenent *et al.*, 2010). More recently, higher-dimensional generalizations of these reset controllers were initially investigated in Prieur *et al.* (2010), later improved in Prieur *et al.* (2013). That specific generalization was actually focusing on a full state feedback architecture and was therefore generalized, in the context of linear plants, to the case of output feedback and Luenberger observers in Fichera *et al.* (2013b), where suitable dwell-time logics were introduced to avoid Zeno phenomena. The arising LMI-based conditions, finally led to a solution of the $\mathcal{H}_\infty$ design problem in Fichera *et al.* (2016) (which is a revised version of the preliminary work in Fichera *et al.*, 2012a). Parallel to these works, Loquen *et al.* (2007) addressed the presence of input saturation in reset systems while Tarbouriech *et al.* (2011) and Fichera *et al.* (2013a) suggested anti-windup actions to manage input saturation effects. Finally, Loquen *et al.* (2008) studied stability of reset systems in the presence of nonzero reference signals.

Even though this survey monograph concentrates on the above-mentioned bulk of literature, there are several recent additional results in the literature, following somewhat different routes, but indicating that reset control is yet an active research field attracting much scientific interest. One route is devoted to strategies where the reset actions are triggered at fixed time instants, often periodic. Stability and $\mathcal{L}_2$ gain properties have been addressed in Heemels *et al.* (2016) for a class of hybrid systems that exhibit linear flow dynamics, periodic time-triggered jumps and arbitrary nonlinear (possibly discontinuous) jump maps, making a link with the lifted system approach from sampled-data control theory. Discrete-time triggering conditions have been provided in Guo *et al.* (2012) in view of a computer-based implementation, for which the triggering condition is replaced by a discrete-time counterpart using a sampled triggering signal. In that paper, the reset controller is designed in three steps involving the baseline controller design, the reset matrix design and the triggering condition tuning. Nearly-periodic situations have been addressed in Hetel *et al.* (2013), considering uncertain intervals between to reset instants, and manipulating the time condition rather than the state condition for the resetting rule.

Recently, van Loon *et al.* (2017) proposed frequency-domain tools for stability analysis of reset control system, with the objective to attract interest of industrials in reset control strategies. Actually, little has been done until now in the direction of frequency-domain tools allowing to derive graphically verifiable stability conditions based on measured frequency response data, with no need to manipulate a parametric model. First attempts in that direction are contained in Hollot *et al.* (2001), in which results regarding BIBO stability were provided for a second-order linear continuous-time closed-loop dynamics. In Zhao and Hua (2017), a Generalized First Order Reset Element (GFORE) has been introduced to better fit for implementation on a physical analog device. The main idea was to establish reset conditions depending on input and output data, rather than less accessible information about the plant states.

Reset control systems have also been extensively studied in the presence of time delays. Stability analysis independent of the delay was proposed in Baños and Barreiro (2009) using Lyapunov–Krasovskii functionals. On the other hand, stability analysis dependent of the delay was addressed in Barreiro and Baños (2010) and Davó *et al.* (2017), using an impulsive delay dynamical systems framework to manage resets as impulsive events. Moreover, using a delay-dependent approach Zhao and Wang (2014) considered piecewise Lyapunov functions to prove stability of the reset observer. Stability analysis of time-delay reset systems is also addressed in an application to networked control system involving time-varying network-induced delays in Baños *et al.* (2014a), but considering discrete-time reset system descriptions. Finally, reset logics for improving the performance of high-gain observers have been studied in Andrieu *et al.* (2016), the idea being to use reset to project trajectories approaching a region of the state-space where high-gain peaking occurs to another region free of peaking phenomena.

A different research route corresponds to the study of reset observers (not addressed in this survey due to their different nature), as reported in Paesa *et al.* (2011) and Paesa *et al.* (2012) and Andrieu *et al.* (2016) with the idea to exploit resetting of some observer states to improve the observer settling time and overshoot/peaking performance.

The main stream of studies relative to reset control systems has been considering interconnection of reset compensators with linear plants. There exist however a few works that explored the interconnection of reset control systems with nonlinear plants. A passivity-based approach has been proposed in Carrasco *et al.* (2010), considering a linear compensator with reset action interconnected with a nonlinear plant. A key advantage of the passivity-based approach is that it allows to derive stability conditions to be checked on the linear compensator without considering the reset action. Passivation-based arguments were also used in Forni *et al.* (2011) using a passivity property of the continuous-time part of the reset controller associated with a suitable non-increase condition for the storage function at jumps to prove stability. Stability analysis for a class of Lipschitz nonlinear systems involving some resetting action at fixed time instants was also performed in Rios *et al.* (2017). LMI-based stability conditions were therein proposed thanks to the use of a 2D vector Lyapunov function issued from the impulsive system representation.

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