

# Fair Play in Assemblies: Supplementary Information

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## Abstract

This Appendix contains supplementary information for “Fair Play in Assemblies.” Specifically, it constructs an equilibrium from Proposition 4 and rules out profitable deviations.

## 1 An Equilibrium with Persistent Rules and Policies

In this version of the model, dynamic utilities from passing policy  $x$  with current rule  $r$  and median  $\mu$  take the form

$$U_i(x, r, \mu; \sigma) = u_i(x) + \delta \sum_{\mu' \in N} \pi(\mu' | \mu) U_i(x^{rp}(x, r, \mu'; \sigma), r^{rp}(x, r, \mu'; \sigma), \mu'; \sigma).$$

for all legislators  $i$ . Note that the proof of proposition 3 in the Appendix contains a discussion of the state space and relevant expected utilities.

Consider the following strategy profile  $\hat{\sigma}$ . The leaders'  $i = 1, 4$  proposal strategies are

$$\hat{\sigma}_i(s^{rp})1 = \begin{cases} R & \text{if } \mu = i \text{ or } \bar{x} \in [\mu - x^*, \mu + x^*] \\ \neg R & \text{otherwise,} \end{cases}$$

where  $x^*$  is defined as

$$x^* = d - \sqrt{\frac{d(1 - p(1 - q)\delta)(d + d(1 - 2p)(1 - q)\delta - 2(1 - p)(1 - q)\delta)}{1 - (1 - q)\delta(2p + (1 - 2p)(1 - q)\delta)}}$$

Note that  $x^*$  converges to 0 as  $\delta$  approaches 0.

Next, each  $j = 2, 3$  accepts the  $i = 1, 4$  procedural proposal if and only if the proposal

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includes minority rights. Specifically, we write

$$\hat{\sigma}_j^{rc}(s^{rc}) = \begin{cases} 1 & \text{if } r^p = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Now consider the policy stage and consider leader 1 in states such that she is the majority leader. In this profile, she proposes the median's ideal point if minority rights are implemented or if 1 is herself the median. If minority rights are not implemented and 1 is not the median, then, she engages in a Romer-Rosenthal bargaining with the median, legislator 2. Formally, we write

$$\hat{\sigma}_1^{pp}(s^{pp}) = \begin{cases} \hat{x}_\mu & \text{if } r = 1 \text{ or } \mu = 1 \\ 0 & \text{if } r = 0, \mu = 2, \text{ and } \bar{x} \in (-\infty, 0] \cup [2d, \infty) \\ \bar{x} & \text{if } r = 0, \mu = 2, \text{ and } \bar{x} \in (0, d] \\ 2d - \bar{x} & \text{if } r = 0, \mu = 2, \text{ and } \bar{x} \in (d, 2d). \end{cases}$$

The leader 4 plays a symmetric strategy when she is the majority leader. Specifically,

$$\hat{\sigma}_4^{pp}(s^{pp}) = \begin{cases} \hat{x}_\mu & \text{if } r = 1 \text{ or } \mu = 4 \\ 1 & \text{if } r = 0, \mu = 3, \text{ and } \bar{x} \in (-\infty, 1 - 2d] \cup [1, \infty) \\ \bar{x} & \text{if } r = 0, \mu = 3, \text{ and } \bar{x} \in [1 - d, 1) \\ 2 - 2d - \bar{x} & \text{if } r = 0, \mu = 3, \text{ and } \bar{x} \in (1 - 2d, 1 - d). \end{cases}$$

For the minority leaders, we define

$$\hat{\sigma}_1^{pa}(s^{pa}) = \begin{cases} 0 & \text{if } |\hat{x}_1 - \hat{x}_\mu| \leq |x^p - \hat{x}_\mu| \\ \mu - |x^p - \hat{x}_\mu| & \text{otherwise} \end{cases}$$

and

$$\hat{\sigma}_4^{pa}(s^{pa}) = \begin{cases} 1 & \text{if } |\hat{x}_4 - \hat{x}_\mu| \leq |x^p - \hat{x}_\mu| \\ \mu + |x^p - \hat{x}_\mu| & \text{otherwise.} \end{cases}$$

Finally, the legislator  $i = 2, 3$  chooses the proposal or status quo that maximizes their dynamic expected utility but with deference to the minority leader when they are indifferent.<sup>1</sup> Then as follows

$$\hat{\sigma}_i^{Mc}(s^{Mc}) = \hat{\sigma}_i^{mc}(s^{mc}) = \begin{cases} c_a & \text{if } U_i(x^a, r, \mu) \geq U_i(a', r, \mu), a' \in A^{mc} \\ c_p & \text{if } U_i(x^p, r, \mu) \geq U_i(a', r, \mu), a' \in A^{mc} \text{ and} \\ & U_i(x^p, r, \mu) > U_i(x^a, r, \mu) \\ c_q & \text{otherwise.} \end{cases}$$

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<sup>1</sup>This is needed for the minority leader's best response to exist.

To put more structure on  $\hat{\sigma}_i^{Mc}(s^{Mc})$ , we ensure that, along the path of play, legislators 2 and 3 always accept their majority leader's policy proposal under majority dominance.

#### Proof of Proposition 4

The open set of parameters of interest is  $d \in \left(\frac{1}{4}, \frac{1}{7}\right)$ ,  $p \in \left(\frac{1}{2}, \frac{7}{8}\right)$  and  $q \in \left(\frac{6}{7}, 1\right)$ . Throughout, I refer to these as the specified set of parameters.

We prove the proposition in three steps. First, we verify that the legislators play optimally with the minority right in place. Second, we do the same before the minority right has been implemented. To do this, we full characterize the four legislators' dynamic utilities. Third, we investigate the conditions under which the majority leaders offer minority rights. All results are proved using the expected utilities of actors 1 and 2 because those of actors 3 and 4 are symmetric. In what follows, we acknowledge an abuse of notation and write  $U_i(x, r, P_i)$  and  $U_i(x, r, P_{-i})$  to denote  $i$ 's dynamic expected utility over the outcomes  $x$  and  $r$  when the median is  $i$ 's own and opposing party, respectively. Doing so helps condense the derivations of the dynamic utilities under  $\hat{\sigma}$  because the median only influences the transition probabilities, which vary depending on the median's party.

**Claim 1** *The dynamic expected utility functions  $U_i(x, R, P_i; \hat{\sigma})$  and  $U_i(x, R, P_{-i}; \hat{\sigma})$  are continuous, strictly concave, and symmetric around  $\hat{x}_i$  for all  $i = 1, 2$ .*

*Proof.* Consider  $U_i(x, R, P_i; \hat{\sigma})$ , when  $x$  is the policy outcome in a state in which minority rights have been enforced and actors play according to  $\hat{\sigma}$  in all other states. We can write

$$U_i(x, R, P_i; \hat{\sigma}) = u_i(x) + \delta (pqU_i(0, R, 1; \hat{\sigma}) + p(1 - q)U_i(d, R, 2; \hat{\sigma}) + (1 - p)qU_i(1 - d, R, 4; \hat{\sigma}) + (1 - p)(1 - q)U_i(1, R, 3; \hat{\sigma})).$$

Because  $u_i$  is continuous, strictly concave and symmetric around  $\hat{x}_i$ , so is  $U_i(x, R, P_i; \hat{\sigma})$ . The same logic demonstrates that the minority leader's dynamic utility  $U_i(x, R, P_{-i}; \hat{\sigma})$  shares the same properties as well.  $\square$

Using Claim 1, we can verify that no actors have profitable deviations in minority-right states. To see this, note that if either the majority leader or the minority leader offers a different policy proposal, the median will always accept the proposal of the actor who did not deviate. Finally, the median has no incentive to accept the status quo because her dynamic utility is maximized at her ideal policy position, which is the outcome under strategy  $\hat{\sigma}$ .

To condense the notation that follows, define the following:

$$Z_i^M = pqU_i(0, R, 1; \hat{\sigma}) + (1 - p)qU_i(1, R, 4; \hat{\sigma})$$

and

$$Z_i^m = pqU_i(1, R, 4; \hat{\sigma}) + (1 - p)qU_i(0, R, 1; \hat{\sigma}).$$

**Claim 2** *The dynamic utility functions  $U_1(0, R, 1; \hat{\sigma})$ ,  $U_1(d, R, 2; \hat{\sigma})$ ,  $U_1(1 - d, R, 3; \hat{\sigma})$ , and  $U_1(1, R, 4; \hat{\sigma})$  take the following functional form:*

$$\begin{aligned} U_1(0, R, 1; \hat{\sigma}) &= -\frac{\delta(1 - p - 2d(1 - p)(1 - q) + d^2(1 - q)(1 + (1 - 2p)\delta))}{(1 - \delta)(1 + (1 - 2p)\delta)}, \\ U_1(1, R, 4; \hat{\sigma}) &= \frac{-1 + \delta(p - d(1 - q)(d - 2p - (2 - d)(1 - 2p)\delta))}{(1 - \delta)(1 + (1 - 2p)\delta)}, \\ U_1(d, R, 2; \hat{\sigma}) &= \frac{-d^2(1 + (1 - 2p)\delta)(1 - q\delta) + 2d(1 - p)(1 - q)\delta - (1 - p)\delta}{(1 - \delta)(1 + (1 - 2p)\delta)} \end{aligned}$$

and

$$U_1(1 - d, R, 3; \hat{\sigma}) = -\frac{(1 - d)^2 - (p - 2dp(1 + q)d^2(1 - 2p - q))\delta + d(2 - d)(1 - 2p)q\delta^2}{(1 - \delta)(1 + (1 - 2p)\delta)}$$

*Proof.* We can write  $U_1(0, R, 1; \hat{\sigma})$ ,  $U_1(d, R, 2; \hat{\sigma})$ ,  $U_1(1 - d, R, 3; \hat{\sigma})$ , and  $U_1(1, R, 4; \hat{\sigma})$ , as a system of four equations with four unknowns. Specifically, the equations are (??)-(??)

□

**Claim 3** *The dynamic utility functions  $U_2(0, R, 1; \hat{\sigma})$ ,  $U_2(d, R, 2; \hat{\sigma})$ ,  $U_2(1 - d, R, 3; \hat{\sigma})$ , and  $U_2(1, R, 4; \hat{\sigma})$  take the following form:*

$$\begin{aligned} U_2(0, R, 1; \hat{\sigma}) &= \frac{-(1 - p)\delta + 2d(1 - p)(2 - q)\delta + -d^2(1 + \delta(4 - q(3 - \delta) - \delta - p(6 - 2q(2 - \delta) - 2\delta)))}{(1 - \delta)(1 + (1 - 2p)\delta)}, \\ U_2(d, R, 2; \hat{\sigma}) &= -\delta \frac{1 - p - 2d(1 - p)(2 - q) + d^2(4 - 2p(2 - q(2 - \delta)) - q(3 - \delta))}{(1 - \delta)(1 + (1 - 2p)\delta)}, \\ U_2(1, R, 4; \hat{\sigma}) &= -\frac{1 - p\delta + d(2 - d - \delta(2pq + d(p(2 - 4q) + q)) - (2 - 3d)(1 - 2p)(1 - q)\delta^2)}{(1 - \delta)(1 + (1 - 2p)\delta)}, \end{aligned}$$

and

$$U_2(1 - d, R, 3; \hat{\sigma}) = -\frac{1 - p\delta - d(4 - 4d + \delta(dq + 2p(2 + q - 2d(1 + q)))) - (2 - 3d)(1 - 2p)q\delta^2}{(1 - \delta)(1 + (1 - 2p)\delta)}$$

*Proof.* We can write  $U_2(0, R, 1; \hat{\sigma})$ ,  $U_2(d, R, 2; \hat{\sigma})$ ,  $U_2(1 - d, R, 3; \hat{\sigma})$ , and  $U_2(1, R, 4; \hat{\sigma})$  as a system of four equations with four unknowns. Specifically, the equations are

$$\begin{aligned} U_2(0, R, 1; \hat{\sigma}) &= -d^2 + U_2(d, R, 2; \hat{\sigma}) \\ U_2(d, R, 2; \hat{\sigma}) &= \delta \left( p(1 - q)U_2(d, R, 2; \hat{\sigma}) + (1 - p)(1 - q)U_2(1 - d, R, 3; \hat{\sigma}) + Z_2^M \right), \\ U_2(1, R, 4; \hat{\sigma}) &= -(1 - d)^2 + \delta \left( p(1 - q)U_2(1 - d, R, 3; \hat{\sigma}) + (1 - p)(1 - q)U_2(d, R, 2; \hat{\sigma}) + Z_2^m \right), \\ \text{and} \\ U_2(1 - d, R, 3; \hat{\sigma}) &= (1 - d)^2 - (1 - 2d)^2 + U_2(1, R, 4; \hat{\sigma}). \end{aligned}$$

Solving returns the solutions above.

□

We now consider the dynamic expected utility calculations of the legislators in states that have not yet implemented minority rights.

**Claim 4** For  $i = 1, 2$ , the dynamic utility function  $U_i(x, \neg R, P_i; \hat{\sigma})$  is such that

$$\begin{aligned} U_i(0, \neg R, 2; \hat{\sigma}) &= \frac{u_i(0)(1 - p(1 - q)\delta) + u_i(1)(1 - q)(1 - p)\delta}{(1 - (1 - q)\delta)(1 + (1 - 2p)(1 - q)\delta)} + \\ &\quad \delta U_i(0, R, 1; \hat{\sigma}) \frac{q(p + (1 - 2p)(1 - q)\delta)}{(1 - (1 - q)\delta)(1 + (1 - 2p)(1 - q)\delta)} + \\ &\quad \delta U_i(1, R, 4; \hat{\sigma}) \frac{q(1 - p)}{(1 - (1 - q)\delta)(1 + (1 - 2p)(1 - q)\delta)} \end{aligned}$$

and

$$\begin{aligned} U_i(1, \neg R, 3; \hat{\sigma}) &= \frac{u_i(1)(1 - p(1 - q)\delta) + u_i(0)(1 - q)(1 - p)\delta}{(1 - (1 - q)\delta)(1 + (1 - 2p)(1 - q)\delta)} + \\ &\quad \delta U_i(0, R, 1; \hat{\sigma}) \frac{q(1 - p)}{(1 - (1 - q)\delta)(1 + (1 - 2p)(1 - q)\delta)} + \\ &\quad \delta U_i(1, R, 4; \hat{\sigma}) \frac{q(p + (1 - 2p)(1 - q)\delta)}{(1 - (1 - q)\delta)(1 + (1 - 2p)(1 - q)\delta)} \end{aligned}$$

*Proof.* We can write  $U_i(0, \neg R, 2; \hat{\sigma})$  and  $U_i(1, \neg R, 3; \hat{\sigma})$  as a system of two equations with two unknowns. Specifically, we have

$$U_i(0, \neg R, 2; \hat{\sigma}) = u_i(0) + \delta \left( p(1 - q)U_i(0, \neg R, 2; \hat{\sigma}) + (1 - p)(1 - q)U_i(1, \neg R, 3; \hat{\sigma}) + Z_i^M \right)$$

and

$$U_i(1, \neg R, 3; \hat{\sigma}) = u_i(1) + \delta \left( p(1 - q)U_i(1, \neg R, 3; \hat{\sigma}) + (1 - p)(1 - q)U_i(0, \neg R, 2; \hat{\sigma}) + Z_i^m \right)$$

Solving provides the solutions above. □

We now fully characterize  $U_i(x, \neg R, P_i; \hat{\sigma})$  for  $i = 1, 2$ . To do so, first define the following intervals:

$$\begin{aligned} I_1 &= (-\infty, 0] & I_2 &= (0, d - x^*] \\ I_3 &= (d - x^*, d + x^*) & I_4 &= [d + x^*, 2d) \\ I_5 &= [2d, 1 - 2d] & I_6 &= (1 - 2d, 1 - d - x^*) \\ I_7 &= (1 - d - x^*, 1 - d + x^*) & I_8 &= [1 - d + x^*, 1) \\ I_9 &= [1, \infty). \end{aligned}$$

**Claim 5** For  $i = 1, 2$ , the dynamic utility function  $U_i(x, \neg R, P_i; \hat{\sigma})$  takes the following form:

1. for all  $x \in I_1 \cup I_5 \cup I_9$ ,

$$U_i(x, \neg R, P_i; \hat{\sigma}) = u_i(x) + \delta \left( p(1 - q)U_i(0, \neg R, 2; \hat{\sigma}) + (1 - p)(1 - q)U_i(1, \neg R, 3; \hat{\sigma}) + Z_i^M \right)$$

2. for all  $x \in I_2$ ,

$$U_i(x, \neg R, P_i; \hat{\sigma}) = \frac{u_i(x) + \delta \left( (1 - p)(1 - q)U_i(1, \neg R, 3; \hat{\sigma}) + Z_i^M \right)}{1 - p(1 - q)\delta},$$

3. for all  $x \in I_3$ ,

$$U_i(x, \neg R, P_i; \hat{\sigma}) = u_i(x) + \delta \left( p(1-q)U_i(d, R, 2; \hat{\sigma}) + (1-p)(1-q)U_i(1, \neg R, 3; \hat{\sigma}) + Z_i^M \right),$$

4. for all  $x \in I_4$ ,

$$U_i(x, \neg R, P_i; \hat{\sigma}) = u_i(x) + \delta \left( p(1-q)U_i(2d-x, \neg R, 2; \hat{\sigma}) + (1-p)(1-q)U_i(1, \neg R, 3; \hat{\sigma}) + Z_i^M \right),$$

5. for all  $x \in I_6$ ,

$$U_i(x, \neg R, P_i; \hat{\sigma}) = u_i(x) + \delta \left( p(1-q)U_i(0, \neg R, 2; \hat{\sigma}) + (1-p)(1-q)U_i(2-2d-x, \neg R, 3; \hat{\sigma}) + Z_i^M \right),$$

where, for all  $x' \in I_8$ ,

$$U_i(x', \neg R, 3; \hat{\sigma}) = \frac{u_i(x) + \delta \left( (1-p)(1-q)U_i(0, \neg R, 2; \hat{\sigma}) + Z_i^m \right)}{1 - p(1-q)\delta},$$

6. for all  $x \in I_7$

$$U_i(x, \neg R, P_i; \hat{\sigma}) = u_i(x) + \delta \left( p(1-q)U_i(0, \neg R, 2; \hat{\sigma}) + (1-p)(1-q)U_i(1-d, R, 3; \hat{\sigma}) \right),$$

and

7. for all  $x \in I_8$

$$\begin{aligned} U_i(x, \neg R, P_i; \hat{\sigma}) = & \frac{u_i(x)(1 + (1-2p)(1-q)\delta)}{1 - p(1-q)\delta} + \\ & \delta U_i(0, R, 1; \hat{\sigma}) \frac{q(p + (1-2p)(1-q)\delta)}{1 - p(1-q)\delta} + \\ & \delta U_i(0, \neg R, 2; \hat{\sigma}) \frac{(1-q)(p + (1-2p)(1-q)\delta)}{1 - p(1-q)\delta} + \\ & \delta U_i(1, R, 4; \hat{\sigma}) \frac{q(1-p)}{1 - p(1-q)\delta} \end{aligned}$$

*Proof.*

1. This follows directly from the definition of  $\hat{\sigma}$  and the expected utility calculations.

2. Consider some  $x \in I_2$ . We can write  $U_i(x, \neg R, P_i; \hat{\sigma})$  as follows

$$U_i(x, \neg R, P_i; \hat{\sigma}) = u_i(x) + \delta \left( p(1-q)U_i(x, \neg R, 3; \hat{\sigma}) + (1-p)(1-q)U_i(1, \neg R, 3; \hat{\sigma}) + Z_i^M \right),$$

and solving for  $U_i(x, \neg R, P_i; \hat{\sigma}) = U_i(x, \neg R, 3; \hat{\sigma})$  produces the desired result.

3. This follows directly from  $\hat{\sigma}$ .

4. Again, the expression comes from  $\hat{\sigma}$ . However, note that when  $x \in I_4$ ,  $2d - x$  resides in  $I_2$ . This implies  $U_i(2d - x, \neg R, P_i; \hat{\sigma})$  can be computed using the expression in 2.
5. The expression follows from  $\hat{\sigma}$ . When  $x \in I_6$ , then  $2 - 2d - x$  resides in the closure of  $I_8$ , and we can use the result below to compute  $U_i(2 - 2d - x, \neg R, 3; \hat{\sigma})$ .
6. This follows immediately from  $\hat{\sigma}$ .
7. Given  $x \in I_8$ , we can write

$$U_i(x, \neg R, P_i; \hat{\sigma}) = u_i(x) + \delta \left( p(1 - q)U_i(0, \neg R, 2; \hat{\sigma}) + (1 - p)(1 - q)U_i(x, \neg R, 3; \hat{\sigma}) + Z_i^M \right)$$

and

$$U_i(x, \neg R, 3; \hat{\sigma}) = u_i(x) + \delta \left( p(1 - q)U_i(x, \neg R, 3; \hat{\sigma}) + (1 - p)(1 - q)U_i(0, \neg R, 2; \hat{\sigma}) + Z_i^m \right).$$

Solving these two equations for  $U_i(x, \neg R, P_i; \hat{\sigma})$  and  $U_i(x, \neg R, 3; \hat{\sigma})$  produces the desired result.  $\square$

To analyze points of continuity, it will be convenient to define a function  $F_i^k : \text{clo}(I_k) \rightarrow \mathbb{R}$  such that  $F_i^k(x) = U_i(x, \neg R, P_i; \hat{\sigma})$  but  $U_i(x, \neg R, P_i; \hat{\sigma})$  takes the same functional form over the entire interval  $\text{clo}(I_k)$  as it does over  $I_k$ . For example,

$$F_i^2(x) = \frac{u_i(x) + \delta \left( (1 - p)(1 - q)U_i(1, \neg R, 3; \hat{\sigma}) + Z_i^M \right)}{1 - p(1 - q)\delta},$$

for all  $x \in I_2$ , particularly, even at  $x = 0$ .

**Claim 6** *The dynamic expected utility functions  $U_i(x, \neg R, P_i; \hat{\sigma})$  are continuous at points 0,  $d$ ,  $2d$ ,  $1 - 2d$ ,  $1 - d$  and 1.*

*Proof.* The result follows from the Romer-Rosenthal construction of the majority leaders' policy proposal strategies in  $\hat{\sigma}$  under majority dominance. To see these, we illustrate the argument at 0. We can write

$$\begin{aligned} F_i^1(0) &= u_i(0) + \delta \left( p(1 - q)U_i(0, \neg R, 2; \hat{\sigma}) + (1 - p)(1 - q)U_i(1, \neg R, 3; \hat{\sigma}) + Z_i^M \right) \\ &= \frac{u_i(0) + \delta \left( (1 - p)(1 - q)U_i(1, \neg R, 3; \hat{\sigma}) + Z_i^M \right)}{1 - p(1 - q)} \\ &= F_i^2(0), \end{aligned}$$

where the second equality follows because  $F_i^1(0) = U_i(0, \neg R, 2; \hat{\sigma})$  by definition.  $\square$

**Claim 7** *For all policies  $x \in \mathbb{R}$ ,  $U_2(d, R, 2; \hat{\sigma}) \geq U_2(x, \neg R, 2; \hat{\sigma})$  and  $U_2(0, R, 1; \hat{\sigma}) \geq U_2(0, \neg R, 1; \hat{\sigma})$*

*Proof.* Consider the first inequality. Not only does legislator 2 receive some instantaneous policy benefit when comparing  $U_2(d, R, 2; \hat{\sigma})$  to  $U_2(x, \neg R, 2; \hat{\sigma})$ , because  $u_2(d) \geq u_2(x)$ , but he

also receives a larger expected utility in future states when either he or 3 is the median, strictly so in the later case. In the states in which 1 or 4 is the median, 2's instantaneous payoff remains unchanged. For these last two reasons,  $U_2(0, R, 1; \hat{\sigma}) \geq U_2(0, \neg R, 1; \hat{\sigma})$ .  $\square$

In words, Claim 7 guarantees that legislators 2 and 3 will always accept a proposal for the right given the stream of policies under  $\hat{\sigma}$ .

**Claim 8** *The dynamic utility function  $U_1(x, \neg R, P_i; \hat{\sigma})$  is continuous at  $d - x^*$  and  $d + x^*$ .*

*Proof.* We show that  $U_1(x, \neg R, P_i; \hat{\sigma})$  is continuous at  $d - x^*$ . The proof of continuity at  $d + x^*$  is symmetric due to the construction of  $\hat{\sigma}$  in a Romer-Rosenthal type framework.

First, we must show that  $F_1^2(d - x^*) = F_1^3(d - x^*)$ . We can write

$$F_1^2(d - x^*) = u_1(d - x^*) + \delta \left( p(1 - q)U_1(d - x^*, \neg R, 2; \hat{\sigma}) + (1 - p)(1 - q)U_1(1, \neg R, 3; \hat{\sigma}) + Z_1^M \right)$$

and

$$F_1^3(d - x^*) = u_1(d - x^*) + \delta \left( p(1 - q)U_1(d, R, 2; \hat{\sigma}) + (1 - p)(1 - q)U_1(1, \neg R, 3; \hat{\sigma}) + Z_1^M \right).$$

Thus, it suffices to show  $U_1(d - x^*, \neg R, 2; \hat{\sigma}) = U_1(d, R, 2; \hat{\sigma})$ . Let  $x = d - x^*$ , then we have

$$\begin{aligned} U_1(d - x^*, \neg R, 2; \hat{\sigma}) &= U_1(d, R, 2; \hat{\sigma}) \\ &\iff \\ \frac{-x^2 + \delta \left( (1 - p)(1 - q)U_1(1, \neg R, 3; \hat{\sigma}) + Z_1^M \right)}{1 - p(1 - q)\delta} &= \frac{-d^2 + \delta \left( (1 - p)(1 - q)U_1(1 - d, R, 3; \hat{\sigma}) + Z_1^M \right)}{1 - p(1 - q)\delta} \\ &\iff \\ x^2 &= d^2 + \delta(1 - p)(1 - q) (U_1(1, \neg R, 3; \hat{\sigma}) - U_1(1 - d, R, 3; \hat{\sigma})) \end{aligned}$$

With some algebra, it can be shown that

$$\begin{aligned} (U_1(1, \neg R, 3; \hat{\sigma}) - U_1(1 - d, R, 3; \hat{\sigma})) &= \frac{d^2 + d^2\delta - 2d^2p\delta - d^2q\delta + 2d^2pq\delta - 2d + 2dp\delta - 2dpq\delta}{(1 - (1 - q)\delta)(1 + (1 - 2p)(1 - q)\delta)} \\ &= \frac{d(d + d(1 - 2p)(1 - q)\delta - 2(1 - p(1 - q)\delta))}{(1 - (1 - q)\delta)(1 + (1 - 2p)(1 - q)\delta)} \end{aligned}$$



Substituting this into the above equation for  $x^2$  gives us

$$\begin{aligned}
 x^2 &= d^2 + \delta(1-p)(1-q) \left( \frac{d(d + d(1-2p)(1-q)\delta - 2(1-p(1-q)\delta))}{(1 - (1-q)\delta)(1 + (1-2p)(1-q)\delta)} \right) \\
 &\iff \\
 x^2 &= \frac{d(1-p(1-q)\delta)(d + d(1-2p)(1-q)\delta - 2(1-p)(1-q)\delta)}{(1 - (1-q)\delta)(1 + (1-2p)(1-q)\delta)} \\
 &\iff \\
 x &= \pm \sqrt{\frac{d(1-p(1-q)\delta)(d + d(1-2p)(1-q)\delta - 2(1-p)(1-q)\delta)}{1 - (1-q)\delta(2p + (1-2p)(1-q)\delta)}},
 \end{aligned}$$

which holds by the definition of  $x$  and  $x^*$ .  $\square$

**Claim 9** *Under the specified set of parameters, then the dynamic utility function  $U_1(x, \neg R, P_i; \hat{\sigma})$  is strictly increasing when  $x \leq \hat{x}_1$  and decreasing in all its “pieces” when  $x \geq \hat{x}_1$ , and has a unique maximizer at  $\hat{x}_1$ .*

*Proof.* Differentiation reveals that

$$D_x U_1(x, \neg R, P_i; \hat{\sigma}) = \begin{cases} -2x & \text{if } x \in I_1 \\ \frac{-2x}{1-p(1-q)\delta} & \text{if } x \in I_2 \\ -2x & \text{if } x \in I_3 \\ -\frac{2(x-2dp(1-q)\delta)}{1-p(1-q)\delta} & \text{if } x \in I_4 \\ -2x & \text{if } x \in I_5 \\ -2x + \frac{2(1-p)(1-q)(2-2d-x)\delta}{1-p(1-q)\delta} & \text{if } x \in I_6 \\ -2x & \text{if } x \in I_7 \\ -\frac{2x(1+(1-2p)(1-q)\delta)}{1-p(1-q)\delta} & \text{if } x \in I_8 \\ -2x & \text{if } x \in I_9 \end{cases}$$

I claim that in each of the 9 intervals that comprise  $U_1(x, \neg R, P_i; \hat{\sigma})$ ,  $U_1(x, \neg R, P_i; \hat{\sigma})$  is strictly decreasing except when  $x \leq 0$ . By the definition of  $D_x U_1(x, \neg R, P_i; \hat{\sigma})$ , we only need to verify the monotonicity of  $U_1(x, \neg R, P_i; \hat{\sigma})$  in  $I_4$  and  $I_6$ . To do this, we must ensure that for all  $x \in I_4$ ,

$$\begin{aligned}
 -(x - 2dp(1-q)\delta) &< 0 \iff x > 2dp(1-q)\delta \\
 &\iff x > 2 \left( \frac{1}{4} \right) \left( \frac{7}{8} \right) \left( 1 - \frac{6}{7} \right) \delta \\
 &\iff x > \frac{1}{16} \delta.
 \end{aligned}$$

Because  $x > d + x^*$ ,  $x^* > 0$ , and  $d > \frac{1}{7}$ , this inequality holds, so  $U_1(x, \neg R, P_i; \hat{\sigma})$  is strictly

decreasing in this interval. We must also guarantee that for all  $x \in I_6$  we have

$$\begin{aligned}
 x > \frac{(1-p)(1-q)(2-2d-x)\delta}{1-p(1-q)\delta} &\iff x \left(1 + \frac{(1-p)(1-q)\delta}{1-p(1-q)\delta}\right) > \frac{(1-p)(1-q)(2-2d)\delta}{1-p(1-q)\delta} \\
 &\iff x > \frac{(1-p)(1-q)(2-2d)\delta}{1+(1-2p)(1-q)\delta} \\
 &\iff x > \frac{\left(1-\frac{1}{2}\right)\left(1-\frac{6}{7}\right)\left(2-2\frac{1}{7}\right)\delta}{1+\left(1-2\frac{1}{2}\right)\left(1-\frac{6}{7}\right)\delta} \\
 &\iff x > \frac{6}{49}\delta,
 \end{aligned}$$

where the implication follows because the right hand side of the inequality is decreasing in  $p$ ,  $q$ , and  $d$ .

With the previous result in mind, we establish that  $\hat{x}_1 = 0$  is a maximizer of  $U_1(x, \neg R, P_i; \hat{\sigma})$  under the set of parameters specified. Note that by Claim 6 and 8,  $U_1(x, \neg R, P_i; \hat{\sigma})$  is continuous over the interval  $(-\infty, 1-d-x^*]$ . Because  $U_1(x, \neg R, P_i; \hat{\sigma})$  is continuously differentiable at  $\hat{x}_1$  and is strictly concave, taking second derivatives, we can conclude  $\hat{x}_1$  is a local maximizer. To verify that  $\hat{x}_1$  is a global maximizer, we must verify that  $U_1(0, \neg R, P_i; \hat{\sigma}) \geq F_1^7(1-d-x^*)$  and  $U_1(0, \neg R, P_i; \hat{\sigma}) \geq F_1^8(1-d+x^*)$ . This is in fact the case because we have shown above that  $U_1(x, \neg R, P_i; \hat{\sigma})$  is strictly decreasing and continuous in  $I_7$  and  $I_8 \cup I_9$ .

First, we demonstrate that  $U_1(0, \neg R, P_i; \hat{\sigma}) \geq F_1^7(1-d-x^*)$ . We write

$$U_1(0, \neg R, P_i; \hat{\sigma}) = \delta \left( p(1-q)U_1(0, \neg R, 2; \hat{\sigma}) + (1-p)(1-q)U_1(1, \neg R, 3; \hat{\sigma}) + Z_1^M \right)$$

and

$$F_1^7(1-d-x^*) = u_1(1-d-x^*) + \left( p(1-q)U_1(0, \neg R, 2; \hat{\sigma}) + (1-p)(1-q)U_1(1-d, R, 3; \hat{\sigma}) + Z_1^M \right).$$

Therefore, we must have

$$\begin{aligned}
 \delta(1-p)(1-q)U_1(1, \neg R, 3; \hat{\sigma}) &\geq -(1-d-x^*)^2 + \delta(1-p)(1-q)U_1(1-d, R, 3; \hat{\sigma}) \\
 &\iff \\
 (1-d-x^*)^2 &\geq \delta(1-p)(1-q) (U_1(1-d, R, 3; \hat{\sigma}) - U_1(1, \neg R, 3; \hat{\sigma})) \\
 &\iff \\
 (1-d-x^*)^2 &\geq \delta(1-p)(1-q) \left( -\frac{d(d(1+(1-2p)(1-q)\delta) - 2(1-p(1-q)\delta))}{(1-(1-q)\delta)(1+(1-2p)(1-q)\delta)} \right).
 \end{aligned}$$

Inspecting the right hand side of the above inequality reveals that it is increasing in  $d$ , and decreasing in  $p$  and  $q$  when  $d < 1/2$ . Therefore, substituting the  $d = 1/4$ ,  $p = 1/2$ , and  $q = 6/7$ , we must show

$$(1-d-x^*)^2 \geq \frac{(49-4\delta)\delta}{224(7-\delta)} \leq \frac{15}{448}.$$

But this must be true because  $1-d-x^* > 1/2$ .

Second, we demonstrate that  $U_1(0, \neg R, P_i; \hat{\sigma}) \geq F_1^8(1 - d + x^*)$ . We can write

$$F_1^8(1 - d + x^*) = u_1(1 - d + x^*) + \left( p(1 - q)U_1(0, \neg R, 2; \hat{\sigma}) + (1 - p)(1 - q)U_1(1 - d + x^*, \neg R, 3; \hat{\sigma}) + Z_1^M \right).$$

We must show

$$\begin{aligned} \delta(1 - p)(1 - q)U_1(1, \neg R, 3; \hat{\sigma}) &\geq -(1 - d + x^*)^2 + \delta(1 - p)(1 - q)U_1(1 - d + x^*, \neg R, 3; \hat{\sigma}) \\ &\iff \\ (1 - d + x^*)^2 &\geq \delta(1 - p)(1 - q) (U_1(1 - d + x^*, \neg R, 3; \hat{\sigma}) - U_1(1, \neg R, 3; \hat{\sigma})), \end{aligned}$$

but this also must be true because  $U_1(1 - d, 1; \hat{\sigma}) \geq U_1(1 - d + x^*, 0; \hat{\sigma})$  when  $p > 1/2$ .  $\square$

**Claim 10** *Under the specified set of parameters, the dynamic utility function  $U_2(x, \neg R, P_i; \hat{\sigma})$  satisfies the following properties.*

1. *It is symmetric around  $\hat{x}_2$  in the interval  $(0, 2d)$ .*
2. *It is continuous and strictly increasing in all of its “pieces” for all  $x < d$  and strictly decreasing in all of its “pieces” for all  $x > d$ .*
3. *It is discontinuous at  $d - x^*$  and  $d + x^*$ , namely,  $F_2^3(d - x^*) > F_2^2(d - x^*)$  and  $F_2^3(d + x^*) > F_2^4(d + x^*)$ .*
4.  $U_2(0, \neg R, P_i; \hat{\sigma}) \geq F_2^7(1 - d - x^*)$ .
5.  $U_2(0, \neg R, P_i; \hat{\sigma}) \geq F_2^8(1 - d + x^*)$ .
6. *It has a global maximizer at  $\hat{x}_2 = d$ .*

*Proof.* Differentiation of  $U_2(x, \neg R, P_i; \hat{\sigma})$  gives us

$$D_x U_2(x, \neg R, P_i; \hat{\sigma}) = \begin{cases} 2(d - x) & \text{if } x \leq I_1 \\ \frac{2(d-x)}{1-p(1-q)\delta} & \text{if } x \in I_2 \\ 2(d - x) & \text{if } x \in I_3 \\ \frac{2(d-x)}{1-p(1-q)\delta} & \text{if } x \in I_4 \\ 2(d - x) & \text{if } x \in I_5 \\ 2(d - x) + \frac{2(1-p)(1-q)(2-3d-x)\delta}{1-p(1-q)\delta} & \text{if } x \in I_6 \\ 2(d - x) & \text{if } x \in I_7 \\ \frac{2(d-x)(1+(1-2p)(1-q)\delta)}{1-p(1-q)\delta} & \text{if } x \in I_8 \\ 2(d - x) & \text{if } x \in I_9 \end{cases} \quad (1)$$

Symmetry in the interval  $I_3$  follows from the symmetry of  $u_2$ . Symmetry in the intervals  $I_2$  and  $I_4$  follows from the symmetry of  $u_2$  and the Romer-Rosenthal bargaining in  $\hat{\sigma}$ .

The second claim follows directly from Eq. (1), but we illustrate the proof for the piece  $I_6$ . We require

$$\begin{aligned} 2(d-x) + \frac{2(1-p)(1-q)(2-3d-x)\delta}{1-p(1-q)\delta} < 0 &\iff x \left(1 + \frac{(1-p)(1-q)\delta}{1-p(1-q)\delta}\right) > d + \frac{(1-p)(1-q)(2-3d)\delta}{1-p(1-q)\delta} \\ &\iff x > \frac{2(1-p)(1-q)\delta + d(1-(3-2p)(1-q)\delta)}{1+(1-2p)(1-q)\delta} \end{aligned}$$

for all  $x \in I_6$ . The right hand side of the above inequality is increasing in  $d$  and decreasing in  $p$  and  $q$ . Substituting  $1/4$  for  $d$ ,  $1/2$  for  $p$  and  $6/7$  for  $q$ , we must have, for all  $x \in I_6$

$$x > \frac{1}{4} + \frac{\delta}{14},$$

but this must be true because  $x > 1 - 2d > 1/2$ .

To demonstrate the third claim, we show  $F_2^3(d-x^*) > F_2^2(d-x^*)$ . The relationship  $F_2^3(d+x^*) > F_2^4(d+x^*)$  follows from the symmetry of  $U_2(x, \neg R, P_i; \hat{\sigma})$  in the interval  $(0, 2d)$ . We can write

$$F_2^3(d-x^*) = -(x^*)^2 + \delta \left( p(1-q)U_2(d, R, 2; \hat{\sigma}) + (1-p)(1-q)U_2(1, \neg R, 3; \hat{\sigma}) + Z_2^M \right)$$

and

$$F_2^2(d-x^*) = -(x^*)^2 + \delta \left( p(1-q)U_2(d-x^*, \neg R, 2; \hat{\sigma}) + (1-p)(1-q)U_2(1, \neg R, 3; \hat{\sigma}) + Z_2^M \right).$$

Therefore, we must show  $U_2(d, R, 2; \hat{\sigma}) > U_2(d-x^*, \neg R, 2; \hat{\sigma})$ , but this follows from Claim 7.

To prove the fourth claim, write

$$U_2(0, \neg R, P_i; \hat{\sigma}) = -d^2 + \delta \left( p(1-q)U_2(0, \neg R, 2; \hat{\sigma}) + (1-p)(1-q)U_2(1, \neg R, 3; \hat{\sigma}) + Z_2^M \right)$$

and

$$F_2^7(1-d-x^*) = -(1-2d-x^*)^2 + \delta \left( p(1-q)U_2(0, \neg R, 2; \hat{\sigma}) + (1-p)(1-q)U_2(1-d, R, 3; \hat{\sigma}) + Z_2^M \right).$$

We require

$$\begin{aligned} -d^2 + \delta(1-p)(1-q)U_2(1, \neg R, 3; \hat{\sigma}) &\geq -(1-2d-x^*)^2 + \delta(1-p)(1-q)U_2(1-d, R, 3; \hat{\sigma}) \\ &\iff \\ d^2 &\leq (1-2d-x^*)^2 + \delta(1-p)(1-q) (U_2(1, \neg R, 3; \hat{\sigma}) - U_2(1-d, R, 3; \hat{\sigma})) \\ &\iff \\ d^2 - (1-2d-x^*)^2 &\leq \delta(1-p)(1-q) \left( \frac{d(d(3+(1+2p)(1-q)\delta) - 2(1-p(1-q)\delta))}{(1-(1-q)\delta)(1+(1-2p)(1-q)\delta)} \right) \end{aligned}$$

Consider the left hand side of the inequality above. It is increasing in  $\delta$  and decreasing in  $p$  and

$q$ . To see why, note that  $x^*$  is increasing in  $\delta$ . Because  $d < 1/4$  and  $0 < x^* < d$ ,  $1 - 2d - x^* > 0$  and decreasing in  $\delta$  for all  $\delta \in [0, 1]$  when the other parameters reside in the intervals specified above. Therefore  $-(1 - 2d - x^*)^2$  is increasing in  $\delta$  as well. A similar argument illustrates the expression's relationship with  $p$  and  $q$ . Now the right hand side of the equation is increasing in  $p$  and  $q$  and decreasing in  $\delta$ . Because of this, the right hand side is maximized and the left hand side is minimized at  $\delta = 1$ ,  $p = 1/2$  and  $q = 6/7$ . Substituting these into the expression gives us

$$d^2 - \left(1 - 3d + \frac{1}{2}\sqrt{\frac{13}{3}}\sqrt{d^2 - \frac{d}{7}}\right)^2 < \frac{19d^2 - 13d}{84}$$

Because the above inequality holds when  $d \in \left(\frac{1}{7}, \frac{1}{4}\right)$ , we obtain the desired result.

For the fifth claim, write

$$F_2^8(1-d+x^*) = -(1-2d+x^*)^2 + \delta \left( p(1-q)U_2(0, \neg R, 2; \hat{\sigma}) + (1-p)(1-q)U_2(1-x^*, \neg R, 3; \hat{\sigma}) + Z_2^M \right).$$

So we must show that

$$\begin{aligned} -d^2 + \delta(1-p)(1-q)U_2(1, \neg R, 3; \hat{\sigma}) &> -(1-2d+x^*)^2 + \delta(1-p)(1-q)U_2(1-x^*, \neg R, 3; \hat{\sigma}) \\ &\iff \\ d^2 &< (1-2d+x^*)^2 + \delta(1-p)(1-q)(U_2(1, \neg R, 3; \hat{\sigma}) - U_2(1-x^*, \neg R, 3; \hat{\sigma})) \end{aligned}$$

but this follows from the previous result because  $(1-2d+x^*)^2 > (1-2d-x^*)^2$  and  $U_2(1-x^*, \neg R, 3; \hat{\sigma}) < U_2(1-d, R, 3; \hat{\sigma})$ .

To prove the sixth and final claim, note that the first three parts of the claim establish  $\hat{x}_2$  as a local maximizer in the interval  $(0, 2d)$ . Continuity at 0 and part 2 imply there does not exist  $x' \leq 0$  such that  $U_2(x', \neg R, P_i; \hat{\sigma}) > U_2(d, \neg R, P_i; \hat{\sigma})$ . Continuity at  $2d$  and  $1-2d$  and part 2 imply there does not exist  $x' \in [2d, 1-d-x^*]$  such that  $U_2(x', \neg R, P_i; \hat{\sigma}) > U_2(d, \neg R, P_i; \hat{\sigma})$ . Parts 4 and 2 imply there does not exist  $x' \in I_7$  such that  $U_2(x', \neg R, P_i; \hat{\sigma}) > U_2(d, \neg R, P_i; \hat{\sigma})$ . Parts 5 and 2 and continuity at 1 imply there does not exist  $x' \in [1-d+x^*, \infty)$  such that  $U_2(x', \neg R, P_i; \hat{\sigma}) > U_2(d, \neg R, P_i; \hat{\sigma})$ .  $\square$

For reference, Figure 1 illustrates Claim 10 and graphs the shape of  $U_2(x, \neg R, 2; \hat{\sigma})$  for some parameters considered here.

**Claim 11** *Under the specified set of parameters, in the policy stage under majority dominance, leader 1 does not have a profitable deviation from  $\hat{\sigma}$ . That is, for all  $s^{pp} \in S^{pp}$  such that  $r = \neg R$ ,  $U^{pp}(\hat{\sigma}_1^{pp}(s^{pp}), s^{pp}; \hat{\sigma}) \geq U^{pp}(x', s^{pp}; \hat{\sigma})$  for all  $x' \in \mathbb{R}$ .*

*Proof.* Consider some state  $s^{pp} \in S^{pp}$  such that  $r = 0$ . If  $\mu = 1$ ,  $\hat{\sigma}^{pp}$  specifies that the leader to implement her unique ideal policy, as required. Now suppose  $\mu = 2$ . We now rule out profitable deviations in five cases.

Case 1: Suppose  $\bar{x} \in I_1$ . Then legislator 2 will accept 1's dynamic (and instantaneous)

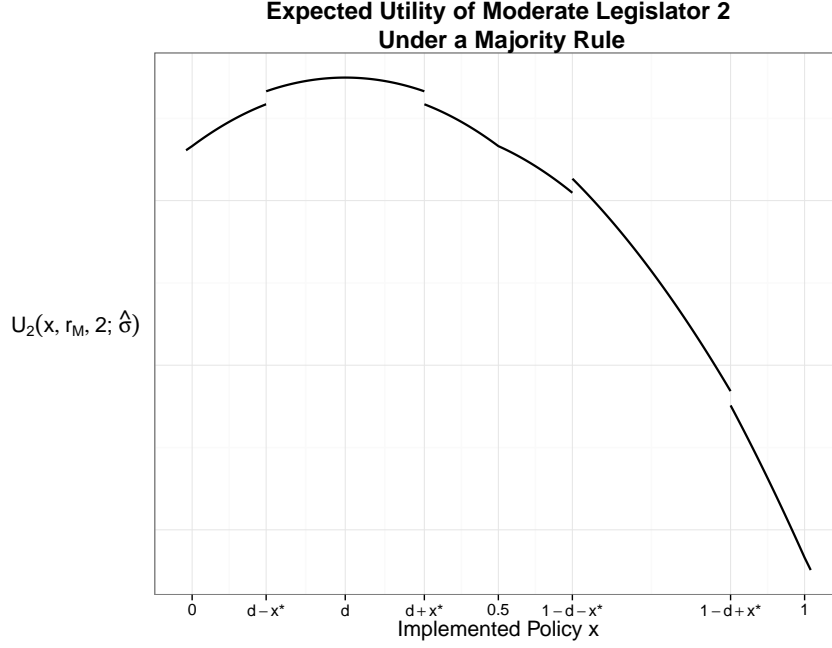


Figure 1: The expected utility of moderate legislator 2 is discontinuous at  $d - x^*$ ,  $d + x^*$ ,  $1 - d - x^*$  and  $1 - d + x^*$  in the equilibrium from Proposition 4. This occurs because, when the default policy resides in either  $(d - x^*, d + x^*)$  or  $(1 - d - x^*, 1 - d + x^*)$  the majority leader in the next period will enact the minority right.

ideal point of  $\hat{x}_1$  because  $U_2(x, 0, P_i; \hat{\sigma})$  is continuous and strictly increasing on the interval  $I_1$ . The policy  $\hat{x}_1$  is indeed 1's proposal under  $\hat{\sigma}$ .

Case 2: Suppose  $\bar{x} \in (0, 2d)$ . Legislator 2 will not accept any proposal  $x^p \notin (0, 2d)$  because  $U_2(\bar{x}, \neg R, 2; \hat{\sigma}) > U_2(0, \neg R, 2; \hat{\sigma}) > U_2(x^p, \neg R, 2; \hat{\sigma})$  by Claim 10. In addition,  $U_2(x, \neg R, 2; \hat{\sigma})$  is symmetric around  $d$  in the interval  $(0, 2d)$  and strictly increasing in  $(0, d)$ . Hence, a best response for leader 1 will be to propose  $d - |d - \bar{x}|$ , which is indeed her strategy specified in  $\hat{\sigma}$ .

Case 3: Suppose  $\bar{x} \in [2d, 1 - d - x^*]$ . By symmetry,  $U_2(0, \neg R, 2; \hat{\sigma}) = U_2(2d, \neg R, 2; \hat{\sigma})$ . Because  $U_2(x, \neg R, 2; \hat{\sigma})$  is continuous and strictly decreasing over  $[2d, 1 - d - x^*]$ , legislator 2 will accept 1's ideal point. This is 1's proposal specified by  $\hat{\sigma}$ .

Case 4: Suppose  $\bar{x} \in I_7$ . Legislator 2 will accept 1's ideal point because, by Claim 10,  $U_2(0, \neg R, 2; \hat{\sigma}) \geq F_2^7(1 - d - x^*)$  and  $U_2(x, \neg R, 2; \hat{\sigma})$  is strictly decreasing over  $I_7$ . This is 1's proposal specified by  $\hat{\sigma}$ .

Case 5: Suppose  $\bar{x} \in [1 - d + x^*, \infty)$ . Legislator 2 will accept 1's ideal point because, by Claim 10,  $U_2(0, \neg R, 2; \hat{\sigma}) \geq U_2(1 - d + x^*, \neg R, 2; \hat{\sigma})$  and  $U_2(0, \neg R, 2; \hat{\sigma})$  is strictly decreasing and continuous over  $[1 - d + x^*, \infty)$ . This is 1's proposal specified by  $\hat{\sigma}$ .  $\square$

**Claim 12** *Under the specified set of parameters, in the policy stage, legislator 2 does not have a profitable deviation from  $\hat{\sigma}$  for any state  $s \in S^{Mc}$  such that  $\mu = 2$  and  $r = \neg R$ .*

*Proof.* This follows from 1's stream of proposal offers as they leave the median better off than or indifferent to the default policy. Because of his expected utility function detailed in Claims 4

and 10, he will not benefit by rejecting  $i$ 's proposal in every period in which he is the median.  $\square$

**Claim 13** *Under the specified set of parameters, it is a best response for leader 1 to grant minority rights when she is the median, that is,  $U_1^{rp}(R, (\bar{x}, \bar{r}, 1); \hat{\sigma}) \geq U_1^{rp}(\neg R, (\bar{x}, \bar{r}, 1); \hat{\sigma})$ . for all  $\bar{r} \in \{R, \neg R\}$  and all  $\bar{x} \in \mathbb{R}$ .*

*Proof.* It suffices to show that  $U_1(0, R, 1; \hat{\sigma}) \geq U_1(0, \neg R, 1; \hat{\sigma})$  due to the construction of  $\hat{\sigma}$ . We can write

$$\begin{aligned}
 & U_1(0, R, 1; \hat{\sigma}) - U_1(0, \neg R, 1; \hat{\sigma}) \geq 0 \\
 & \iff \\
 & \delta \left( Z_1^M + p(1-q)U_1(d, R, 2; \hat{\sigma}) + (1-p)(1-q)U_1(1-d, R, 3; \hat{\sigma}) \right) - \\
 & \delta \left( Z_1^M + p(1-q)U_1(0, \neg R, 2; \hat{\sigma}) + (1-p)(1-q)U_1(1, \neg R, 3; \hat{\sigma}) \right) \geq 0 \\
 & \iff \\
 & \Delta_1 \equiv p(U_1(d, R, 2; \hat{\sigma}) - U_1(0, \neg R, 2; \hat{\sigma})) + (1-p)(U_1(1-d, R, 3; \hat{\sigma}) - U_1(1, \neg R, 3; \hat{\sigma})) \geq 0 \\
 & \iff \\
 & p(-d^2 + \delta(1-q)\Delta_1) + (1-p)(-(1-d)^2 + 1 + \delta(1-q)\Delta_1) \geq 0 \\
 & \iff \\
 & -pd^2 - (1-p)(1-d)^2 + (1-p) + \delta(1-q)\Delta_1 \geq 0 \\
 & \iff \\
 & \sum_{\tau=1}^{\infty} (-pd^2 - (1-p)(1-d)^2 + (1-p))\delta^{\tau-1}(1-q)^{\tau-1} \geq 0 \\
 & \iff \\
 & -pd^2 - (1-p)(1-d)^2 + (1-p) \geq 0,
 \end{aligned}$$

where the second to last biconditional follows because we can write  $\Delta_1 = -pd^2 - (1-p)(1-d)^2 + (1-p) + \delta(1-q)\Delta_1$ . The final inequality holds when  $d < 1/4$  and  $p < 7/8$ .  $\square$

**Claim 14** *Under the specified set of parameters, when the median is legislator 2 and the default rule is majority dominance, leader 1's best response is to implement the minority right if and only if the status quo resides in  $I_3$ . That is,  $U_1^{rp}(R, (\bar{x}, \neg R, 2); \hat{\sigma}) \geq U_1^{rp}(\neg R, (\bar{x}, \neg R, 2); \hat{\sigma})$  if and only if  $\bar{x} \in I_3$ .*

*Proof.* If leader 1 implements minority rights when legislator 2 is the median, her expected utility is  $U_1(d, R, 2; \hat{\sigma})$ . If legislator 2 does not implement minority rights, the policy outcome  $x$  will reside in  $[0, d]$ . Further,  $x > d - x^*$  if and only if  $\bar{x} \in I_3$ . Because  $U_1(x, \neg R, 2; \hat{\sigma})$  is continuous and strictly decreasing in  $x$  when  $x \in [0, d]$ , it suffices to verify that  $U_1(d, R, 2; \hat{\sigma}) = U_1(d - x^*, \neg R, 2; \hat{\sigma})$ . However, this follows from the proof in Claim 8.  $\square$