Lobbying, Inside and Out

Online Appendix

A Equilibrium definition

To define the Perfect Bayesian Equilibrium of the game, denote $U^P(\zeta^P, l_o^P; \tau, l_o^Q, b)$ the pro-change SIG P's utility as a function of the decision-maker's policy choice $b \in [0,1]$, the the outside lobbying strategy of the SIG supportive of the status quo Q, and its own type. Denote $U^Q(\zeta^Q, l_o^Q; \tau, l_o^P, b)$ Q's utility as a function of the decision-maker's policy choice b, its opponent's outside lobbying strategy, and its own type. Denote $U^D(b; l_o^Q, l_o^P)$ the decision-maker's utility as a function of the SIGs' outside lobbying strategies and its own choice of bill content.

A PBE in pure strategies consists of: 1) P's decision to engage in outside lobbying: $l_o^{P*}(b, l_o^Q; \tau) \in \{0,1\}$; 2) Q's decision to engage in outside lobbying: $l_o^{Q*}(b, \zeta^P; \tau) \in \{0,1\}$; 3) D's policy choice: $b^*(\zeta^P, \zeta^Q) \in [0,1]$, 4) the SIGs' signaling strategy: $\zeta^{J*}(\tau) \in \{H, L\} \times \mathbb{R}_+$ for all $\tau \in \{H, L\}$, $J \in \{P, Q\}$; 5) and beliefs $\mu^J(\zeta^J)$ that the resolve of $J \in \{P, Q\}$ is high, which together satisfy the following conditions:

- C1: $l_o^{P*}(b, l_o^Q; \tau) = 1$ if and only if (iff): $EU^P(\zeta^P, 1; \tau, l_o^Q, b) \geq EU^P(\zeta^P, 0; \tau, l_o^Q, b)$ for all $\zeta^P \in \{H, L\} \times \mathbb{R}_+$, $l_o^Q \in \{0,1\}$, $b \in [0,1]$ (where the expectation is over outcomes).

- C2: $l_o^{Q*}(b, \zeta^P; \tau) = 1$ iff: $E(U^Q(\zeta^Q, 1; \tau, l_o^{P*}(b, 1; \tau), b)|\zeta^P) \geq E(U^Q(\zeta^Q, 0; \tau, l_o^{P*}(b, 0; \tau), b)|\zeta^P)$ for all $\zeta^Q \in \{H, L\} \times \mathbb{R}_+$, $\tau \in \{H, L\}$, $\zeta^P \in \{H, L\} \times \mathbb{R}_+$, $b \in [0,1]$ (where the expectation is over outcomes and $l_o^{P*}(b, l_o^Q; \tau)$ conditional on ζ^P).

- C3: $b^*(\zeta^P, \zeta^Q) \in arg\max_{b \in [0,1]}$ $E(U^D(b; l_o^{Q*}, l_o^{P*})|\zeta^P, \zeta^Q)$ for all $\zeta^Q \in \{H, L\} \times \mathbb{R}_+$, $\zeta^P \in \{H, L\} \times \mathbb{R}_+$ (where the expectation is over outcomes, $l_o^{Q*}(b, \zeta^P; \tau)$, and $l_o^{P*}(b, l_o^Q; \tau)$ conditional on ζ^Q and ζ^P).

- C4: Signals satisfy $\zeta^Q(\tau) \in arg\max_{m \in \{H,L\},\ l_i^Q \geq 0}$ $EU^Q(\zeta^Q, l_o^{Q*}; \tau, l_o^{P*}, b^*)$ for Q and, for P, $\zeta^P(\tau) \in arg\max_{m \in \{H,L\},\ l_i^P \geq 0}$ $EU^P(\zeta^P, l_o^{P*}; \tau, l_o^{Q*}, b^*)$ (where the expectation is over outcomes, b^*, l_o^{-J*}, τ^{-J}, $-J$ denotes the opposite SIG).

- C5: Beliefs $\mu^Q(\zeta^Q)$ and $\mu^P(\zeta^P)$ satisfy Bayes' rule whenever possible.

B Proofs for the main analysis

Recall $\zeta^J(\tau) \in \{H, L\} \times \mathbb{R}_+$, is the signal of SIG $J \in \{P, Q\}$ as a function of its type $\tau \in \{H, L\}$. Throughout, I assume without loss of generality that when an SIG plays a separating strategy, it announces its type: $m(\tau) = \tau,\ \tau \in \{H, L\}$. The decision-maker's posterior that P's (resp., Q's) resolve is high following its signal is $\mu^P(\zeta^P)$ (resp., $\mu^Q(\zeta^Q)$). As I restrict attention to pure strategy in the main analysis, the posterior always satisfies $\mu^J(\zeta^J) \in \{0, \pi^J, 1\},\ J \in \{P, Q\}$. Denote $b(\zeta^P, \zeta^Q) \in [0, 1]$ the decision-maker's policy choice as a function of SIGs' signals. Denote $l_o^Q(b, \zeta^P; \tau) \in \{0, 1\}$ Q's outside lobbying strategy as a function of the decision-maker's proposal, pro-change SIG's signal, and its own type. Similarly, denote $l_o^P(b, l_o^Q; \tau) \in \{0, 1\}$ the pro-change SIG's outside lobbying strategy as a function of the decision-maker's proposal, Q's outside lobbying activities, and its own type. Starred strategies denote equilibrium strategies.

In the proofs, I focus on the SIGs' inside lobbying strategy with players playing their best response down the game tree. This implies in particular that the SIG supportive of the status quo chooses $l_o^{Q*}(b, \zeta^P; \tau) = 1$ if and only if $-(1 - E(p(l_o^P)|\zeta^P))\gamma_\tau^Q b - c > -\gamma_\tau^Q b$, $\forall b \in [0, 1]$ with $p(0) = \overline{p}$ and $p(1) = \underline{p}$. Finally, in the proof, I consider the cases when the decision-maker suffers a disutility loss of $k \geq 0$ when Q engages in outside lobbying activities. Hence, the decision-maker's utility function $u^D(y; l_o^Q) = y - l_o^Q \times k$. The baseline model is simply a special case for which $k = 0$.

B.1 SIG supportive of the status quo

I first prove the results regarding the influence of the SIG supportive of the status quo Q (with ζ^P the pro-change SIG's signal which reveals no information as $\pi^P = 0$). Since it has low resolve, the pro-change SIG never engages in outside activities and the probability the bill passes is $\overline{p}$ if $l_o^Q = 1$. As discussed in the main text, the decision-maker chooses either $b = \frac{c}{\overline{p}\gamma_H^Q} := b_H$ or $b = 1$ in this case.

Lemma B.1. *Q plays a separating strategy (i.e., $\zeta^Q(H) \neq \zeta^Q(L)$) on the equilibrium path only if $b(\zeta^P, \zeta^Q(H)) < b(\zeta^P, \zeta^Q(L))$.*

Proof. First, notice that by Assumptions 1-3, the decision-maker's best response after observing $\zeta^Q(L)$ is $b(\zeta^P, \zeta^Q(L)) = 1$. If $b(\zeta^P, \zeta^Q(H)) = b(\zeta^P, \zeta^Q(L)) = 1$, Q's signal has no effect on the decision-maker's policy choice. Using our equilibrium restriction, this cannot be an equilibrium. $\square$

Lemma B.2. *In a separating equilibrium, Q's strategy satisfies: $l_i^Q(L) = 0$ and $l_i^Q(H) > 0$.*

Proof. By Lemma B.1 (i.e., $b(\zeta^P, \zeta^Q(H)) < 1$), $Q(L)$'s incentive compatibility constraint (IC) is satisfied only if $l_i^Q(H) > l_i^Q(L)$. $l_i^Q(L) = 0$ follows by the Intuitive Criterion. □

Lemma B.3. *A separating equilibrium exists if and only if:*

$$1 - \overline{p} - k \leq \frac{c}{\overline{p}\gamma_H^Q} \leq (1 - \overline{p})\frac{c}{\overline{p}\gamma_L^Q}$$

In a separating equilibrium, the decision-maker chooses $b = b_H$ after signal $\zeta^Q(H)$ and $b = 1$ after signal $\zeta^Q(L)$.

Proof. Necessity. Suppose $\zeta^Q(H) = (H, l_i^Q(H)) \neq \zeta^Q(L) = (L, l_i^Q(L))$. When $1 - \overline{p} - k > b_H$, the decision-maker strictly prefers 1 to b_H. Her best response is then $b^*(\zeta^P, \zeta^Q(H)) = 1 = b^*(\zeta^P, \zeta^Q(L))$. By Lemma B.1, a separating equilibrium cannot exist then.

Assume $b_H \geq 1 - \overline{p} - k$ so $b(\zeta^P, \zeta^Q(H)) = b_H$. A type L's (IC) is: $-\gamma_L^Q \geq -\gamma_L^Q b_H - l_i^Q(H)$. By the Intuitive Criterion, $l_i^Q(H) = \gamma_L^Q(1 - b_H) =: \overline{l_i^Q}(b_H)$. A type H's (IC) is (using the reasoning in the text): $-\gamma_H^Q b_H - l_i^Q(H) \geq -(1 - \overline{p})\gamma_H^Q - c$. Plugging in $\overline{l_i^Q}(b_H)$ and using $-\gamma_H^Q b_H = -(1 - \overline{p})\gamma_H^Q b_H - c$, simple algebra yields that a necessary condition is $\gamma_L^Q \leq (1 - \overline{p})\gamma_H^Q$, or $\frac{c}{\overline{p}\gamma_H^Q} \leq (1 - \overline{p})\frac{c}{\overline{p}\gamma_L^Q}$ as claimed.

Sufficiency. Suppose $1 - \overline{p} - k \leq \frac{c}{\overline{p}\gamma_H^Q} \leq (1 - \overline{p})\frac{c}{\overline{p}\gamma_L^Q}$, and consider the following assessment: i) $Q(L)$'s ($Q(H)$'s) signal is $\zeta^Q(L) = (L, 0)$ ($\zeta^Q(H) = (H, \overline{l_i^Q}(b_H))$);[1] ii) The decision-maker's posterior is: $\mu^Q(\zeta^Q) = 0$ if $\zeta^Q = (m, l_i^Q)$ satisfies $l_i^Q < l_i^Q(H)$ and 1 otherwise for all m; iii) the decision-maker's policy choice is: $b(\zeta^P, \zeta^Q) = 1$ if $\zeta^Q = (m, l_i^Q)$ satisfies $l_i^Q < \overline{l_i^Q}(b_H)$ and $b(\zeta^P, \zeta^Q) = b_H$ otherwise; (iv) all players play their best response down the game tree (see above). It can be checked that beliefs satisfy Bayes' rule, the decision-maker's policy choice is a best response given her belief, and Q's (IC) holds for both types. Hence, the assessment described above is an equilibrium. □

Proof of Lemma 2

Follow directly from the proof of Lemma B.3 after imposing $k = 0$. □

Proof of Proposition 1

Follows directly from the proof of Lemma B.3 (strategies are independent of k). □

For the next lemma, it is useful to denote $\overline{\pi^Q}(b_H; k) := \frac{1 - b_H}{\overline{p} + k}$. Notice that this function takes value higher than one whenever $b_H \leq 1 - \overline{p} - k$.

[1]Recall that I use the notation $J(\tau)$ to designate a type-τ ($\tau \in \{H, L\}$ SIG $J \in \{P, Q\}$.

Lemma B.4. *A pooling equilibrium exists if and only if* $b_H \notin \left[1-\overline{p}-k, (1-\overline{p})\frac{c}{\overline{p}\gamma_L^Q}\right)$ *or* $\pi^Q > \overline{\pi^Q}(b_H;k)$. *In a pooling equilibrium, the decision-maker's equilibrium policy choice—*$b^*(\zeta^P,\zeta^Q)$*—satisfies:*

1. $b^*(\zeta^P,\zeta^Q) = b_H$ *if* $\pi^Q \geq \overline{\pi^Q}(b_H;k)$*;*
2. $b^*(\zeta^P,\zeta^Q) = 1$ *otherwise.*

Proof. I just prove necessity (sufficiency follows from a similar reasoning as above). I first characterize the decision-maker's policy choice in a pooling assessment. Denote Q's signal $\zeta^Q(\tau) := \zeta^Q = (m, l_i^Q)$ for $\tau \in \{H,L\}$, some $m \in \{H,L\}$ and $l_i^Q \geq 0$ (to be determined). By Bayes' rule, $\mu^Q(\zeta^Q) = \pi^Q$. When the decision-maker chooses $b = 1$, her expected utility is $\pi^Q(1-\overline{p}-k)+(1-\pi^Q)$; with $b = b_H$, her expected utility is b_H. Simple algebra yields $b(\zeta^P,\zeta^Q) = 1$ if and only if $\pi^Q \leq \overline{\pi^Q}(b_H,k)$. Note that this condition is always satisfied when $b_H \leq 1-\overline{p}-k$ (since $\pi^Q \in (0,1)$).

I now show that a pooling equilibrium does not exist when $1-\overline{p}-k < b_H < (1-p)\frac{c}{\overline{p}\gamma_L^Q}$ and $\pi^Q < \overline{\pi^Q}(b_H;k)$. By way of contradiction, suppose it does. The equilibrium policy choice then satisfies $b^*(\zeta^P,\zeta^Q) = 1$. For simplicity (though it is no essential), suppose that there is no inside lobbying expenditures then. Consider now the out-of-equilibrium signal $\widehat{\zeta^Q} = (H, \overline{l_i^Q}(b_H)+\epsilon)$, with $\epsilon > 0$ appropriately chosen. By Lemma B.3, $Q(L)$ prefers the equilibrium payoff to sending signal $\widehat{\zeta^Q}$ even if $b(\zeta^P,\widehat{\zeta^Q}) = b_H$. Hence, by the Intuitive Criterion, the decision-maker's out-of-equilibrium belief satisfies: $\mu^Q(\widehat{\zeta^Q}) = 1$ so $b(\zeta^P,\widehat{\zeta^Q}) = b_H$. It can be checked that for ϵ small enough, a type H then prefers the 'inside lobbying strategy' $\widehat{\zeta^Q}$ to the equilibrium strategy. Hence, the equilibrium does not survive the Intuitive Criterion, a contradiction. A pooling equilibrium exists for all other parameter values as the decision-maker always prefers $b = 1$ to b_H (if $b_H \leq 1-\overline{p}-k$), chooses $b = b_H$ absent additional information ($\pi^Q \geq \overline{\pi^Q}(b_H;k)$), or there is no credible signal for $Q(H)$ to reveal its resolve ($b_H \geq (1-\overline{p})\frac{c}{\gamma_L^Q\overline{p}}$). ☐

Proof of Lemma 3

Again, it follows directly from the proof of Lemma B.4 after imposing $k = 0$ and noting that $\overline{\pi^Q}(b_H) = \overline{\pi^Q}(b_H;0)$. ☐

The statement of Proposition 2 does not depend on k so the proposition is proved directly and the proof extends to the case when $k > 0$.

Proof of Proposition 2

Point 1. Consider the following belief structure: $\mu^Q(\zeta^Q) = 0$ when $\zeta^Q = (m, l_i^Q)$ for $m \in \{H, L\}$ and $l_i^Q \in [0, \tilde{l_i^Q})$ with $\tilde{l_i^Q} > 0$, and $\mu^Q(\zeta^Q) = \pi^Q$, otherwise. Given this belief structure, the decision-maker's best response is: $b(\zeta^P, \zeta^Q) = 1,\ \forall \zeta^Q \in \{H, L\} \times [0, \tilde{l_i^Q})$ and $b(\zeta^P, \zeta^Q) = b_H,\ \forall \zeta^Q \in \{H, L\} \times [\tilde{l_i^Q}, \infty)$. A type H's (IC) is: $-\gamma_H^Q b_H - \tilde{l_i^Q} \geq -(1 - \overline{p})\gamma_H^Q - c$. A type L's (IC) is: $-\gamma_L^Q b_H - c\tilde{l_i^Q} \geq -\gamma_L^Q$. Both (IC) are satisfied whenever $\tilde{l_i^Q} \leq \overline{l_i^Q}(b_H)$. So any signaling strategy satisfying $\zeta^Q = (m, l_i^Q)$ with $l_i^Q \leq \overline{l_i^Q}(b_H)$ can be part of a pooling equilibrium. Given $b^*(\zeta^P, \zeta^Q) = b_H$, outside lobbying activities satisfy $l_o^{Q*}(b_H, \zeta^P; \tau) = 0,\ \tau \in \{H, L\}$.

Point 2. Since $b^*(\zeta^P, \zeta^Q) = 1$ (i.e., $\pi^Q \leq \overline{\pi^Q}(b_H)$), $l_i^{Q*} = 0$ (as the decision-maker already chooses the SIG's least preferred policy). Obviously, $l_o^{Q*}(1, \zeta^P; H) = 1$. □

B.2 Pro-change SIG

I now prove the results regarding the pro-change SIG influence (with ζ^Q denoting Q's signal, which contains no information given the degenerate prior). Recall that I assume $\pi^Q = 1$ so Q is known to have high resolve.

Proof of Lemma 4

Follows from a similar reasoning as in the proof of Lemma 2. $\square$

When $k \geq 0$, the inequality stated in Lemma 4 simply becomes $1 - \overline{p} - k \leq b_H$. I assume that this inequality holds in what follows.

Lemma B.5. *The pro-change SIG plays a separating strategy on the equilibrium path (i.e. $\zeta^P(H) \neq \zeta^P(L)$) only if: $b(\zeta^P(H), \zeta^Q) = 1$ and $b(\zeta^P(L), \zeta^Q) = b_H$.*

Proof. First, it should be noted that the decision-maker's policy choice when she learns P's type is either $b = 1$ or $b = b_H$. Indeed, $P(H)$ engages in outside lobbying only if the bill b satisfies $\gamma_H^P(1 - \underline{p})b - c \geq \gamma_H^P(1 - \overline{p})b$, or equivalently $b \geq \frac{c}{\gamma_H^P(\overline{p} - \underline{p})}$. Under Assumption 3, $\frac{c}{\gamma_H^P(\overline{p}-\underline{p})} > \frac{c}{\underline{p}\gamma_H^Q}$, this means that for any $b \geq \frac{c}{\gamma_H^P(\overline{p}-\underline{p})}$ which yields $l_o^P(b, 1; H) = 1$, anticipating P's strategy, Q chooses to engage in outside lobbying even if it knows P is of high resolve: $l_o^Q(b, \zeta^P(H); H) = 1$ (indeed anticipating $l_o^P(b, 1; H) = 1$, Q engages in outside lobbying whenever $-\gamma_H^Q(1 - \underline{p})b - c \geq -\gamma_H^Q b \Leftrightarrow b \geq \frac{c}{\underline{p}\gamma_H^Q}$). The decision-maker's expected payoff is then $(1 - \underline{p})b - k$ for any bill satisfying $b \geq \frac{c}{\gamma_H^P(\overline{p}-\underline{p})}$. She then obviously prefers $b = 1$ in this set.

The rest of proof proceeds by contradiction. Suppose $b^*(\zeta^P(H), \zeta^Q) = 1 = b^*(\zeta^P(L), \zeta^Q)$ or $b^*(\zeta^P(H), \zeta^Q) = b_H = b^*(\zeta^P(L), \zeta^Q)$ then given the equilibrium restriction a separating equilibrium does not exist. Suppose then that $b(\zeta^P(H), \zeta^Q) = b_H$ and $b(\zeta^P(L), \zeta^Q) = 1$. By Assumptions 1 and 2, only $P(H)$ ever engages in outside lobbying activities. This implies that for the above strategy to be the decision-maker's best response, it must be that $1 - \overline{p} - k \geq b_H$ (so $b(\zeta^P(L), \zeta^Q) = 1$) and $1 - \underline{p} - k \leq b_H$ (so $b(\zeta^P(H), \zeta^Q) = b_H$). The first inequality is not satisfied by assumption and, further, given $\overline{p} > \underline{p}$, both inequalities cannot be satisfied simultaneously. $\square$

A direct consequence of Lemma B.5 is that whenever the pro-change SIG plays a separating strategy, a type H's on-path equilibrium outside lobbying strategy satisfies $l_o^{P*}(b^*, 1; H) = 1$ since $b^*(\zeta^P(H), \zeta^Q) = 1$ so $l_o^{Q*}(\cdot) = 1$.

Lemma B.6. *If $b_H \geq 1 - \overline{p}$, in a separating equilibrium, the pro-change SIG's strategy satisfies $l_i^P(H) = 0$.*

Proof. The proof is by contradiction. Suppose $\zeta^P(H) = (H, l_i^P(H))$ with $l_i^P(H) > 0$. By the Intuitive Criterion, this implies $l_i^P(L) = 0$. A type L pro-change SIG's incentive compatibility constraint (IC) is then

$$\gamma_L^P b_H \geq (1 - \overline{p})\gamma_L^P - l_i^P(H)$$

For a type H, the (IC) is

$$\gamma_H^P b_H \leq (1 - \underline{p})\gamma_H^P - c - l_i^P(H)$$

Under the assumption, a type L pro-change SIG has no incentive to send a signal ζ^P satisfying $l_i^P > 0$. Applying the Intuitive Criterion, there then exists a profitable deviation to $\widehat{l_i^P} \in (0, l_i^P(H))$ (i.e., $\mu^P((m, \widehat{l_i^P})) = 1$ so $b^*((m, \widehat{l_i^P}), \zeta^Q) = 1$), leading to a contradiction. □

Lemma B.7. *There exists a unique $\underline{\gamma} : \mathbb{R}_+^2 \to \mathbb{R}_+$ such that a separating equilibrium exists if and only if (i) the compromise bill b_H satisfies $b_H \leq 1 - \underline{p} - k$; and (ii) the resolves of the pro-change SIG satisfy: $\gamma_H^P \geq \max\left\{\underline{\gamma}(b_H, \gamma_L^P), \frac{c}{\underline{p} - \overline{p}}\right\}$.*
In a separating equilibrium, the decision-maker chooses $b = 1$ after signal $\zeta^{P*}(H)$ and $b = b_H$ after signal $\zeta^{P*}(L)$.

Proof. Necessity. Suppose $\zeta^P(H) \neq \zeta^P(L)$. When $b_H > 1 - \underline{p} - k$, the decision-maker's best response is: $b(\zeta^P, \zeta^Q) = b_H$ for all ζ^P. By Lemma B.5, a separating equilibrium does not exist. Suppose $b_H \leq 1 - \underline{p} - k$ so the decision-maker's best response satisfies $b(\zeta^P(H), \zeta^Q) = 1$ and $b(\zeta^P(L), \zeta^Q) = b_H$. We need to consider two cases: (i) $b_H \geq 1 - \overline{p}$ and (ii) $b_H < 1 - \overline{p}$.
In the first case, using Lemma B.6, a type H pro-change SIG's (IC) is: $(1 - \underline{p})\gamma_H^P - c \geq b_H \gamma_H^P - l_i^P(L)$. A type L's (IC) is: $b_H \gamma_L^P - l_i^P(L) \geq (1 - \overline{p})\gamma_L^P$. By the Intuitive Criterion, $l_i^P(L) = \max\left\{(b_H - (1 - \underline{p}))\gamma_H^P + c, 0\right\}$. Therefore, both (IC) are automatically satisfied whenever $\frac{c}{\gamma_H^P} \leq (1 - \underline{p}) - b_H$ (so $l_i^P(L) = 0$). When $\frac{c}{\gamma_H^P} > (1 - \underline{p}) - b_H$, a type L's (IC) is satisfied if and only if $\gamma_H^P \geq \underline{\gamma}(b_H, \gamma_L^P)$, with $\underline{\gamma}(b_H, \gamma_L^P) := \frac{c - \gamma_L^P(b_H - (1 - \overline{p}))}{1 - \underline{p} - b_H}$.
In the second case, $P(L)$ would never truthfully reveal its type if $l_i^P(L) > 0$. However, there now exists a possible equilibrium with $l_i^P(H) > 0$, which, using the (IC) in the proof of Lemma B.6 and the Intuitive Criterion, equals: $l_i^P(H) = \gamma_L^P((1 - \overline{p}) - b_H)$. The (IC) of a type H is now satisfied only if $\gamma_H^P((1 - \underline{p}) - b_H) - c \geq \gamma_L^Q((1 - \overline{p}) - b_H)$, or $\gamma_H^P \geq \frac{c - \gamma_L^P(b_H - (1 - \overline{p}))}{1 - \underline{p} - b_H}$.
To complete the proof, we show that $\underline{\gamma}(\cdot)$ is binding if and only if $b_H > 1 - \overline{p}$, i.e., $\underline{\gamma}(b_H, \gamma_L^P) >$

$\frac{c}{\overline{p}-\underline{p}} \Leftrightarrow b_H > 1-\overline{p}$. Notice that the function $G(b_H) = \frac{c-\gamma_L^P(b_H-(1-\overline{p}))}{1-\underline{p}-b_H}$ is increasing with b_H. Indeed, $G'(b_H)$ as the same sign as $\gamma_L^P(1-\overline{p}) - \left(\gamma_L^P(1-\underline{p}) - c\right)$, which is strictly positive under Assumption 2.(ii). Further, the function G satisfies $G(1-\overline{p}) = \frac{c}{\overline{p}-\underline{p}}$. Note that this also implies that a separating equilibrium always exists when $1-\overline{p}-k \leq b_H \leq 1-\overline{p}$.

Sufficiency. Follows from a similar reasoning as in the proof of Lemma B.3. □

Proof of Lemma 5

The proof follows from Lemma B.7 making note of two observations. First, the necessary condition becomes $b_H \leq 1-\underline{p}$ after imposing $k=0$. Second, when $k=0$, we have $b_H \geq 1-\overline{p}$ by assumption. Hence, using the proof of Lemma B.7, the condition $\gamma_H^P \geq \underline{\gamma}(b_H, \gamma_L^P)$ is binding. □

Proof of Proposition 3

Direct from the proof of Lemma B.7 after noticing that when $k=0$ only case (i) in that proof applies. □

The proof of Remark 1 is immediate when $b_H < 1-\overline{p}$ (a case that can only occurs if $k>0$). When $b_H \geq 1-\overline{p}$, it follows from noticing that $l_i^P(L) = \max\{c-\gamma_H^P(1-\underline{p}-b_H), 0\} \leq c$ by Lemma 5, with equality only in the knife-edge case when $b_H = 1-\underline{p}$ (note that this can never happen when $k>0$ since a necessary condition for a separating equilibrium to exist is then: $1-\underline{p}-k \geq b_H$).

Before stating a helpful preliminary Lemma, recall that in a pooling equilibrium, if it exists, signals satisfy $\zeta^{P*}(H) = \zeta^{P*}(L) = \zeta^{P*}$. For the next result, denote $\widehat{l_i^P}(b_H) = \gamma_L^P(b_H-(1-\overline{p}))$ and recall that $\overline{l_i^P}(b_H) = \gamma_H^P(b_H-(1-\underline{p}))+c$.

Lemma B.8. *Denote* $\overline{\pi^P}(b_H;k) = \frac{b_H-(1-\overline{p}-k)}{\overline{p}-\underline{p}}$.

1) If $\pi^P \leq \overline{\pi^P}(b_H;k)$*, then a pooling equilibrium with exists if and only if* $b_H \geq \min\left\{1-\underline{p}-\frac{c}{\gamma_H^P}, 1-\underline{p}-k\right\}$.

The policy-maker chooses $b^*(\zeta^{P*}, \zeta^Q) = b_H$.

The pro-change SIG's equilibrium strategy satisfies on the equilibrium path:

(i) $\zeta^{P*} = (m^*, l_i^{P*})$*, with* $m^* \in \{H, L\}$ *and* $l_i^{P*} = 0$ *if* $b_H > 1-\underline{p}-k$ *and* $l_i^{P*} \in \left[0, \min\left\{\overline{l_i^P}(c_L^P, b_H), \widehat{l_i^P}(b_H)\right\}\right]$ *otherwise;*

(ii) $l_o^{P*}(\tau) = 0$ *for* $\tau \in \{H, L\}$.

2) If $\pi^P \geq \overline{\pi^P}(b_H;k)$, then a pooling equilibrium with exists if and only if either (a) $b_H \geq 1-\underline{p}-\frac{c}{\gamma_H^P}$ and $\gamma_H^P < \underline{\gamma}(b_H,\gamma_L^P)$ or (b) $b_H \leq 1-\overline{p}$.

The decision-maker chooses $b^*(\zeta^{P*},\zeta^Q)=1$.

The pro-change SIG's equilibrium strategy satisfies on the equilibrium path:

(i) $\zeta^{P*}=(m^*,l_i^{P*})$, with $m^* \in \{H,L\}$ and $l_i^{P*}=0$ if $b_H \geq 1-\overline{p}-\frac{c}{\gamma_H^P}$ and $l_i^{P*} \in \big[0,\min\big\{-\overline{l_i^P}(c_L^P,b_H),-\widehat{l_i^P}(b_H)\big\}\big]$ if $b_H \leq 1-\overline{p}$;

(ii) $l_o^{P*}(H)=1$ and $l_o^{P*}(L)=0$.

Proof. Absent additional information about P's type, the decision-maker chooses between $b=b_H$ and $b=1$. By Assumption 3 and following a similar reasoning as in the proof of Lemma B.5, there is no bill b such that $l_o^P(b,1;H)=1$ and $l_o^Q(b,\zeta^{P*};H)=0$ (i.e., for all bills such that $P(H)$ engages in outside lobbying, Q strictly prefers $l_o^Q=1$ to $l_o^Q=0$).

Suppose a pooling equilibrium exists, the decision-maker's policy choice then satisfies $b^*(\zeta^{P*},\zeta^Q)=b_H$ if and only if $\pi^P(1-\underline{p})+(1-\pi^P)(1-\overline{p})-k \leq b_H$. After rearranging, this is equivalent to $b^*(\zeta^{P*},\zeta^Q)=b_H$ if and only if $\pi^P \leq \overline{\pi^P}(b_H;k)$ and $b^*(\zeta^{P*},\zeta^Q)=1$ if and only if $\pi^P \geq \overline{\pi^P}(b_H;k)$ with $\overline{\pi^P}(b_H;k)=\frac{b_H-(1-\overline{p}-k)}{\overline{p}-\underline{p}}$ as claimed. (Note that $\overline{\pi^P}(b_H;k)\geq 1$ when $b_H \geq 1-\underline{p}-k$ so $b^*(\zeta^{P*},\zeta^Q)=b_H$ then and $\overline{\pi^P}(b_H;k)\leq 0$ when $b_H \leq 1-\overline{p}-k$ so we would have $b^*(\zeta^{P*},\zeta^Q)=1$ then).

Point 1) In this case, $b^*=b_H$. This directly implies that if a pooling equilibrium exists, then $l_o^{P*}(\tau)=0$, $\tau\in\{H,L\}$. I now discuss conditions for existence as well as the inside lobbying strategies when a pooling equilibrium exists. To do so, we need to consider different cases.

Suppose first $k \geq \frac{c}{\gamma_H^P}$ so $\min\Big\{1-\underline{p}-\frac{c}{\gamma_H^P},1-\underline{p}-k\Big\}=1-\underline{p}-k$. If $b_H \geq 1-\underline{p}-k$, the decision-maker chooses $b=b_H$ for all beliefs about the pro-change SIG's type. It can easily be checked that the unique equilibrium is pooling then with (i) $b^*(\zeta^{P*},\zeta^Q)=b_H$ and (ii) there is no inside lobbying expenditures on path. If $b_H<1-\underline{p}-k<1-\underline{p}-\frac{c}{\gamma_H^P}$, I claim there always exists a signal such that $P(H)$ can credibly reveal its type and a pooling equilibrium does not exist. The next paragraph carries on proving the claim.

Suppose now, and until the end of the proof of point 1), that $k<\frac{c}{\gamma_H^P}$ so $\min\Big\{1-\underline{p}-\frac{c}{\gamma_H^P},1-\underline{p}-k\Big\}=1-\underline{p}-\frac{c}{\gamma_H^P}$. Suppose first that $b_H<1-\underline{p}-\frac{c}{\gamma_H^P}$. Note that this implies that $P(H)$ prefers $b=1$ to $b=b_H$. I show that a pooling equilibrium does not exist then. To do so, I need to consider two subcases. First subcase, if $b_H>1-\overline{p}$, then a low-type SIG has never any incentive to pretend to be a high-type. Hence, after any cheap talk message $m\neq m^*$, the policy-maker's posterior must be that the pro-change SIG has high resolve with probability one. $P(H)$ strictly prefers $b=1$ to $b=b_H$ and so has an incentive to deviate by sending message $m\neq m^*$. Hence, a pooling equilibrium cannot exist then. Second subcase: $b_H \leq 1-\overline{p}$. Then we are in case (ii) detailed in the proof of Lemma B.7. Using this proof, for any inside lobbying expenditures satisfying $l_i^P > \gamma_L^P(1-\overline{p}-b_H)$, then the posterior of the decision-maker must satisfy that that the pro-change SIG has high resolve with probability one. Using the proof of Lemma B.7, we can see that $P(H)$ has a profitable deviation (for appropriate level of inside lobbying expenditures above $\gamma_L^P(1-\overline{p}-b_H)$) and a pooling equilibrium does not exist then. Combining both cases, I obtain that $b_H \geq 1-\underline{p}-\frac{c}{\gamma_H^P}$ is necessary for existence. Suppose now that $b_H \geq 1-\underline{p}-\frac{c}{\gamma_H^P}$ (which implies $b_H > 1-\overline{p}$). Both types obtain their preferred policy and it is quite obvious they have no incentive to deviate. Further, inside lobbying expenditures can appear on path when $b_H \leq 1-\underline{p}-k$. To see that, consider the following belief structure for the decision-maker. The decision-maker's posterior satisfies $\mu^P(\zeta^P)=1$, when $\zeta^P=(m,l_i^P)$ for $m\in\{H,L\}$ and $l_i^P \in [0,\tilde{l_i^P})$, with $\tilde{l_i^P}>0$, and $\mu^P(\zeta^P)=\pi^P$, otherwise. Given this belief structure and assumption, the decision-maker's best response is: $b(\zeta^P,\zeta^Q)=1,\ \forall\zeta^P\in\{H,L\}\times[0,\tilde{l_i^P})$ and $b(\zeta^P,\zeta^Q)=b_H,\ \forall\zeta^P\in\{H,L\}\times[\tilde{l_i^P},\infty)$. A type L's (IC) is: $\gamma_L^P b_H-\tilde{l_i^P}\geq\gamma_L^P(1-\overline{p})$. A type H's (IC) is: $\gamma_H^P b_H-\tilde{l_i^P}\geq\gamma_H^P(1-\underline{p})-c$. Both (IC) are satisfied whenever $\tilde{l_i^P}\leq\min\left\{\overline{l_i^P}(b_H),\widehat{l_i^P}(b_H)\right\}$. Consequently, when $b_H\geq 1-\underline{p}-\frac{c}{\gamma_H^P}$, any signaling strategy satisfying $\zeta^P(L)=\zeta^P(H)=(m,l_i^P)$, with $l_i^P\leq\min\left\{\overline{l_i^P}(b_H),\widehat{l_i^P}(b_H)\right\}$ can be part of a pooling equilibrium. As an aside, note that $\min\left\{\overline{l_i^P}(b_H),\widehat{l_i^P}(b_H)\right\}=\widehat{l_i^P}(b_H)$ whenever $\gamma_H^P\leq\frac{c-\gamma_L^P(b_H-(1-\overline{p}))}{(1-\underline{p})-b_H}$ or $\gamma_H^P\leq\underline{\gamma}(b_H,\gamma_L^P)$.

Point 2) In this case, $b^*=1$. Note that this directly implies that, if the pooling equilibrium exists, on path $l_o^P(H)=1$ and $l_o^P(L)=0$. I now discuss conditions for existence as well as the inside lobbying strategies when a pooling equilibrium exists. To do so we need to consider different cases.

Suppose first that $b_H\in(1-\overline{p},1-\underline{p}-\frac{c}{\gamma_H^P})$. Then, $P(L)$ prefers $b=b_H$ to $b=1$ and $P(H)$ prefers $b=1$ to $b=b_H$. Following the same step as in point 1), there exists a cheap talk message such that the decision-maker's posterior is that the SIG has *low* resolve with probability one after say message. This cheap talk message constitutes a profitable deviation for $P(L)$, hence a pooling equilibrium does not exist then.

If $b_H\leq 1-\overline{p}$, both types prefer $b=1$ to $b=b_H$. It is quite direct that a pooling equilibrium exists then. I further show that we can observer inside lobbying expenditures on path. Consider the following belief structure for the decision-maker. The decision-maker's posterior satisfies $\mu^P(\zeta^P)=0$, when $\zeta^P=(m,l_i^P)$ for $m\in\{H,L\}$ and $l_i^P\in[0,\tilde{l_i^P})$, with $\tilde{l_i^P}>0$, and $\mu^P(\zeta^P)=\pi^P$, otherwise. Given this belief structure, the decision-maker's best response is: $b(\zeta^P,\zeta^Q)=b_H,\ \forall\zeta^P\in\{H,L\}\times[0,\tilde{l_i^P})$ and $b(\zeta^P,\zeta^Q)=1,\ \forall\zeta^P\in\{H,L\}\times[\tilde{l_i^P},\infty)$. A type L's (IC) is: $\gamma_L^P b_H\leq\gamma_L^P(1-\overline{p})-\tilde{l_i^P}$. A type H's (IC) is: $\gamma_H^P b_H\leq\gamma_H^P(1-\underline{p})-c-\tilde{l_i^P}$. Both (IC) are satisfied whenever $\tilde{l_i^P}\leq\min\left\{-\overline{l_o^P}(b_H),-\widehat{l_o^P}(b_H)\right\}$.

Consequently, when $b_H \geq 1 - \overline{p}$, any signaling strategy satisfying $\zeta^P(L) = \zeta^P(H) = (m, l_i^P)$, with $l_i^P \leq \min\left\{-\overline{l_o^P}(b_H), -\widehat{l_o^P}(b_H)\right\}$ can be part of a pooling equilibrium.

Lastly, suppose that $b_H \geq 1 - \underline{p} - \frac{c}{\gamma_H^P}$. Then both types prefer $b = b_H$ to $b = 1$. However, the decision-maker would change her policy only if she learns that she faces a low type. Using the proof of Lemma B.7, a credible signal exists whenever $\gamma_H^P \geq \underline{\gamma}(b_H, \gamma_L^P)$. By the Intuitive Criterion, a pooling equilibrium does not exist then. So existence requires $\gamma_H^P < \underline{\gamma}(b_H, \gamma_L^P)$ in this case. If this condition is satisfied, it is immediate that inside lobbying expenditures equal zero on path (since both types would rather obtain policy b_H than pay any cost to get policy $b = 1$). $\square$

Proof of Lemma 6

Recall that $\overline{\pi^P}(b_H) = \overline{\pi^P}(b_H, 0)$. In the case when $k = 0$, we necessarily have $b_H > 1 - \overline{p}$ and $1 - \underline{p} - \frac{c}{\gamma_H^P} < 1 - \underline{p}$. Hence, the conditions for existence described in Lemma B.8 reduce to (i) $b_H \geq 1 - \underline{p} - \frac{c}{\gamma_H^P}$ (see points 1) and 2)) and (ii) $\pi^P \leq \overline{\pi^P}(b_H)$ (existence is guaranteed then by point 1)) or when $\pi^P > \overline{\pi^P}(b_H)$, $\gamma_H^P < \underline{\gamma}(b_H, \gamma_L^P)$. These are the conditions stated in the text of the lemma. The decision-maker's strategy follows directly from Lemma B.8. $\square$

Proof of Proposition 4

Follows from Lemma B.8. Notice that $b_H > 1 - \overline{p}$ by assumption. So there is no inside lobbying expenditures on path when $b^* = 1$ (ignoring arguments). $\square$

C Allowing for mixed strategies

In this section, I allow for mixed strategies. I start with the pro-change SIG under the conditions of the "Pro-change SIG influence" section and then consider the SIG supportive of the status quo under the conditions detailed in the section titled "Influence of SIGs supportive of the status quo." For simplicity, I only consider the case when one type plays a non-degenerate mixed strategy. Focusing on this type of 'semi-separating assessment' is without significant loss of generality as, a.e., the indifference condition cannot be satisfied for two types simultaneously. To limit the number of cases considered, I also restrict attention to the case of the baseline analysis with Assumptions 1-3 holding true and $k = 0$.

Pro-change SIG

When it comes to the pro-change SIG, the conditions for existence of a semi-separating equilibrium is more restrictive (in the sense of set inclusion) than for a separating equilibrium. For the same reason as in the main text, a semi-separating equilibrium does not exist when $b_H < 1 - \overline{p}$. A semi-separating equilibrium cannot exist when a type H's resolve is relatively high since this type strictly prefers to reveal its type then (therefore, we can never satisfy the indifference condition). As in the main text, a semi-separating equilibrium does not exist when resolve satisfies: $\gamma_H^P < \underline{\gamma}(b_H, \gamma_L^P)$. Allowing for mixed strategies thus does not improve information transmission because the set of decision-maker's undominated strategy at the policy stage is $\{b_H, 1\}$. Any other policy reduces the decision-maker's policy payoff conditional on being passed. Since the set of possible policy choice on path remains unchanged when the pro-change SIG plays a mixed strategy, the latter faces the same type of incentives as in a separating assessment. Therefore, all the results described in the main text hold when mixed strategies are allowed.

Proposition C.1. *Suppose $b_H \geq 1 - \overline{p}$. A semi-separating equilibrium exists if and only if: i. $b_H \leq 1 - \underline{p}$; and ii. $\underline{\gamma}(b_H; \gamma_L^P) \leq \gamma_H^P \leq \frac{c}{1-\underline{p}-b_H}$.*

Proof. I only prove necessity. Sufficiency follows from the usual argument.
Point i. follows from a similar reasoning as in the proof of Lemma 5. I, thus, focus on point ii.
Suppose that the type H always sends signal $\zeta^P(L) = (L, l_i^P(L))$ for some $l_i^P(L) \geq 0$ to be determined in equilibrium. $P(H)$ randomizes between $\zeta^P(H) = (H, 0)$ and $\zeta^P(L)$. Denote α the probability that the decision-maker chooses $b = 1$ after signal $\zeta^P(L)$. By a similar reasoning as in Lemma 5, $P(H)$

and $P(L)$'s incentive compatibility constraints (IC) are then respectively:

$$\gamma_H^P(1-\underline{p})-c=\alpha(\gamma_H^P(1-\underline{p})-c)+(1-\alpha)\gamma_H^P b_H-l_i^P(L) \tag{C.1}$$

$$\alpha\gamma_L^P(1-\overline{p})+(1-\alpha)\gamma_L^P b_H-l_i^P(L)\geq\gamma_L^P(1-\overline{p}) \tag{C.2}$$

Notice that the constraint $l_i^P(L)\geq 0$ implies that condition (C.1) is satisfied only if $\gamma_H^P b_H\geq\gamma_H^P(1-\underline{p})-c$ so this inequality is a first necessary condition (otherwise, $P(H)$ can credibly reveal its type with a cheap talk message). Since $l_i^P(L)=(1-\alpha)(c-\gamma_H^P((1-\underline{p})-b_H))$, plugging this value into condition (C.2) yields, after some algebra, that a second necessary condition is $\gamma_H^P\geq\underline{\gamma}(b_H,\gamma_L^P)$. Suppose instead that $P(H)$ always sends signal $\zeta^P(H)=(H,0)$, whereas the type L randomizes between $\zeta^P(L)=(L,l_i^P(L))$ for some $l_i^P(L)\geq 0$ and $\zeta^P(H)=(H,0)$. Denote α' the probability that the decision-maker chooses $b=1$ after signal $\zeta^P(H)$. By a similar reasoning as in Lemma 5, a type H and type L pro-change SIG's (IC) are then respectively:

$$\alpha'(\gamma_H^P(1-\underline{p})-c)+(1-\alpha')\gamma_H^P b_H\geq\gamma_H^P b_H-l_i^P(L) \tag{C.3}$$

$$\alpha'\gamma_L^P(1-\overline{p})+(1-\alpha')\gamma_L^P b_H=\gamma_L^P b_H-l_i^P(L) \tag{C.4}$$

As above, $\gamma_H^P b_H\geq\gamma_H^P(1-\underline{p})-c$ is a necessary condition for existence (otherwise cheap talk messages are credible to reveal types). Suppose it holds in what follows. From (C.4), we obtain $l_i^P(L)=\alpha'\gamma_L^P(b_H-(1-\overline{p}))$. Plugging this into (C.3), we get after some simple algebra that a second necessary condition is again: $\gamma_H^P\geq\frac{c-\gamma_L^P(b_H-(1-\overline{p}))}{1-\underline{p}-b_H}=\underline{\gamma}(b_H,\gamma_L^P)$. □

SIG supportive of the status quo

As for the pro-change SIG, allowing the SIG supportive of the status quo to use mixed strategies does not affect the results described in the main text. As above, the intuition behind this result is that the decision-maker never proposes a policy different than b_H or 1 when Q randomizes. Any other policy choice only reduces the decision-maker's policy payoff conditional on being implemented. Consequently, Q faces the same incentive as in a separating assessment and all the results described in the main text carry through when I allow for mixed strategies.

Proposition C.2. *A semi-separating equilibrium exists if and only if:*

$$1-\overline{p}\leq\frac{c}{\overline{p}\gamma_H^Q}\leq(1-\overline{p})\frac{c}{\overline{p}\gamma_L^Q}$$

Proof. I only prove necessity. Sufficiency proceeds from the usual argument.

I consider an assessment in which a type $\tau \in \{H, L\}$ sends signal $\zeta^Q(\tau) = (\tau, l_i^Q(\tau))$ for some $l_i^Q(\tau) \geq 0$ with positive probability. Using a similar reasoning as in Lemma B.1, it can be checked that a semi-separating equilibrium exists only if $E(b(\zeta^P, \zeta^Q(H))) < E(b(\zeta^P, \zeta^Q(L)))$ (with expectations as the decision-maker may be randomizing). As a consequence, Lemma B.2 still holds in this setting so the signal most used by $Q(H)$ must satisfy: $l_i^Q(H) > 0 = l_i^Q(L)$. Using a similar reasoning as in Lemma 2, this directly implies that a necessary condition for existence of a semi-separating equilibrium is $1 - \overline{p} \leq b_H$ so the decision-maker's best response to $\zeta^Q(H)$ is $b^*(\zeta^P, \zeta^Q(H)) = b_H$.

Suppose that the type L randomizes (so the type H plays $\zeta^Q(H)$ with probability 1). After signal $\zeta^Q(H)$, the decision-maker's posterior must satisfy: $\mu^Q(\zeta^Q(H))(1-\overline{p}) + (1 - \mu^Q(\zeta^Q(H))) = b_H$ so she is indifferent between $b = 1$ and b_H.[2] Consequently, a necessary condition for this equilibrium to exist is $\pi^Q \leq \frac{1-b_H}{\overline{p}} = \overline{\pi^Q}(b_H)$ (otherwise, $\mu^Q(\zeta^P(H)) > \overline{\pi^Q}(b_H)$ and the decision-maker cannot be made indifferent). Denote β the probability that the decision-maker chooses b_H after signal $\zeta^Q(H)$. Using a similar reasoning as in Lemma 2, the type H and type L's (IC) constraints are respectively:

$$\beta(-\gamma_H^Q(1-\overline{p}) - c) + (1-\beta)(-\gamma_H^Q b_H) - l_i^Q(H) \geq -\gamma_H^Q(1-\overline{p}) - c \tag{C.5}$$

$$\beta(-\gamma_L^Q) + (1-\beta)(-\gamma_L^Q b_H) - l_i^Q(H) = -\gamma_L^Q b_H \tag{C.6}$$

From (C.6), we obtain $l_i^Q(H) = (1-\beta)\gamma_L^Q(1-b_H)$. Plugging the inside lobbying expenditures into condition (C.5), a necessary condition is (after simple algebra): $\frac{c}{\overline{p}\gamma_H^Q} \leq (1-\overline{p})\frac{c}{\overline{p}\gamma_L^Q}$.

Suppose that the type H randomizes (so the type L plays $\zeta^Q(L)$ with probability 1). After signal $\zeta^Q(L)$, the decision-maker's posterior must satisfy: $\mu^Q(\zeta^Q(L))(1-\overline{p}) + (1 - \mu^Q(\zeta^Q(L))) = b_H$ so she is indifferent between $b = 1$ and b_H. Consequently, a necessary condition for this equilibrium to exist is $\pi^Q \geq \overline{\pi^Q}(b_H)$. Denote β' the probability that the decision-maker chooses b_H after signal $\zeta^Q(L)$. Using a similar reasoning as in Lemma 2, the type H and type L's (IC) constraints are respectively:

$$-\gamma_H^Q b_H - l_i^Q(H) = \beta'(-\gamma_H^Q b_H) + (1-\beta')(-\gamma_H^Q(1-\overline{p}) - c) \tag{C.7}$$

$$\beta'(-\gamma_L^Q b_H) + (1-\beta')(-\gamma_L^Q) \geq -\gamma_L^Q b_H - l_i^Q(H) \tag{C.8}$$

From (C.7), we obtain $l_i^Q(H) = (1-\beta')(\gamma_H^Q(1-\overline{p}-b_H)+c)$. Plugging the inside lobbying expenditures into condition (C.8), a necessary condition is again (after simple algebra): $\frac{c}{\overline{p}\gamma_H^Q} \leq (1-\overline{p})\frac{c}{\overline{p}\gamma_L^Q}$. □

[2] If the decision-maker strictly prefers 1 to b_H, then the necessary condition $E(b(\zeta^P, \zeta^Q(H))) < b(\zeta^P, \zeta^Q(L)))$ is violated. If the decision-maker strictly prefers b_H, the results are exactly the same as Lemma 2.

D Robustness

As discussed in the text, relaxing Assumptions 1.(i) and .(ii) implies that Q and P, respectively, cannot influence policy choices. Similarly, relaxing Assumption 2.(ii) is equivalent to the case when the pro-change SIG always has high resolve. These cases are, thus, not considered here. Below, I describe formally how the analysis changes when I relax the other two assumptions (Assumption 2.(i) and Assumption 3) assuming that outside lobbying imposes no disutility for the decision-maker (compared to Appendix B, $k = 0$). I point out below when the lack of direct cost matters.

Relaxing Assumption 2.(i)

Here, I assume that $\gamma_H^Q > \gamma_L^Q > \frac{c}{\overline{p}}$ and analyses the case of influence of SIGs supportive of the status quo then (i.e., $\pi^P = 0$ so P is known to have low resolve). Now, when the decision-maker learns that Q is a low type, she needs to choose between compromising by offering $b_L = \frac{c}{\gamma_L^Q \overline{p}} < 1$ and attempting to pass a comprehensive reform.

I, first, show that a separating equilibrium does not exist in this amended setting. First, to be willing to reveal its type, it must be that a high resolve SIG gains by doing so. This can happen only if the decision-maker is willing to compromise. In other words, a necessary condition for a separating equilibrium to exist is still $b_H \geq 1 - \overline{p}$ (note that if the decision-maker is willing to compromise with $Q(H)$, she also compromises with $Q(L)$ since $b_L > b_H$). Second, $Q(H)$ must have sufficient incentives to reveal its type. Note that the gain from imitation for $Q(L)$ is $\gamma_L^Q(b_L - b_H)$ so for inside lobbying to be a credible signal is must be that $l_i^Q(H) = \gamma_L^Q(b_L - b_H)$ (with equality coming from the Intuitive Criterion). The gain from differentiation for $Q(H)$ is: $\gamma_H^Q b_H - (\gamma_H^Q(1 - \overline{p})b_L + c)$. Indeed, as in the main text, $Q(H)$ would engage in outside lobbying if it pretends to be $Q(L)$ as the decision-maker proposes b_L then. Therefore, for $Q(H)$ to be willing to reveal its type, it must be that $\gamma_H^Q b_H - (\gamma_H^Q(1 - \overline{p})b_L + c) \geq l_i^Q(H)$, or after rearranging $\gamma_H^Q(1 - \overline{p}) > \gamma_L^Q \Leftrightarrow b_H \leq (1 - \overline{p})b_L$. Since $b_L < 1$, the two inequalities—$b_H \geq 1 - \overline{p}$ and $b_H \leq (1 - \overline{p})b_L$—cannot be satisfied simultaneously and a separating equilibrium does not exist.

Regarding a pooling equilibrium (Lemma 3 and Proposition 2), the decision-maker absent any additional information has now three choices. She can fully compromise by choosing $b = b_H$ and avoiding any outside lobbying activity, partially compromises by picking $b = b_L$, and risking outside lobbying by $Q(H)$, or not compromise at all by attempting to pass $b = 1$. Note that when the decision-maker picks $b = b_H$, she gets b_H. When she chooses $b = 1$, she obtains $1 - \overline{p}$. There are, thus, two cases to consider: (a) $b_H \geq 1 - \overline{p}$ and (b) $b_H < 1 - \overline{p}$.

In case (a), we can define $\overline{\pi^Q}(b_H, b_L) = \frac{b_L - b_H}{\overline{p}}$ such that the decision-maker picks $b = b_H$ absent additional information whenever $\pi^Q \geq \overline{\pi^Q}(b_H, b_L)$. A pooling equilibrium exists when $Q(H)$ does not have a profitable deviation. This is always guaranteed when $\pi^Q \geq \overline{\pi^Q}(b_H, b_L)$ since it obtains its favourite bill. When $\pi^Q < \overline{\pi^Q}(b_H, b_L)$, it must be (by the Intuitive Criterion) that the conditions for the existence of a separating equilibrium are not satisfied, which is always the case since there is no separating equilibrium when Assumption 2.(i) is relaxed. In a pooling equilibrium, the decision-maker chooses (i) $b = b_H$ when $\pi^Q \geq \overline{\pi^Q}(b_H, b_L)$ or (ii) $b = b_L$ otherwise. In case (i), due to out-of-equilibrium belief, inside lobbying expenditures can occur in equilibrium (as long as they satisfy $l_i^Q \leq \gamma_H^Q(1 - \overline{p})(b_L - b_H)$). In case (ii), there is no inside lobbying expenditures and $Q(H)$ engages in outside lobbying activities.

In case (b)—$b_H < 1 - \overline{p}$—, the decision-maker, absent additional information, chooses between the comprehensive reform and partial compromise b_L. The decision-maker chooses the former when $\pi^Q > \frac{b_L - (1-\overline{p})}{\overline{p}}$ since the gain from partially compromising is limited then (note that this condition is always satisfied when $b_L < 1 - \overline{p}$). When the decision-maker picks $b = 1$, a pooling equilibrium always exists then with no inside lobbying expenditures and both types engaging in outside lobbying on path. Indeed for a pooling equilibrium to fail to exist, it would have to be that $Q(L)$ credibly reveals its low resolve to obtain b_L (assuming $b_L > 1 - \overline{p}$), which is impossible since $Q(H)$ has more to gain from obtaining a more moderate bill.[3] If $\pi^Q \leq \frac{b_L - (1-\overline{p})}{\overline{p}}$, D chooses $b = b_L$ absent additional information. No type has an incentive to deviate (it is the best policy they can obtain) so a pooling equilibrium always exists. Further, both types may incur inside lobbying expenditures driven by the decision-maker's out-of-equilibrium beliefs (as long as these expenditures satisfy $l_i^Q \leq \gamma_L^Q(1 - \overline{p})(1 - b_L)$) and $Q(H)$ also engages in outside lobbying.

The analysis above reveals that the baseline set-up stacks the desk in favour of inside lobbying expenditures being a useful proxy for SIGs supportive of the status quo to influence policy choices. Here, inside lobbying expenditures are never correlated with influence (there is no separating equilibrium) since the decision-maker only picks a policy based on her prior (only pooling equilibria

[3] Formally, there must exist l_i^Q such that $-\gamma_L^Q b_L - l_i^Q \geq -\gamma_L^Q(1 - \overline{p}) - c$ (so that $Q(L)$ is willing to reveal its type) and $-\gamma_H^Q(1-\overline{p})b_L - c - l_i^Q \leq -\gamma_H^Q(1-\overline{p}) - c$ (so that $Q(H)$ has no incentive to mimic). The first inequality is equivalent to $l_i^Q \leq \gamma_L^Q b_L - \gamma_L^Q(1-\overline{p}) - c = \gamma_L^Q(1-\overline{p})(1-b_L)$ (using the definition of b_L: $-\gamma_L^Q b_L = -\gamma_L^Q(1-\overline{p})b_L - c$). The second inequality is equivalent to $l_i^Q \geq \gamma_H^Q(1-\overline{p})(1-b_L)$. Given $\gamma_H^Q > \gamma_L^Q$, both inequalities cannot be satisfied simultaneously.

exist). Further, there is no longer a clear division between inside and outside lobbying since both can jointly appear on path as discussed in the last paragraph, complicating even further empirical analysis. As a final note, let me stress that a separating equilibrium may exist when outside lobbying is costly for the policy-maker as long as this cost is sufficiently large (conditions for existence are $1-\overline{p}-k \leq b_H \leq (1-\overline{p})b_L$). There, we would recover the positive correlation between inside lobbying expenditures and compromise.

Relaxing Assumption 3

Here, I assume that $\frac{\gamma_H^P}{\gamma_H^Q} \geq \frac{\underline{p}}{\overline{p}-\underline{p}}$, with still $\gamma_L^P < \frac{c}{\overline{p}-\underline{p}}$ and briefly details how the analysis proceeds for the case of pro-change SIG influence then (i.e., $\pi^Q = 1$ so Q is known to have high resolve). As in the main text, I suppose that $b_H > 1-\overline{p}$ so the pro-change SIG's actions have a chance to influence policy choices.

With Assumption 3 relaxed, when the decision-maker proposes $\overline{b_H} = \frac{c}{\gamma_H^Q \underline{p}} \in (b_H, 1)$, $P(H)$ would defend $\overline{b_H}$ should Q engage in outside lobbying (i.e., P's outside lobbying strategy satisfies $l_o^P(\overline{b_H}, 1; H) = 1$). That is, the decision-maker can now credibly compromise with the Q by choosing $\overline{b_H}$ when she learns that P is a high type (Q chooses $l_o^Q(\overline{b_H}, \zeta^P(H); H) = 0$ when $\zeta^P(H) \neq \zeta^P(L)$ since $-\gamma_H^Q\overline{b_H} = -(1-\underline{p})\gamma_H^Q\overline{b_H} - c$). There are then two cases to consider: (a) $\overline{b_H} \leq 1-\underline{p}$ and (b) $\overline{b_H} > 1-\underline{p}$ (implicitly assuming the decision-maker chooses $b=1$ when indifferent, though this is of limited importance).

In case (a), the conditions for and strategies in a separating equilibrium remain the same as in Lemma 5 and Proposition 3 since the decision-maker would still choose $b=1$ upon learning that P is of high resolve (note though that that condition (i) of Lemma 5 is automatically satisfied).

The analysis of pooling equilibria (Lemma 6 and Proposition 4) is substantially unchanged. We need, however, to consider the possibility that absent information, Q does not engage in outside lobbying if the bill proposed is $b^p = \frac{c}{(\pi^P\underline{p}+(1-\pi^P)\overline{p})\gamma_H^Q}$ and it expects $P(H)$ to defend it, which requires $\gamma_H^P(1-\underline{p})b^p - c \geq \gamma_H^P(1-\overline{p})b^p$, or equivalently $\frac{\gamma_H^P}{\gamma_H^Q} \geq \frac{(\pi^P\underline{p}+(1-\pi^P)\overline{p})}{\overline{p}-\underline{p}}$.[4] When this last inequality does not hold, then we are back in the main text. Let's suppose it holds. If $b^p \leq \pi^P(1-\underline{p}) + (1-\pi^P)(1-\overline{p})$ and the decision-maker chooses $b=1$ in a pooling equilibrium, then the condition for the existence of a pooling equilibrium is summarized in Lemma 6.(ii) and the strategies in Proposition 4.(2). If $b^p > \pi^P(1-\underline{p}) + (1-\pi^P)(1-\overline{p})$, then a low-type has no incentive to reveal its type and a pooling

[4]The inequality $\frac{\gamma_H^P}{\gamma_H^Q} \geq \frac{(\pi^P\underline{p}+(1-\pi^P)\overline{p})}{\overline{p}-\underline{p}}$ never holds in the main text by Assumption 3.

equilibrium exists as long as a high-type prefers this compromise to the full reform, in other words $\gamma_H^P(1-\underline{p}) - c \leq \gamma_H^P b^p$ (this is the same condition as in Lemma 6 but with b^p replacing b_H). Due to the out-of-equilibrium belief of the decision-maker, inside lobbying expenditures are possible on path just as in Proposition 4.(1). Hence, we recover similar patterns as in the main text, only with a more favourable compromise bill from P's standpoint.

The more interesting case is, thus, case (b). And the analysis of it takes the form of the proof of Proposition 5.

Proof of Proposition 5

In this case, upon learning that P is a type-L, the decision-maker picks $b = b_H$. When she learns P is of high resolve, she chooses $b = \overline{b_H} > b_H$. Q, in no case, engages in outside lobbying and neither does the pro-change SIG. This now means that $P(L)$ has strong incentives to imitate $P(H)$. She gets a better bill without risk. On the other hand, $P(H)$ has incentive to separate from $P(L)$ for the same reason. That is, we are in the context of a classical signaling game, and a separating equilibrium always exists. $P(H)$ would engage in inside lobbying expenditures, equal to $P(L)$'s benefit from differentiation: $l_i^P(H) = \gamma_L^P(\overline{b_H} - b_H)$. Since a high-resolve has always more to gain from obtaining a more comprehensive bill, both types' incentive compatibility constraints are always satisfied. In this separating equilibrium, the empirical patterns radically change compared to the main text. Indeed, we find a positive correlation between the content of the bill and the inside lobbying expenditures.

I now show that a pooling equilibrium does not exist in case (b). Absent additional information, the decision-maker chooses between $b = 1$ and $b = b_H$ or $b = b^p$ depending if b^p, defined above, is credible (i.e., if $\frac{\gamma_H^P}{\gamma_H^Q} \geq \frac{(\pi^P \underline{p} + (1-\pi^P)\overline{p})}{\overline{p} - \underline{p}}$ is satisfied).[5] When the decision-maker picks $b = b_H$ or $b = b^p$ absent additional information, $P(H)$ would have incentives to reveal its type to obtain $b = \overline{b_H} > b^p$ and would always find a credible signal to do so. By the Intuitive Criterion, a pooling equilibrium fails to exist then. So a pooling equilibrium could exist only if the decision-maker chooses $b = 1$ based on her prior, which requires that $\pi^P(1-\underline{p}) + (1-\pi^P)(1-\overline{p}) \geq b_H$ (or b^p if the latter is credible). Existence also requires that (i) $P(H)$ has no opportunity to credibly reveal its type and obtains $\overline{b_H}$ and (ii) $P(L)$ cannot credibly reveal its type and obtains b_H. The first condition requires that there is **no** inside lobbying expenditures l_i^P such that: $\gamma_L^P(1-\overline{p}) \geq \gamma_L^P \overline{b_H} - l_i^P$ and $\gamma_H^P(1-\underline{p}) - c \leq \gamma_H^P \overline{b_H} - l_i^P$ (with one inequality strict). No such expenditures exist only if: $\gamma_L^P(\overline{b_H} - (1-\overline{p})) \geq \gamma_H^P(\overline{b_H} - (1-\underline{p})) + c \Leftrightarrow$

[5] The decision-maker cannot pick $b = \overline{b_H}$ since Q would start outside lobbying activities then (it is only indifferent if it is certain P is of high resolve), her expected payoff would $\left(\pi^P(1-\underline{p}) + (1-\pi^P)(1-\overline{p})\right) \times \overline{b_H}$, which is strictly less than $\left(\pi^P(1-\underline{p}) + (1-\pi^P)(1-\overline{p})\right) \times 1$.

$\gamma_H^P(1-\underline{p})-c-\gamma_L^P(1-\overline{p}) \geq (\gamma_H^P-\gamma_L^P)\overline{b_H}$. The second condition requires (by the same reasoning) that the following inequality holds: $\gamma_H^P(b_H-(1-\underline{p}))+c \geq \gamma_L^P(b_H-(1-\overline{p})) \Leftrightarrow \gamma_H^P(1-\underline{p})-c-\gamma_L^P(1-\overline{p}) \leq (\gamma_H^P-\gamma_L^P)b_H$. This two inequalities cannot be satisfied simultaneously (this is a consequence of the increasing differences assumption in the set-up) and a pooling equilibrium does not exist then. $\square$

E Policy choices with competing SIGs

In this section, I analyze formally the case when there is competition for influence with signaling between the SIG supportive of the status quo and the pro-change SIG (i.e., $\pi^J \in (0,1),\ J \in \{P,Q\}$). Throughout, I assume that Assumptions 1-3 hold. Further, I impose that $k=0$ as in the baseline analysis to limit the number of cases.

Pro-change SIG and policy choices

I first study how the pro-change SIG's signal influences the decision-maker's policy choice when it plays a separating strategy on the equilibrium path: $\zeta^P(L) \neq \zeta^P(H)$.[6] First, as in the main text, the pro-change SIG obtains more favorable political decision in equilibrium when its resolve is high. Unlike the main text, one needs to compare the expected policy choice.

Lemma E.1. *When the pro-change SIG plays a separating strategy on the equilibrium path,* $E(b(\zeta^P(L),\zeta^Q(\tau)))$ $> E(b(\zeta^P(H),\zeta^Q(\tau))$ *(the expectation is over Q's type).*

Assuming that the pro-change SIG plays a separating strategy, the next Proposition shows that there still exist parameter values such that inside lobbying expenditures are negatively correlated with policy choice when the pro-change SIG plays a separating strategy.

Proposition E.1. *When the pro-change SIG plays a separating strategy on the equilibrium path,*
1. When Q plays a pooling strategy on the equilibrium path, $l_i^{P*}(H) = 0 \leq l_i^{P*}(L)$ *(with strict inequality whenever* $b_H > \pi^Q(1-\underline{p}) + (1-\pi^Q) - \frac{c}{\gamma_H^P}$*);*
2. When Q plays a separating strategy on the equilibrium path, $l_i^{P*}(H) = 0 \leq l_i^{P*}(L)$ *(with strict inequality whenever* $b_H > 1-\underline{p}-\frac{c}{\gamma_H^P}$*);*
3. In all cases, $l_o^{P*}(H) = 1$ *and* $l_o^{P*}(L) = 0$.

For a separating equilibrium to exist, it must be that the decision-maker prefers the compromise bill b_H to the comprehensive reform when she learns the pro-change SIG is of low resolve. Note that $P(L)$ also prefers the compromise bill then (D and P have aligned interests) and so it has no incentive to pretend to have high resolve. As a result, just like in the main text, $P(H)$ does not want to incur inside lobbying expenditures to reveal that it is willing to engage in costly outside lobbying. As such, only a type L uses inside lobbying expenditures to credibly "plead poverty" and encourage

[6] Lemmas E.5 and E.6 show that a separating strategy is an equilibrium strategy for some parameter values.

the decision-maker to compromise. Even so, $P(H)$'s expected outside lobbying cost is always higher than $P(L)$'s inside lobbying expenditures.

Finally, as in the main text, a pro-change SIG may still incur inside lobbying expenditures (depending on parameter values and the decision-maker's out-of-equilibrium belief) when it plays a pooling strategy on the equilibrium path ($\zeta^P(L) = \zeta^P(H)$).

Proposition E.2. *There exists a non-empty open set of lobbying costs such that a pooling equilibrium exists with the decision-maker choosing $b^* = b_H$, the pro-change SIG, on the equilibrium path, incurring strictly positive inside lobbying expenditures and never engaging in outside lobbying.*

Propositions E.1 and E.2 indicate that inside lobbying expenditures are still associated with compromise. In turn, outside lobbying activities are always correlated with full reform. Empirical analyses of pro-change SIG influence focusing exclusively on inside lobbying expenditures are, thus, likely to suffer from the same issues as in the main text, underestimating the strength (and extent) of SIG influence. Outside lobbying expenditures, on the other hand, fare slightly better to measure the extent of P influence. As long as P plays a separating strategy, it incurs these expenditures only after the decision-maker attempts to pass the full reform. The problem, as in the main text, is that for some parameter values, outside activity occurs in a pooling equilibrium (details available upon request) so these activities also generate some attenuation bias.

SIG supportive of the status quo and policy choices

I now study how uncertainty about the pro-change SIG's resolve changes Q's incentives to play a separating strategy.

Let's consider first the case when the pro-change SIG plays a separating strategy. When the decision-maker learns that the pro-change SIG's resolve is high, the expected payoff from proposing $b = 1$ increases (since the proposal is more likely to pass). Consequently, there is a risk that the decision-maker does not compromise with $Q(H)$. This has two effects. First, it reduces $Q(H)$'s benefit from differentiation. However, this uncertainty also reduces $Q(L)$'s benefit from imitation, and so a type H can incur lower inside lobbying expenditures to credibly reveal its type. The two effects fully cancel out and the condition for a separating strategy to be incentive compatible remains exactly the same as in the main text.

Let's now consider the case when P plays a pooling strategy. Then, $P(H)$ would intervene at the outside lobbying stage only if the decision-maker proposes the full reform. This means that the compromise bill is unchanged, but the value of deviating and pretending to be $Q(L)$ decreases for $Q(H)$. Rather than obtaining the bill to removed with probability $\overline{p}$ when there is no uncertainty, this probability drops to $\pi^P \underline{p} + (1-\pi^P)\overline{p}$ with uncertainty. As a result, the gain from differentiation increases. Since, on the other hand, $Q(L)$ is unaffected (uncertainty only affects the outcome following outside lobbying, and this type never engages in such activities), separation becomes easier for Q. In other words, uncertainty relaxes the incentive compatibility constraint.

The reasoning is summarized in Lemma E.2 with case (a) corresponding to P pooling and case (b) to P separating.

Lemma E.2. *Q plays a separating strategy on the equilibrium path*

(a) when the pro-change SIG plays a pooling strategy, if and only if: $\pi^P(1-\underline{p})+(1-\pi^P)(1-\overline{p}) \leq b_H$ *and* $\gamma_H^Q\left[\pi^P(1-\underline{p})+(1-\pi^P)(1-\overline{p})-(1-\overline{p})b_H\right] \geq \gamma_L^Q(1-b_H)$*;*

(b) when the pro-change SIG plays a separating strategy, if and only if: $1-\overline{p} \leq b_H$ *and* $\gamma_H^Q(1-\overline{p}) \geq \gamma_L^Q$

While the conditions for separation change slightly, the strategies remain as in the main text. Only $Q(H)$ incurs inside lobbying expenditures in exchange for a more favorable political decision (closer to the status quo) in expectation.

Proposition E.3. *Q plays a separating strategy on the equilibrium path, equilibrium strategies satisfy:*

1. $l_i^{Q*}(H) > 0$ *and* $l_i^{Q*}(L) = 0$*;*

2. $E(b^*(\zeta^{Q*}(H), \zeta^{P*}(\tau))) < E(b^*(\zeta^{Q*}(L), \zeta^{P*}(\tau)))$ *(where the expectation is over the pro-change SIG's type).*

The next proposition establishes conditions under which both Q and P play a separating strategy on the equilibrium path and engage in both types of activities. Even though both types of lobbying are *strategic substitutes*, outside lobbying sometimes complements inside lobbying on the equilibrium path.

Proposition E.4. *There exists a non-empty open set of parameter values such that on the equilibrium path, both SIGs play a separating strategy and Q engages in both inside and outside lobbying with positive probability.*

The main empirical implications discussed in the main text holds when there is competition for influence. Q's inside lobbying expenditures still tilt political decisions in its favor in a separating equilibrium: they are always associated with better bills *in expectations* (Proposition E.3). Indeed, Proposition E.4 highlights that inside lobbying expenditures do not guarantee the SIG supportive of the status quo influences policy choice due to the possible presence of a pro-change SIG with high-resolve. Q might fail to bias the content of a bill and be forced to resort to outside lobbying activities even when it plays a separating strategy in equilibrium. That is, even in a separating equilibrium, threats may fail. Further, like in the main text, a separating equilibrium does not always exists. Since the SIG supportive of the status quo can still influence policy in a pooling equilibrium (details available upon request), analyses which exclusively consider inside lobbying expenditures underestimate the extent of SIG influence. As there exist pooling equilibria with inside lobbying expenditures, inside lobbying expenditures as a proxy for influence entail downwardly biased estimate of the extent of SIG influence.[7] All these issues, some already raised, some specific to this environment, lead to the usual conclusion: inside lobbying expenditures are a poor proxy of the power of SIGs supportive of the status quo. Using outside lobbying expenditures can help the researcher identify when these SIGs fail to influence policy even if it plays a separating strategy. This class of expenditures, thus, still provides an unbiased measure of the extent of SIG influence.

Proofs

I first study when the pro-change SIG's best response is to play a separating strategy: $\zeta^P(L) \neq \zeta^P(H)$. The following lemmas provide the key elements to prove Lemma E.1 and Proposition E.1.

Lemma E.3. *Q and P play a separating strategy on the equilibrium path only if: $b(\zeta^P(H), \zeta^Q(H)) = 1$ and $b(\zeta^P(L), \zeta^Q(H)) = b_H$.*

Proof. The proof is by contradiction. Recall that $b = 1$ is always a best response when the decision-maker learns that Q has low resolve: $b(\zeta^P, \zeta^Q(L)) = 1$ for all ζ^P.

Suppose $b(\zeta^P(H), \zeta^Q(H)) = 1$ and $b(\zeta^P(L), \zeta^Q(H)) = 1$. The signal then does not influence the decision-maker's policy choice and, given the equilibrium restriction, a separating assessment cannot be an equilibrium.

Suppose $b(\zeta^P(H), \zeta^Q(H)) = b_H$ and $b(\zeta^P(L), \zeta^Q(H)) = b_H$. In this case, the pro-change SIG's signal has no influence on policy choice and it does not play a separating strategy given the equilibrium restriction. Hence we have reached a contradiction. □

[7] When Q does not separate, the decision-maker's policy choice depends on her evaluation of the threat of outside lobbying (π^Q) and the pro-change SIG's strategy. Q might incur inside lobbying expenditures on the equilibrium path, but they have no impact on policy choices: Proposition 2 applies. So the attenuation bias identified in the main text is still present.

When the pro-change SIG plays a separating strategy on the equilibrium path, denote $p(l_o^P = 1|\zeta^P(\tau))$ the probability that a type $l \in \{H, L\}$ pro-change SIG engages in outside lobbying after sending signal $\zeta^P(\tau)$.

Lemma E.4. *The pro-change SIG plays a separating strategy on the equilibrium path only if: i)* $p(l_o^P = 1|\zeta^P(H)) > 0$ *and ii)* $p(l_o^P = 1|\zeta^P(L)) = 0$.

Proof. Point ii. follows directly from Assumption 2.(ii). Point i. is always satisfied when Q plays a separating strategy by Lemma E.3. When Q does not separate ($\zeta^Q(H) = \zeta^Q(L) = \zeta^Q$), the proof is by contradiction. Suppose $p(l_o^P = 1|\zeta^P(H)) = 0$. This implies: $b^*(\zeta^Q, \zeta^P(H)) = b^*(\zeta^Q, \zeta^P(L)) = b_H$. But then the pro-change SIG does not play a separating strategy under the equilibrium restriction. $\square$

Note that the two previous lemmas indicate that if Q plays a separating strategy, then $b(\zeta^P(H), \zeta^Q(H)) = 1$, $b(\zeta^P(H), \zeta^Q(L)) = 1$, whereas $b(\zeta^P(L), \zeta^Q(H)) = b_H$ and $b(\zeta^P(L), \zeta^Q(L)) = 1$. If Q plays a pooling strategy, then $b(\zeta^P(H), \zeta^Q) = 1$. Our equilibrium restriction then implies that after signal $\zeta^P(L)$, D's best response must satisfy $b(\zeta^P(L), \zeta^Q) = b_H$.

The next two lemmas determine when a separating strategy is the pro-change SIG's best response to other players' actions.

Lemma E.5. *When Q does not separate, a pro-change SIG's separating strategy is a best response to other players' actions if and only if the following conditions are satisfied:*
(i.) $\pi^Q(1-\overline{p}) + (1-\pi^Q) \leq b_H \leq \pi^Q(1-\underline{p}) + (1-\pi^Q)$
and (ii.)

(a) $b_H \leq \pi^Q(1-\underline{p}) + (1-\pi^Q) - \frac{c}{\gamma_H^P}$, *or*

(b) $\pi^Q(1-\underline{p}) + (1-\pi^Q) - \pi^Q\frac{c}{\gamma_H^P} < b_H$ *and* $\gamma_H^P \geq \frac{\pi^Q c - \gamma_L^P[b_H - \pi^Q(1-\overline{p}) - (1-\pi^Q)]}{\pi^Q(1-\underline{p}) + (1-\pi^Q) - b_H}$.

Proof. Recall that Lemmas E.3 and E.4 imply that when the pro-change SIG plays a separating strategy on the equilibrium path, the decision-maker's policy choices must satisfy: $b(\zeta^P(H), \zeta^Q) = 1$ and $b(\zeta^P(L), \zeta^Q) = b_H$.

First, consider necessity. Taking into account that $Q(L)$ does not engage in outside lobbying and using Lemma 3 in the main text, the decision-maker's best-response after $\zeta^P(L)$ is $b(\zeta^P(L), \zeta^Q) = b_H$ if and only if $b_H \geq \pi^Q(1-\overline{p}) + (1-\pi^Q)$ (this condition can also be expressed in term of D's prior: $\pi^Q \geq \overline{\pi^Q}(b_H) = \frac{1-b_H}{\overline{p}}$). Similarly, the decision-maker's best-response after $\zeta^P(H)$ is $b(\zeta^P(H), \zeta^Q) = 1$ if and only if $b_H \leq \pi^Q(1-\underline{p}) + (1-\pi^Q)$ (this condition can be expressed in term of D's prior again:

$\pi^Q \leq \widehat{\pi^Q}(b_H) = \frac{1-b_H}{\underline{p}}$). This proves that condition (i.) is necessary.

For condition (ii.), consider the type L and type H's incentive compatibility constraints (IC), which are, respectively:

$$\gamma_H^P[\pi^Q(1-\underline{p}) + (1-\pi^Q)] - \pi^Q c - l_i^P(H) \geq \gamma_H^P b_H - l_i^P(L) \tag{E.1}$$

$$\gamma_L^P[\pi^Q(1-\overline{p}) + (1-\pi^Q)] - l_i^P(H) \leq \gamma_L^P b_H - l_i^P(L) \tag{E.2}$$

Using the Intuitive Criterion, it can be verified that $l_i^P(L)l_i^P(H) = 0$. By the reasoning above, we can focus on the case when $\pi^Q(1-\overline{p}) + (1-\pi^Q) \leq b_H$ so $P(L)$ never has incentive to pretend to $P(H)$ and $l_i^P(H) = 0$.

With this preliminary result in mind, consider now case (a)—so that $\pi^Q(1-\overline{p}) + (1-\pi^Q) \leq b_H \leq \pi^Q(1-\underline{p}) + (1-\pi^Q) - \frac{c}{\gamma_H^P}$, it can easily be checked that both (IC) are satisfied with $l_i^P(L) = l_i^P(H) = 0$ (after some algebra).

In case (b), $l_i^P(H) = 0 < l_i^P(L) = \gamma_H^P[b_H - \pi^Q(1-\underline{p}) - (1-\pi^Q)] + \pi^A c$ and a necessary condition is $\gamma_H^P \geq \frac{\pi^Q c - \gamma_L^P[b_H - \pi^Q(1-\overline{p}) - (1-\pi^Q)]}{\pi^Q(1-\underline{p}) + (1-\pi^Q) - b_H}$.

For sufficiency, consider the following assessment in case (b) (other cases can be treated similarly): i) A type L (type H) pro-change SIG sends signal $\zeta^P(L) = (L, l_i^P(L))$ ($\zeta^P(H) = (H, 0)$), with $l_i^P(L) = \gamma_H^P[b_H - \pi^Q(1-\underline{p}) - (1-\pi^Q)] + \pi^A c$; ii) The decision-maker's posterior is: $\mu^P(\zeta^P) = 0$ if $\zeta^P = (L, l_i^P)$, with $l_i^P \geq l_i^P(L)$, and 1 otherwise; iii) the decision-maker's policy choice is: $b(\zeta^P, \zeta^Q) = b_H$ if $\zeta^P = (L, l_i^P)$, with $l_i^P \geq l_i^P(L)$ and $b(\zeta^P, \zeta^Q) = 1$, otherwise; (iv) all players play their best response down the game tree (see Section A). It can be checked that beliefs satisfy Bayes' rule, the decision-maker's policy choice is a best response given her belief, and the pro-change SIG's (IC) hold. Hence, the conditions described in the text of the Lemma corresponds to sufficient condition for the pro-change SIG to play a separating strategy. Note that we have not proven that it is sufficient for the pro-change SIG to separate in a PBE since we have assumed, but not shown that Q does not separate. □

Lemma E.6. *When Q plays a separating strategy, a pro-change SIG's separating strategy is a best response to other players' actions if and only if the following conditions are satisfied: (i.) $1-\overline{p} \leq b_H \leq 1-\underline{p}$ and (ii.) $\gamma_H^P \geq \frac{c-\gamma_L^P(b_H-(1-\overline{p}))}{1-\underline{p}-b_H}$.*

Proof. I only prove necessity, sufficiency follows from the usual argument. By Lemma E.3, recall that the decision-maker's best response after the SIGs' signals must satisfy $b(\zeta^P(H), \zeta^Q(H)) = 1$, $b(\zeta^P(H), \zeta^Q(L)) = 1$, $b(\zeta^P(L), \zeta^Q(H)) = b_H$, and $b(\zeta^P(L), \zeta^Q(L)) = 1$. As it is now common,

$b(\zeta^P(H), \zeta^Q(H)) = 1$ requires $1 - \underline{p} \geq b_H$ and $b(\zeta^P(L), \zeta^Q(H)) = b_H$ requires $1 - \overline{p} \leq b_H$. This gives condition (i.).

Condition (ii.) follows from the pro-SIG's (IC) which are for a type L and type H, respectively:

$$\pi^Q(\gamma_H^P(1-\underline{p}) - c) + (1-\pi^Q)\gamma_H^P - l_i^P(H) \geq \pi^Q\gamma_H^P b_H + (1-\pi^Q)\gamma_H^P - l_i^P(L) \tag{E.3}$$

$$\pi^Q\gamma_L^P(1-\overline{p}) + (1-\pi^Q)\gamma_L^P - l_i^P(H) \leq \pi^Q\gamma_L^P b_H + (1-\pi^Q)\gamma_L^P - l_i^P(L) \tag{E.4}$$

Using condition (i.), this directly implies $l_i^P(H) = 0 \leq l_i^P(L) = \max\{0, \pi^Q(\gamma_H^P(b_H - (1-\underline{p})) + c)\}$. Using a similar reasoning as in Lemma 5, it can be checked that condition (ii.) is also a necessary condition for existence. □

Proof of Lemma E.1

The proof follows directly from Lemmas E.5-E.6. □

Proof of Proposition E.1

Point 1. follows directly from the proof of Lemma E.5. Point 2. follows from the proof of Lemma E.6. Point 3. follows from Lemma E.4. □

Proof of Proposition E.2

Suppose $\pi^Q(1 - \underline{p} - \frac{c}{\gamma_H^P}) + (1-\pi^Q) < b_H < \pi^Q(1-\underline{p}) + (1-\pi^Q)$ and $\pi^Q > \frac{1-b_H}{\pi^P\underline{p}+(1-\pi^P)\overline{p}}$. In this case, there exists an equilibrium in which both SIGs play a pooling strategy and the decision-maker chooses $b(\zeta^P, \zeta^Q) = b_H$.[8]

Consider the following belief structure for the decision-maker: $\mu^P(c_L^P|\zeta^P) = 1$ if $\zeta^P = \{\hat{\tau}, l_i^P\}$, $\forall\hat{\tau} \in \{H, L\}$, $l_i^P < \tilde{l_i^P}$ for some $\tilde{l_i^P} > 0$, and $\mu(\zeta^P) = \pi^P$, otherwise. Given this belief structure, for the equilibrium ζ^Q, the decision-maker's best response is: $b(\zeta^P, \zeta^Q) = 1$, $\forall\zeta^P \in \{H, L\} \times [0, \tilde{l_i^P})$, whereas $b(\zeta^P, \zeta^Q) = b_H$ $\forall\zeta^P$ satisfying $\zeta^P \in \{H, L\} \times [\tilde{l_i^P}, \infty)$. Using the type L and type H's incentive compatibility constraint (and a similar reasoning as in the proof of Proposition 4), it can be checked that for all inside lobbying expenditures satisfying $\tilde{l_i^P} \leq \min\{\gamma_H^P(b_H - \pi^Q(1-\underline{p}) - (1-\pi^Q)) + c, \gamma_L^P(b_H - \pi^Q(1-\overline{p}) - (1-\pi^Q))\}$, the pro-change SIG's (IC)'s are satisfied and a pooling equilibrium

[8] $P(H)$ prefers b_H to $b = 1$ and Q gets the compromise policy. And the condition on π^Q guarantees that the decision-maker prefers b_H rather than betting on the comprehensive reform $b = 1$.

with strictly positive inside lobbying expenditures by the pro-change SIG exists.

For completeness, I show that Proposition E.2 does not require that Q plays a pooling strategy on the equilibrium path.

Suppose $1-\underline{p} > b_H > \pi^Q(1-\underline{p}) + (1-\pi^Q) - \pi^Q\frac{c}{\gamma_H^Q}$ and $\pi^P(1-\underline{p}) + (1-\pi^P)(1-\overline{p}) < b_H < (\pi^P(1-\underline{p}) + (1-\pi^P)(1-\overline{p}))\frac{c}{\overline{p}\gamma_L^Q}$. In this case, the decision-maker's best response to $\zeta^Q(H)$ absent information about P is $b(\zeta^P, \zeta^Q(H)) = b_H$, a policy choice both types of P prefers to a comprehensive reform under the parameter values. Further, a separating strategy is incentive compatible for Q (see Lemma E.2). So under these parameters, there exists an equilibrium in which P pools and Q separates.

Consider the following belief structure for the decision-maker in a pooling assessment: $\mu^P(\zeta^P) = 1$ $\forall \zeta^P = (\hat{\tau}, l_i^P)$ satisfying $(\hat{\tau}, l_i^P) \in \{H, L\} \times [0, \tilde{l_i^P})$ and $\mu^P(\zeta^P) = \pi^P$, otherwise. Given this belief structure and the assumptions above, the decision-maker's best response is: $b(\zeta^P, \zeta^Q(H)) = 1$, $\forall \zeta^P \in \{H, L\} \times [0, \tilde{l_i^P})$. The type L and type H's (IC) are then, respectively:

$$(1-\pi^Q)\gamma_L^Q + \pi^Q\gamma_L^Q(1-\overline{p}) \leq (1-\pi^Q)\gamma_L^Q + \pi^Q\gamma_L^Q b_H - \tilde{l_i^P}$$
$$(1-\pi^Q)\gamma_H^Q + \pi^Q\big(\gamma_H^Q(1-\underline{p}) - c\big) \leq (1-\pi^Q)\gamma_H^Q + \pi^Q\gamma_H^Q b_H - \tilde{l_i^P}$$

It can be checked that for all inside lobbying expenditures satisfying $\tilde{l_i^P} \leq \pi^Q \min\{\gamma_H^Q(b_H - (1-\underline{p})) + c, \gamma_L^Q(b_H - (1-\overline{p}))\}$, the two (IC) constraints are satisfied and there exists an equilibrium in which the pro-change SIG plays a pooling strategy and incurs inside lobbying expenditures on the equilibrium path when Q separates. □

In what follows, I study conditions under which a separating strategy ($\zeta^Q(H) \neq \zeta^Q(L)$) is a best response for Q when there is uncertainty about the pro-change SIG's type. As before, I assume that all players play their best response down the game tree.

Proof of Lemma E.2

Using the same reasoning as in Section B.1, it can be checked that only $Q(H)$ incurs inside lobbying expenditures, whether the pro-change SIG separates or pools.

Suppose the pro-change SIG plays a pooling strategy. Then, under the assumptions, the decision-maker always chooses between $b = 1$ and $b = b_H$. When the decision-maker proposes a full reform

$(b = 1)$ and Q chooses $l_o^Q = 1$, then with probability π^P, P is of high resolve (so $l_o^P(1, 1; H) = 1$) and the bill is upheld with probability $1-\underline{p}$, with probability $1-\pi^P$, P is of low resolve (so $l_o^P(1, 1; L) = 0$) and the bill is upheld with $1-\overline{p}$. Hence, the decision-maker prefers to compromise with $Q(H)$ only if $\pi^P(1-\underline{p}) + (1-\pi^P)(1-\overline{p}) \leq b_H$. Further, the (IC) constraints of $Q(H)$ and $Q(L)$ are, respectively:

$$-\gamma_H^Q b_H - l_i^Q(H) \geq -\gamma_H^Q(\pi^P(1-\underline{p}) + (1-\pi^P)(1-\overline{p})) - c$$
$$-\gamma_L^Q b_H - l_i^Q(H) \leq -\gamma_L^Q$$

By the Intuitive Criterion, $l_i^Q(H) = \gamma_L^Q(1-b_H)$. Hence, $Q(H)$'s (IC) is satisfied only if: $\gamma_H^Q\big[\pi^P(1-\underline{p}) + (1-\pi^P)(1-\overline{p}) - (1-\overline{p})b_H\big] \geq \gamma_L^Q(1-b_H)$.

Consider the case when the pro-change SIG plays a separating strategy on the equilibrium path. By Lemma 2, a necessary condition for both SIGs to play a separating strategy is that D's best response satisfies: $b(\zeta^P(H), \zeta^Q(H)) = 1$ and $b(\zeta^P(L), \zeta^Q(H)) = b_H$, or equivalently $1-\overline{p} \leq b_H (\leq 1-\underline{p})$.

A separating strategy must also be incentive compatible for Q. By Lemma E.3, we also know that $b(\zeta^P(H), \zeta^Q(L)) = b(\zeta^P(L), \zeta^Q(L)) = 1$. By a similar logic as in Lemma B.2, it must be that $l_i^Q(L) = 0$ and $l_i^Q(H) > 0$. $Q(L)$'s (IC) is:

$$-\pi^P\gamma_L^Q - (1-\pi^P)\gamma_L^Q b_H - l_i^Q(H) \leq -\gamma_L^Q$$

By the Intuitive Criterion, $l_i^Q(H) = (1-\pi^P)\gamma_L^Q(1-b_H)$.

Now consider the type H's (IC):

$$\pi^P(-\gamma_H^Q(1-\underline{p}) - c) + (1-\pi^P)(-\gamma_H^Q b_H) - l_i^Q(H) \geq \pi^P(-\gamma_H^Q(1-\underline{p}) - c) + (1-\pi^P)(-\gamma_H^Q(1-\overline{p}) - c)$$

Substituting for $l_i^Q(H)$ and rearranging, we get that a necessary condition for a separating strategy to be incentive compatible is thus: $\gamma_L^Q \leq (1-\overline{p})\gamma_H^Q$.

Sufficiency follows from the usual argument. □

Proof of Proposition E.3

Follows directly from the proof of Lemma E.2. □

Proof of Proposition E.4

Suppose an equilibrium with both SIGs separating exists. By Lemmas E.2 and E.3, $Q(H)$ incurs inside lobbying expenditures and engages in outside lobbying when the pro-change SIG has high resolve. We thus just need to show that there exist parameter values such that an equilibrium in which both SIGs play a separating strategy exists. Consider parameter values satisfying the following conditions: $b_H \leq 1 - \underline{p}$ (so $b(\zeta^P(H), \zeta^Q(H)) = 1$), $1 - \overline{p} \leq b_H \leq (1 - \overline{p})\frac{c}{\overline{p}\gamma_L^Q}$ (so a separating strategy is incentive compatible for the Q), and $\gamma_H^P \geq \frac{c - \gamma_L^P(b_H - (1-\overline{p}))}{1 - \underline{p} - b_H}$ (so a separating strategy is incentive compatible for the pro-change SIG). Simple observation reveals that these conditions are not mutually exclusive. By Lemmas E.2 and E.6, under these parameter values there exist an equilibrium in which both SIGs play a separating strategy on the equilibrium path. □

F Micro-founding the impact of outside lobbying activities

In this section, I micro-found the influence of outside lobbying on public opinion. To do so, I use a simplified version of the War of Information (Gül and Pesendorfer, 2012). To highlight this mechanism, I only consider the influence of the SIG supportive of the status quo with the pro-change SIG's type known. I slightly deviate from the case in the main text by assuming that P has high resolve: $\pi^P = 1$. This allows me to study the pro-change group's choice in the war of information without having to consider its signals. Q's resolve is its private information: $\tau^Q \in \{L, H\}$ with common prior probability $Pr(\tau^Q = H) = \pi^Q$. Finally, since Q's type announcement is not credible without inside lobbying expenditures (see Lemma B.2), I assume without loss of generality that Q's signal takes the form of inside lobbying expenditures (i.e., $\zeta^Q := l_i^Q$).

I consider a four-player game with the decision-maker, the SIG supportive of the status quo, the pro-change SIG, and a representative voter. There are two states of the world: $\omega \in \{\underline{\Delta}, \overline{\Delta}\}$. In state $\overline{\Delta}$, the decision-maker's legislative proposal improves the voter's utility relative to the status quo. In state $\underline{\Delta}$, the decision-maker's proposal decreases the voter's utility. No player knows the state of the world at the beginning of the game. However, it is common knowledge that players' common prior is biased in favor of the decision-maker's proposal (see Assumption F.1 below).[9]

Adapting Gül and Pesendorfer's (2012) war of information, the SIGs' outside lobbying activities reveal information to the representative voter. The voter receives no, one, or two signals of the state of the world depending on other players' actions. The voter then sides with the decision-maker or the SIG according to her belief regarding ω. Providing information to the voter is costly for the SIGs, with common knowledge cost c as in the main text.[10]

The augmented game proceeds as follows:

0. Nature draws Q's type ($\tau^Q \in \{H, L\}$).
1. After observing its type, the SIG sends a signal: $l_i^Q \geq 0$.
2. The decision-maker chooses the content of the bill: $b \in [0, 1]$.
3. Q decides whether to start a war of information: $l_o^Q \in \{0, 1\}$.

   If there is no war, the representative voter sides with the decision-maker or Q according to her

[9] The reverse case is also possible. In this case, the timing of outside lobbying activities is different from the main text: P would start outside lobbying activities and Q would intervene later.

[10] A major difference with Gül and Pesendorfer (2012) is that the SIGs can provide information only once to the voter.

prior.

4. If $l_o^Q = 1$, the voter receives a signal of the state of the world: $w_1 \in \{\underline{\delta}, \overline{\delta}\}$. P observes the signal.

5. P decides whether to continue the war of information: $l_o^P \in \{0, 1\}$:

   i. If P stops the war of information ($l_o^P = 0$), the voter sides with the decision-maker or SIG according to her posterior after observing w_1;

   ii. If it continues the war of information ($l_o^P = 1$), the representative voter receives a second signal $w_2 \in \{\underline{\delta}, \overline{\delta}\}$ and she sides with the decision-maker or Q according to her posterior after observing w_1 and w_2.

The outcome of the game y depends on the voter's choice. When the voter sides with the decision-maker, her bill passes: $y = b$. When the voter sides with Q, the bill fails and the status quo remains in place: $y = 0$.

In the context of a war of information, the representative voter receives a signal $w_t = \overline{\delta}$ with probability $\rho > 1/2$ when the state of the world is $\overline{\Delta}$ and with probability $1 - \rho$ when the state of the world is $\underline{\Delta}$, $t \in \{1, 2\}$. The voter receives a signal $w_t = \underline{\delta}$ with probability $1 - \rho$ when the state of the world is $\overline{\Delta}$ and with probability ρ when the state of the world is $\underline{\Delta}$, $t \in \{1, 2\}$. Signals are independent conditional on the state of the world. Denote π_0 players' common prior that the state of the world is $\underline{\Delta}$. $\pi_1(w_1)$ is the voter's and the decision-maker's posterior that the state is $\underline{\Delta}$ after observing signal $w_1 \in \{\underline{\delta}, \overline{\delta}\}$. Finally, $\pi_2(w_1, w_2)$ is the voter's posterior that the state is $\underline{\Delta}$ after observing w_1 and w_2 (when this occurs on the equilibrium path). As the equilibrium concept is PBE, the voter's posterior satisfies Bayes' Rule.

The decision-maker, P, and Q's utility functions assume the following respective form (similar to the main text):

$$u^D(y, d) = y \tag{F.1}$$

$$u^P(y, l_i^P, l_o^P; \tau) = \gamma_\tau^P y - l_i^P - c l_o^P \tag{F.2}$$

$$u^Q(y, l_i^Q, l_o^Q; \tau) = -\gamma_\tau^Q y - l_i^Q - c l_o^Q \tag{F.3}$$

As in the main text, γ_τ^J captures the resolve of the SIG, with $\gamma_H^J > \gamma_L^J$. I include inside lobbying by the pro-change SIG for completeness, though it is of no use here since its type is perfectly known.

The utility function of the representative voter is:

$$u_v(y;\omega) = h(\omega)v(y) \tag{F.4}$$

with $v(\cdot)$ continuous and strictly increasing with $v(0) = 0$, and $h(\omega)$ a function with the following properties: $h(\overline{\Delta}) > 0$ and $h(\underline{\Delta}) < 0$. For example, the function $h(\cdot)$ can assume the following form:

$$h(\omega) = \begin{cases} -1 & \text{if } \omega = \underline{\Delta} \\ 1 & \text{if } \omega = \overline{\Delta} \end{cases}$$

To make the problem interesting, I impose some restrictions on the common prior and informativeness of the signals received by the voter:

Assumption F.1. *ρ and π_0 satisfy:*

$$\pi_1(\underline{\delta}) = \frac{\rho\pi_0}{\rho\pi_0 + (1-\rho)(1-\pi_0)} > \frac{h(\overline{\Delta})}{h(\overline{\Delta}) - h(\underline{\Delta})} > \pi_0$$

The first inequality in Assumption F.1 states that, after receiving a signal $w_1 = \underline{\delta}$, the voter prefers the status quo to any bill b and so always sides with the SIG supportive of the status quo absent additional information. This assumption is satisfied when the prior is not too biased in the decision-maker's direction (π_0 is not too low) and the signal is sufficiently informative (ρ is sufficiently high). When this inequality does not hold, the SIG supportive of the status quo never engages in outside lobbying and the decision-maker always chooses $b = 1$. The second inequality implies that the prior is favorable to the decision-maker. Absent additional signal, the voter sides with the decision-maker.

Notice that if Q engages in outside lobbying (starts a war of information) and the voter receives signal $w_1 = \overline{\delta}$, P has no incentive to continue the war of information. Indeed, $\pi_1(\overline{\delta}) < \pi_0 < \frac{h(\overline{\Delta})}{h(\overline{\Delta})-h(\underline{\Delta})}$ so the voter always sides with the decision-maker absent additional information. Providing information to the voter is costly for P and does not change the voter's decision.[11]

The following notations are useful for the analysis of this set-up. First, denote $p_0(NF) := \pi_0\rho + (1-\pi_0)(1-\rho)$ the probability that the voter receives a signal $w_1 = \underline{\delta}$. This is also the ex-ante probability that Q wins the war of information if P stops the war of information after signal $w_1 = \underline{\delta}$. Denote $p_0(F) := \pi_0\rho^2 + (1-\pi_0)(1-\rho)^2$ the ex ante probability (before the start of the war of information) that Q wins the war of information when P continues the war of information after

[11]We have $\pi_2(\overline{\delta}, \overline{\delta}) < \pi_2(\overline{\delta}, \underline{\delta}) = \pi_0 < \frac{h(\overline{\Delta})}{h(\overline{\Delta})-h(\underline{\Delta})}$ so the voter always sides with the decision-maker.

signal $w_1 = \underline{\delta}$. In this last case, denote as well $p_1(\underline{\delta}) := \pi_1(\underline{\delta})\rho + (1-\pi_1(\underline{\delta}))(1-\rho)$ the interim probability that Q wins the war of information given that the voter has received a signal $w_1 = \underline{\delta}$. In what follows, I impose the following assumption:

Assumption F.2. *Q's resolve satisfies: $\gamma_L^Q < \frac{c}{p_0(NF)} < \gamma_H^Q$.*

Assumption F.2 is the equivalent to Assumption 1.(i) and 2.(i) in the context of the war of information. It guarantees that to start a war of information is a strictly dominated strategy for $Q(L)$.

Assumption F.3. *The P's resolve satisfies: (i) $\gamma_L^P < \frac{c}{p_1(\underline{\delta})} < \gamma_H^P$ and (ii) $\frac{\gamma_H^P}{\gamma_H^Q} < \frac{p_0(F)}{1-p_1(\underline{\delta})}$.*

Point i. is the equivalent to Assumption 1.(ii) and 2.(ii) in the context of the war of information (I include $P(L)$ for completeness). Point (ii) is a rewriting of Assumption 3.

In this amended set-up, I obtain that, like in the main text, a separating equilibrium does not exist for all parameter values.

Proposition F.1. *A separating equilibrium exists if and only if:*

$$1 - p_0(F) \leq \frac{c}{\gamma_H^Q p_0(NF)} \quad \text{and} \quad \gamma_L^Q \leq \gamma_H^Q(1-p_0(F)) + \frac{\gamma_H^Q c(p_0(NF) - p_0(F))}{\gamma_H^Q p_0(NF) - c}$$

Proof. I only show that the conditions are necessary, sufficiency follows from the usual argument. Suppose that the decision-maker learns Q's type at the lobbying state ($\zeta^Q(H) \neq \zeta^Q(L)$). Upon learning that Q is of low resolve ($\zeta^Q = \zeta^Q(L)$), under Assumption F.3, the decision-maker proposes $b = 1$, which is implemented without outside lobbying activities. Suppose the decision-maker learns that Q has high resolve. The decision-maker chooses between $b = \frac{c}{\gamma_H^Q p_0(NF)}$ and $b = 1$. The decision-maker cannot avoid a war of information if she expects P to answer Q's outside lobbying. Indeed, P engages in outside lobbying only if $\gamma_H^P(1-p_1(\underline{\delta}))b - c \geq 0$ after signal $w_1 = \underline{\delta}$ (the only relevant event from the reasoning above). This requires that the bill satisfies $b \geq \frac{c}{(1-p_1(\underline{\delta}))\gamma_H^P}$. On the other hand, $Q(H)$, anticipating $l_o^P(b, \zeta^P; H) = 1$ (i.e., a continuation of the war of information by $P(H)$) prefers no outside lobbying if and only if $-\gamma_H^Q(1-p_0(F))b - c \leq -\gamma_H^Q b$, or equivalently $b \leq \frac{c}{p_0(F)\gamma_H^Q}$ (note that the relevant winning probability for Q is $p_0(F)$ now). Under Assumption F.3.(ii), these two inequalities cannot be satisfied simultaneously. So upon choosing any b satisfying $b \geq \frac{c}{(1-p_1(\underline{\delta}))\gamma_H^P}$, the decision-maker's expected payoff is $(1-p_0(F))b$, which is maximized for $b = 1$. To avoid outside lobbying by $Q(H)$, the decision-maker must, thus, offer a bill $b = \frac{c}{\gamma_H^Q p_0(NF)}$ (by the usual reasoning).

We can then proceeds along the line of the main text. The existence of a separating equilibrium requires D to compromise upon learning Q has high resolve ($\zeta^Q = \zeta^Q(H)$). For compromise to be a best response, given Assumption F.3.(i), it must be that $\frac{c}{\gamma_H^Q p_0(NF)} \geq 1 - p_0(F)$.
Further, Q's (IC) constraints must be satisfied. $Q(L)$'s (IC) is

$$-\gamma_L^Q \frac{c}{\gamma_H^Q p_0(NF)} - l_i^Q(H) \leq -\gamma_L^Q$$

This implies that $l_i^Q(H) = \gamma_L^Q \left(1 - \frac{c}{\gamma_H^Q p_0(NF)}\right)$.
$Q(H)$'s (IC) is

$$-\gamma_H^Q \frac{c}{\gamma_H^Q p_0(NF)} - l_i^Q(H) \geq -\gamma_H^Q(1 - p_0(F)) - c$$

Again, when $Q(H)$ mimics $Q(L)$, it starts a war of information at the outside lobbying stage. Since the pro-change SIG is of type H and responds, its ex-ante winning probability is $p_0(F)$. After some straightforward, but tedious algebra, I obtain the second necessary condition. $\square$

A separating equilibrium exists only under some parameter values. Basically, like in the main text, $Q(H)$'s resolve must be intermediary.[12] We can extend the reasoning to pooling equilibria to show that (i) under some parameter values, Q incurs positive inside lobbying expenditures on path (as before, this requires that the decision-maker compromises absent additional information) and (ii) under other parameter values, the SIG supportive of the status quo engages in outside lobbying on the equilibrium path (as before, this requires the decision-maker does not compromise absent additional information). Further, for some combination of parameters, pooling equilibria and the separating one described above can co-exist. Hence, like in the main text, I find that inside lobbying expenditures are associated with compromise and outside lobbying with comprehensive reform. The empirical implications are, thus, robust to micro-founding the effect of outside lobbying activities as a war of information.

[12] The conditions, you will notice, are slightly different from the main text (see Lemma 2). Indeed, I assume in this appendix that P is high-resolve, whereas it is low-resolve in the main text. When the pro-change SIG is known to be of low resolve, then the conditions for a separating equilibrium to exist become $1 - p_0(NF) \leq \frac{c}{\gamma_H^Q p_0(NF)} \leq (1 - p_0(NF))\frac{c}{\gamma_L^Q p_0(NF)}$, very much like in the main text.

References

[1] Gül, Faruk and Wolfgang Pesendorfer. 2012. "The War of Information", Review of Economic Studies, 79(2): 707-734.