Online Appendix to
"Political Fear and Loathing on Wall Street"

Date: July 9, 2023

Appendix Contents

A Election Risk Pricing

Consider a tradable instrument that has: (a) an observable price; and (b) a value that depends on the distribution of an underlying asset at a given time in the future, t. For example, let's say that there is a contract paying USD 1 if an event A occurs, and 0 otherwise. The *risk neutral probability* of event $A:\ P_{RN}(A)$ is denoted as

$$\frac{Price\ of\ a\ contract\ paying\ USD\ 1\ if\ A\ occurs}{Price\ of\ a\ contract\ paying\ USD\ 1\ no\ matter\ what}$$

Assuming no arbitrage, $P_{RN}(A)$ satisfies the axioms of probability (its values are strictly positive and they add up to one). If the risk-free interest rate is constant and equal to r, then the price of a contract that pays one dollar at time t if A occurs should be $P_{RN}(A)e^{-rt}$ where $P_{RN}(A)$ denotes expectation with respect to the risk neutral probability. More generally, in the absence of arbitrage, the price of a tradable instrument that pays X at time t should be $E_{RN}(X)e^{-rt}$. In addition,the so-called fundamental theorem of asset pricing states that (assuming no arbitrage) interest-discounted asset prices are martingales with respect to risk neutral probability. Therefore, if a security will be worth X at time t, then its price today should be $E_{RN}(X)e^{-rt}$, where E_{RN} denotes the expectation with respect to the risk neutral probability.[1]

As this simple example shows, the market's forecast of a likely movement in a security's price following an election can be derived from option prices. Let W be a Brownian motion on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The firm's stock price S is assumed to satisfy the stochastic differential equation (SDE)

$$\frac{dS}{S} = \sigma dW + \mu\ dt, \tag{A1}$$

where μ and σ are constants called, respectively, the *drift* and *volatility* of the stock.

Equation A1 may be written as:

$$dS = \sigma S dW + \mu S dt, \tag{A2}$$

with solution

$$S_t = S_0\ exp\ \left[\sigma W_t + \left(\mu - \frac{\sigma^2}{2}\right) t\right]. \tag{A3}$$

[1]The absence of arbitrage is crucial for the existence of a risk-neutral measure. If A and B are disjoint, then $P_{RN}(A \cup B) = P(A) + P(B)$; otherwise, one could: (1) sell (buy) contracts paying 1 if A occurs and 1 if B occurs; (2) buy (sell) a contract paying 1 if $A \cup B$ occurs; and (3) pocket the difference.

Taking the logarithm of (3), we get

$$ln\ S_t = ln\ S_0 + \sigma W_t + \left(\mu - \frac{\sigma^2}{2}\right) t \tag{A4}$$

Let T_e be the election date, and Z_e a random variable representing the jump size of the log stock price after the outcome of the election is revealed. Suppose that Z_e is independent of W. We can now write the process as

$$\frac{dS}{S} = \sigma dW + \mu\ dt + (e^{Z_e} - 1)dN(t), \tag{A5}$$

where $N(t)$ is an indicator function that takes the value of 1 when $t \geq T_e$, and zero otherwise.

To price an option on S under this process, we need to find an equivalent martingale measure and set the option price to the discounted expectation of its value in that measure. Consider a bond B_t that is continuously compounding at the risk-free rate r. The value of this riskless bond is thus e^{rt} at time t.

The expected change of S in a small time interval will be

$$\mu S \Delta t + \mathbb{E}(e^{Z_e} - 1) S \Delta t.$$

For the ratio $\frac{S}{e^{rt}}$ to be a martingale, we need S to grow at the risk-free rate; namely, we need the expected change to be $rS\Delta t$, which implies that

$$\mu + \mathbb{E}(e^{Z_e} - 1) = r. \tag{A6}$$

Therefore, the arbitrage-free price for an European option, O, expiring at time T should be

$$e^{-rt}\mathbb{E}(O(S_T)),$$

with S_T evolving according to (5) with drift given by (6).

The option price can thus be expressed in terms of the risk-neutral probability measure $\mathbb{Q}$ rather than the original probability measure $\mathbb{P}$. We can do this change of measure by using the Girsanov transformation for changing the drift of a Brownian motion (Junghenn 2012: 158-60). Let Z_e be a strictly positive random variable on $(\Omega, \mathcal{F})$ with $\mathbb{E}\{e^{Z_e}\} = 1$. If Ω is finite, the equation

$$\mathbb{Q}(A) = \mathbb{E}(\mathbb{I}_A Z), \quad A \in \mathcal{F} \tag{A7}$$

defines a probability measure $\mathbb{Q}$ on $(\Omega, \mathcal{F})$ such that $\mathbb{Q}(\omega) > 0$ iff $\mathbb{P} > 0$, and $\mathbb{Q}$ is equivalent to $\mathbb{P}$.

Following Leung and Santoli (2014), consider an extension of the Black-Scholes model with a single price jump occurring immediately after the election. Suppose that Z_e is normally distributed, then $\mathbb{E}\{e^{Z_e}\} = 1$, implying that $Z_e \sim N\left(-\frac{\sigma_e^2}{2}, \sigma_e^2\right)$, and that the election price *jump* can be parametrized by σ_e. For $T \geq T_e$, then

$$log\ \frac{S_T}{S_t} \sim N\left(\left(r - \frac{\sigma^2}{2} - \frac{\sigma_e^2}{2(T-t)}\right)(T-t), \sigma^2(T-t) + \sigma_e^2\right), \tag{A8}$$

and the price of a European call with strike K and maturity T is given by

$$C(t, S_t) = C_{BS}\left(T-t, S_t; \sqrt{\sigma^2 + \frac{\sigma_e^2}{T-t}}, K, r\right), 0 \leq t < T_e \tag{A9}$$

where $C_{BS}(\tau, S; \sigma, K, r)$ represents the usual Black-Scholes formula with time to maturity τ and spot price S. Given this price formula, the implied volatility (IV) can be expressed as the deterministic function:

$$I(t; K, t) = \begin{cases} \sqrt{\sigma^2 + \frac{\sigma_e^2}{T-t}} & \text{if } 0 \leq \text{t} < T_e \\ \sigma & \text{if } T_e \leq t < T, \end{cases} \tag{A10}$$

where σ is the diffusive volatility.

As Dubinsky et. al (2019) note, this extension of the Black-Scholes model has two important implications: (1) IVs increases continuously prior to release of new information; (2) IV discontinuously falls after the information is released. Therefore, there should be a detectable pattern in the changes of implied volatility before and after elections. More importantly, these patterns suggest two estimators of σ_e, one based on the IV term structure and the other based on IV dynamics.

Given two options with time to maturity T_1 and T_2 ($T1 < T2$) and an election prior to maturity, then $\sigma^2_{t,T_1} > \sigma^2_{t,T_2}$ and σ_e is given by

$$\sigma^2_{e,term} = \frac{\sigma^2_{t,T_1} - \sigma^2_{t,T_2}}{T_1^{-1} - T_2^{-1}}.$$

Alternatively, let σ_{IV,t_1} and σ_{IV,t_2} represent the implied volatilities of two options at times t_1 and t_2, with identical maturity at time T. Assuming that the election outcome is revealed after the close on date t_1 (or before the open on the next trading date, t_2), then the annualized implied variance should be $\sigma^2 + \frac{\sigma_e^2}{T-t}$ just before the election, and σ^2 after the election. Applying Equation A10 and solving for σ_e^2, one can obtain the following estimator of electoral risk based on the post-electoral decrease in implied volatility:

$$\sigma_{e,time} = \sqrt{(T-t)(\sigma_{IV,t_1}^2 - \sigma_{IV,t_2}^2)} \quad .$$

B Hypothetical Variance Swap Contract

Consider the following hypothetical variance swap contract. One party agrees to pay a fixed amount at maturity (i.e. the price of the variance swap), in exchange for a payment equal to the sum of squared daily log returns of the S&P 500.

The payoff, $p_{\tau,m}$ at expiration of a contract initiated at time τ and with maturity m, and Strike Price, $SP_{\tau,m}$, is given by:

$$p_{\tau,m} = VN_{\tau,m} \times [(RVS_{\tau,m})^2 - (SP_{\tau,m})^2] \tag{B1}$$

where VN, the Variance Notional, is determined as:

$$VN_{\tau,m} = \frac{Vega\ Notional}{2 \times SP_{\tau,m}},$$

and the Realized Volatility Strike of the S&P 500 is calculated using the formula:

$$RVS_{\tau,m} = \sqrt{\frac{252 \times \sum_{i=\tau+1}^{\tau+m} \left(ln \frac{Index_i}{Index_{i-1}} \right)^2}{m}} \times 100.$$

C One-Step Binomial Pricing Framework

Let O_t be a European option on an underlying asset with a current price S_t. Denote the option's strike by K, its expiry by T, and the election day as T_e, where $t < T_e < T$. An option bought ahead of the date when the identity of the winning candidate is revealed (at time $t \leq T_e$) will give someone the right to trade the underlying at a strike price of K after the election takes place. To keep things simple, I assume that the underlying asset will pay no cash dividends during the life of the option. I also ignore transaction costs, margin requirements, and taxes.

Suppose that at expiration, the spot price of the underlying asset can only have two possible values. With probability q, it can increase, and become $S_T^u = uS_t$, where $u > 1$; and with probability $(1-q)$, it can decrease, and become $S_T^d = dS_t$, where $d < 1$. Therefore, for $S_T = \{S_T^u, S_T^d\}$, the option's value at expiration will be $C_T = max(0, S_T - K)$ in the case of a call, and $P_T = max(0, K - S_T)$ in the case of a put. To avoid riskless arbitrage opportunities, O_T should be equal to the value of O_t invested for the time interval $\Delta = T - t$ at the risk-free interest rate, $O_T = O_t e^{r\Delta}$, or equally, $O_t = O_T e^{-r\Delta}$.

As Cox, Ross and Rubinstein (1979) show, the value of the option O_t can be calculated as:

$$O_t = e^{-r\Delta}[pO^u + (1-p)O^d], \tag{C1}$$

where O^u is the value of the option at expiration if the price of the underlying goes to uS_t, O^d is the value of the option at expiration if the price of the underlying goes to dS_t, and:

$$p = \frac{e^{r\Delta} - d}{u - d}. \tag{C2}$$

There are many plausible available choices with regard to the parameters u and d. For instance, the price of the underlying asset could either increase by 1.8% or decrease by 1.5%. Following Cox, Ross, and Rubinstein (1979), I adopt the parametrization, $u = e^{\sigma\sqrt{\Delta}}$, where σ is the volatility of the underlying asset. Assuming that the product of the up move multiplier and the down move multiplier is 1, then $d = e^{-\sigma\sqrt{\Delta}}$.

Equation (C2) can help us elucidate the relationship between option prices and electoral forecasts. First, notice that, as long as the interest rate is positive, then $d < e^{r\Delta} < u$. Therefore, p has the properties of a probability: it will always be greater than zero and less than one. Second, as Cox, Ross, and Rubinstein (1979) note, p is the value that would justify the current price of the underlying asset, S_t, in a risk-neutral world. In the context of a national election examined here, we can interpret p as the probability that the spot price will increase to S_T^u at time $T_e < T$.

So, consider a presidential election between two candidates, L and R. Assume that on day $t < T_e$ during the campaign, the option O_t expires in one month, the riskless interest rate is 2.5%, and the volatility of S_t is 20%. According to those inputs, and using equation (7), $p = 0.68$. Suppose the market expects the underlying asset to increase (decrease) in value if R wins (loses). To the extent that asset prices are sensitive to electoral outcomes, then R's probability of winning, as predicted by public opinion polls should be roughly 68%. Otherwise, the behavior of options prices would be inconsistent with the information in public opinion polls.