



## CALCULATION OF THE VALUE WHICH FOREST LAND AND IMMATURE STANDS POSSESS FOR FORESTRY

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*Republished with permission from Commonwealth Forestry Association. The original article "Berechnung des Wertes welchen Waldboden sowie noch nicht haubare Holzbestände für die Waldwirtschaft besitzen" was published in Allgemeine Forst- und Jagd-Zeitung, vol.15, 1849.*

*The following translation was published in Gane, M. (editor) and Linnard, W. (transl.): "Martin Faustmann and the Evolution of Discounted Cash Flow: Two Articles from the Original German of 1849." Commonwealth Forestry Institute, University of Oxford, Institute paper 42, 1968.*



In the October number of this journal (page 361) Herr Oberförster von Gehren developed his views 'on determination of the money value of bare forest land', and explained the basis on which his method of calculation rests; simultaneously he promoted extensive discussion of this important subject. As we do not agree completely with Herr von Gehren's views and method of calculation, and also as we think it of scientific interest and practical value to present another analysis of the subject, we shall now discuss this theme, comparing our views with those of Herr von Gehren. However, at the start we note expressly that in our calculation we shall consider the subject from the forester's standpoint; i.e. we shall only calculate the value which bare forest land possesses when in forestry use. Also, from the forestry point of view, in order to present a complete solution we must extend our analysis to immature stands. We must not calculate the value of such stands as represented by the sale price of their present timber content, but by their value as determined from their exploitation when mature, i.e. by the value which attaches to them in the forestry system or by their age class in the rotation (management may be intermittent or sustained). The practical importance of this calculation is easy to see. From it we obtain the necessary information on the forest value in such cases as voluntary and enforced sales (expropriations), destruction of the forest by fire, insects, man, etc., and assessment of the most advantageous silvicultural system and

length of rotation. Moreover the method itself is simple and its correctness can be demonstrated by various tests. We shall use Herr von Gehren's numerical example, but we shall calculate with compound interest, as we consider this to be more correct.

For the purpose of our calculation we shall distinguish first between *intermittent* and *sustained* yield management, second the area may be *bare of timber* or *carrying a stand*, and finally the area may be regarded as either a *complete working section* or as a *part of one*. By 'working section' we mean an area of forest which is under the same silvicultural system and rotation and therefore can be regarded as a uniform whole for yield calculation. The distinction is necessary because the money value of a working section does not seem to change in a direct relationship with increase or decrease in its size. Also, we must distinguish between areas which are currently timberless and those carrying a stand because the forest owner can suffer pecuniary loss by the clearing and sale of an immature stand, and therefore, besides the value of the land, may justifiably require compensation for this from the buyer. This latter point is also the reason why we included consideration of the value of immature stands in our analysis. Finally, the justification of the distinction between intermittent and sustained yield management, for valuation purposes, is self-evident, and was also recognised by Herr von Gehren.

## I. INTERMITTENT YIELD MANAGEMENT

We are considering the case in which the whole area is successively felled and planted to create an even-aged stand. Therefore we do not need to consider whether the area in question should be regarded as a whole or as a part of a working section. For by enlarging or reducing such an area, the management and hence the conditions of yield calculation do not alter — only the size of the income and expenditure will change in direct relation to the extent of the area.

### A. Forest land which is bare of trees

Herr von Gehren's calculation is correct. But we also want to calculate the size of the annual land rent (net money yield), which by simple capitalisation gives the land value.

We express it generally in a formula, and thus remove one difference between forest land and agricultural land (viz. the timing of their yields and expenditures) which, according to Herr von Gehren, makes the value calculation of the former more difficult than that of the latter.

### *Forest land rent formula*

For a mathematical solution we must restate our problem thus: what is the net annual money yield which bare forest land can provide in perpetuity? This implies *normal* forest yields, in so far as it is within the power of the forester to achieve them. Two different approaches are possible, but they lead to the same results.

(1) All the incomes and expenditures occurring in the first rotation are converted into equal annual sums, and by subtracting the latter from the former we obtain what we want. Confining the calculation to one rotation is sufficient because it can be assumed that all other rotations will be completely identical as regards income and expenditure.

Let the cash value of the final yield	= $E$ ,
the value of yields from thinnings during the rotation	= $D$ ,
and the value of the latter compounded to the end of the rotation	= $rD$ ;
then the size of the plantation costs necessary at the start of the rotation	= $C$ ,
the annual expenditure for administration, protection, etc.	= $A$ ;
the rotation length	= $u$ ,
the interest rate per cent	= $p$ ,
and finally the annual land rent	= $R$ .

To simplify the calculation we can compound to the end of the rotation all the incomes and expenditures which do not occur annually, and then convert their difference into an equivalent annual money rent which is paid for the first time at the end of the first and for the last time at the end of the last year of the rotation, i.e. as often as there are years in the rotation.

At the end of the rotation, the value of the final yield is  $E$ , of the thinnings  $rD$  and the plantation costs  $C(1.0p)^u$ , the latter value being negative. Therefore the value to be

converted into the annual rent mentioned above is:

$$E + rD - C(1 \cdot 0p)^u$$

If we call this annual rent  $x$ , and we calculate the capital value, which this possesses at the end of the rotation, then we get:

$$x(1 \cdot 0p)^{u-1} + x(1 \cdot 0p)^{u-2} + x(1 \cdot 0p)^{u-3} + \dots \\ \dots x(1 \cdot 0p) + x = E + rD - C(1 \cdot 0p)^u$$

The left hand side of this equation can be summed using the formula for a geometric series, i.e.

$$S = \frac{a(q^n - 1)}{q - 1}$$

giving

$$\frac{x[(1 \cdot 0p)^u - 1]}{0 \cdot 0p} = E + rD - C(1 \cdot 0p)^u$$

hence

$$x = \frac{0 \cdot 0p}{(1 \cdot 0p)^u - 1} [E + rD - C(1 \cdot 0p)^u]$$

However, from the value of this annual rent ( $x$ ) the annual expenditure for administration, etc. ( $A$ ), must be deducted in order to obtain the value of the annual land rent ( $R$ ).

Thus the required forest land rent formula is :

$$R = \frac{0 \cdot 0p}{(1 \cdot 0p)^u - 1} [E + rD - C(1 \cdot 0p)^u] - A$$

(2) We arrive at the same formula if we reduce to the present all the incomes and expenditures occurring until infinity, and seek the annual interest income from the difference of these present values.

The income ( $E + rD$ ) occurs for the first time after  $u$  years, and recurs every  $u$  years in perpetuity; therefore its present value is:

$$K = \frac{E+rD}{(1 \cdot 0p)^u} + \frac{E+rD}{(1 \cdot 0p)^{2u}} + \frac{E+rD}{(1 \cdot 0p)^{3u}} + \dots + \frac{E+rD}{(1 \cdot 0p)^\infty}$$

$$= \frac{E+rD}{(1 \cdot 0p)^u} \div \left[ 1 - \frac{1}{(1 \cdot 0p)^u} \right] = \frac{E+rD}{(1 \cdot 0p)^u - 1}$$

(According to the summation formula for an infinite geometric series,  $S = \frac{a}{1-q}$ ).

The establishment cost occurs at the start of the first year and recurs every  $u$  years. Its present value ( $K'$ ) may therefore be calculated like the sum ( $K$ ) of the income ( $E + rD$ ) series, by substituting  $C$  for  $E + rD$ , and adding  $C$  to the value of  $K$ ; i.e.

$$K' = C + \frac{C}{(1 \cdot 0p)^u - 1} = \frac{C(1 \cdot 0p)^u}{(1 \cdot 0p)^u - 1}$$

The expenditure ( $A$ ) for administration, etc., occurs at the end of each year, always in the same amount; therefore its present value is

$$K'' = \frac{A}{0 \cdot 0p}$$

Therefore the net present value, whose annual interest-income corresponds to the annual land rent, is:

$$K - K' - K'' = \frac{E+rD}{(1 \cdot 0p)^u - 1} - \frac{C(1 \cdot 0p)^u}{(1 \cdot 0p)^u - 1} - \frac{A}{0 \cdot 0p}$$

$$= \frac{1}{(1 \cdot 0p)^u - 1} [E+rD - C(1 \cdot 0p)^u] - \frac{A}{0 \cdot 0p}$$

Accordingly, the annual interest-income, i.e. the forest land rent, is expressed here, as in the previous case, by the formula:

$$R = \frac{0 \cdot 0p}{(1 \cdot 0p)^u - 1} [E+rD - C(1 \cdot 0p)^u] - A$$

From this it is easy to find the value of the bare forest land  $B$  by simple capitalisation of the annual forest land rent:

$$B = \frac{R}{0 \cdot 0p} = \frac{E + rD - C(1 \cdot 0p)^u}{(1 \cdot 0p)^u - 1} - \frac{A}{0 \cdot 0p}$$

This formula refers to a single unit of area (morgen or acre). If the number of acres is called  $F$ , then the total land rent =  $R \cdot F$ . and the land value =  $B \cdot F$ .

Now let us apply this formula to the numerical example chosen by Herr von Gehren. During the rotation the following money incomes occur with their values compounded to the end of the rotation period, when  $p = 4$  and  $u = 80$ :

Compounded value at the end of the rotation					
	Year of rotation	Money yield in that year	Formula	Factors <sup>1</sup>	Compounded value
Thin - nings	20	1000 Pf.	$1000(1 \cdot 04)^{60}$	$= 1000 \times 10 \cdot 51962$	$= 10,520 \text{ Pf.}$
	30	1880 Pf.	$1880(1 \cdot 04)^{50}$	$= 1880 \times 7 \cdot 10668$	$= 13,359 \text{ Pf.}$
	40	1380 Pf.	$1380(1 \cdot 04)^{40}$	$= 1380 \times 4 \cdot 80102$	$= 6625 \text{ Pf.}$
	50	1160 Pf.	$1160(1 \cdot 04)^{30}$	$= 1160 \times 3 \cdot 23339$	$= 3751 \text{ Pf.}$
	60	1955 Pf.	$1955(1 \cdot 04)^{20}$	$= 1955 \times 2 \cdot 19112$	$= 4284 \text{ Pf.}$
	70	2433 Pf.	$2433(1 \cdot 04)^{10}$	$= 2433 \times 1 \cdot 48024$	$= 3601 \text{ Pf.}$
Total		9808 Pf.			Total 42,140 Pf.
Final yield <sup>2</sup>	80	42,379 Pf.			42,379 Pf.

Accordingly  $D = 9808$ ;  
and  $rD = 42,104$

$$\left[ r = \frac{42,140}{D} = \frac{42,140}{9808} = 4 \cdot 2965 \right]$$

and  $E = 42,379$ .

<sup>1</sup> We have taken the factors from the tables for calculating interest appended to Herr von Gehren's *Anleitung zur Waldwerthberechnung* (Guide to forest value calculation (Kassel, 1835).

<sup>2</sup> We have taken the whole yield at stand age 80 as the final yield.

The plantation costs at the start of the first year of the rotation are 540 Pf.; thus  $C = 540$  and  $C(1.0p)^u =$

$$540 (1.04)^{80} = 540 \times 23.04979 = 12,447.$$

Herr von Gehren did not bring into his calculation the annual costs for administration, etc., as he put them equal to the annual income from minor products, which was likewise left out of the calculation. We take the former to be 48 Pf. or  $A = 48$ .

If we substitute these values then we get:

$$\begin{aligned} R &= \frac{0.04}{(1.04)^{80} - 1} (42,379 + 42,140 - 12,447) - 48 \\ &= 0.04 \times 0.04535 \times 72,072 - 48 = 130.72 - 48 \\ &= 82.72 \text{ Pf.} \end{aligned}$$

Accordingly, the annual land rent is 83 Pf. or 6 Sgr. 10.72 Pf. (25 Kreuzer), and the land value works out at

$$8272 \times \frac{100}{4} = 2068 \text{ Pf., or 5 Thlr. 23 Sgr. 4 Pf. (10 fl. 3 kr.).}$$

Herr von Gehren obtained a land value about three times higher than this because he calculated with geometric mean interest and we used compound interest. On page 364 Herr von Gehren also gives the land value calculated by compound interest, viz. 9 Thlr. 2 Sgr. 3 Pf., which is different from ours because he omitted the costs of administration, etc. (= 48 Pf.); the further difference of 1 Pf. is due to the decimal places being ignored. In this article we do not wish to investigate whether this small land value is caused by estimates of yields and timber prices which are too low, or by an incorrect ratio of assortments, because here we are only concerned with principles. If it is assumed that they are correct, then this low result indicates that the rotation chosen was too long.

We do not set very great value on our land-rent formula for this present application, but it does have great value for the next and subsequent applications.

## *B. Land currently carrying a stand*

The land value is naturally just as great in this case as in the previous one. However, if land carrying wood is sold, and if the vendor therefore has to sell the present stand at its current market value, then the buyer should compensate the vendor for the loss which results, as well as for the land value. If the existing forestry is lucrative, i.e. the selected rotation age is the most advantageous financially, then a money loss will be incurred by the forest owner who cuts an immature stand; otherwise it would indicate that earlier felling was financially preferable and the selected rotation length was excessive, i.e. not the most attractive. Before maturity, the stands (if they already possess a market value) should be regarded as a product of the land which is not fully ripe, the harvesting of which causes loss to the forest owner in the same way as cutting wheat before time does to the farmer. Just think of a Scots pine stand say 10 years old, whose present market value is indisputably smaller than that which it possesses as the bearer of the future final yield. The latter is the economic value of the stand which we can express by a money capital, just like the economic value of the land. Several viewpoints can lead us to an algebraic expression for this; we shall use three different methods, in order to check on the results obtained and to demonstrate their correctness.

### *Formula for the stand value*

Let the age of the stand be  $n$  years, and to start with let us assume that it is felled before any thinnings have taken place. Also, let the stand be normal.

(1) Obviously a forest owner cannot expect more for a stand than will completely compensate him for the  $n$  years' land rent not drawn and the expenditure disbursed (plantation and administration costs, etc.). The owner could have lent out the capital corresponding to the land value instead of putting it into forestry: then he would have drawn the land rent as interest on capital and would have saved the plantation and administration costs. However, the sum of the annual land rentals ( $nR$ ), the plantation expenditure ( $C$ ) and the annual administration costs ( $nA$ ), viz. ( $nR + C + nA$ ) does not provide him with a satisfactory stand valua-



tion; he is also entitled to compensation for the loss of interest which he has suffered because he has forgone the annual land rent and has spent money which (from the moment of its disbursement) ceases to be available for lending as would have been the case if he had been dealing with interest-producing money capital instead of rent-producing forest land. From this it is clear that the stand value ( $H$ ) is formed by the capital value at the end of the  $n$ th year obtained from the following expressions:

(a) *The annual land rent ( $R$ )*. This occurs at the end of the first year for the first time, and then annually for  $n$  years at the same level. Therefore its capital value is:

$$K = R(1 \cdot 0p)^{n-1} + R(1 \cdot 0p)^{n-2} + \dots + R(1 \cdot 0p) + R = \frac{R[(1 \cdot 0p)^n - 1]}{0 \cdot 0p}$$

(b) *The annual expenditures for administration, etc. ( $A$ )*. They occur in the same way as the annual land rent; therefore their capital value is:

$$K' = A(1 \cdot 0p)^{n-1} + A(1 \cdot 0p)^{n-2} + \dots + A(1 \cdot 0p) + A = \frac{A[(1 \cdot 0p)^n - 1]}{0 \cdot 0p}$$

(c) *The plantation costs ( $C$ )*. They occur once, at the start of the first year, and so their capital value is:

$$K'' = C(1 \cdot 0p)^n$$

From the above, however,  $H$  must equal  $K + K' + K''$ , i.e.

$$H = \frac{R[(1 \cdot 0p)^n - 1]}{0 \cdot 0p} + \frac{A[(1 \cdot 0p)^n - 1]}{0 \cdot 0p} + C(1 \cdot 0p)^n$$

If, instead of  $R$ , we substitute its value found previously, we obtain :

$$\begin{aligned}
 \frac{R[(1 \cdot 0p)^n - 1]}{0 \cdot 0p} &= \left\{ \frac{0 \cdot 0p}{(1 \cdot 0p)^u - 1} [E + rD - C(1 \cdot 0p)^u] - A \right\} \\
 &\quad \times \frac{(1 \cdot 0p)^n - 1}{0 \cdot 0p} \\
 &= \frac{(1 \cdot 0p)^n - 1}{(1 \cdot 0p)^u - 1} [E + rD - C(1 \cdot 0p)^u] - \frac{A[(1 \cdot 0p)^n - 1]}{0 \cdot 0p}.
 \end{aligned}$$

When this value is inserted into the expression for  $H$ , then:

$$+ \text{ and } - \frac{A[(1 \cdot 0p)^n - 1]}{0 \cdot 0p} \quad \text{cancel out, and so}$$

$$H = \frac{(1 \cdot 0p)^n - 1}{(1 \cdot 0p)^u - 1} [E + rD - C(1 \cdot 0p)^u] + C(1 \cdot 0p)^n.$$

Finally, if we eliminate  $C$  as a common factor, then :

$$\begin{aligned}
 H &= (E + rD) \frac{(1 \cdot 0p)^n - 1}{(1 \cdot 0p)^u - 1} + C \left[ (1 \cdot 0p)^n - \frac{(1 \cdot 0p)^n - 1}{(1 \cdot 0p)^u - 1} (1 \cdot 0p)^u \right] \\
 &= (E + rD) \frac{(1 \cdot 0p)^n - 1}{(1 \cdot 0p)^u - 1} + C \frac{(1 \cdot 0p)^{u+n} - (1 \cdot 0p)^n - (1 \cdot 0p)^{u+n} + (1 \cdot 0p)^u}{(1 \cdot 0p)^u - 1} \\
 &= (E + rD) \frac{(1 \cdot 0p)^n - 1}{(1 \cdot 0p)^u - 1} + C \frac{(1 \cdot 0p)^u - (1 \cdot 0p)^n}{(1 \cdot 0p)^u - 1}
 \end{aligned}$$

which is the required formula for the stand value.

If thinnings have already taken place, then a part of the land rent has been drawn in them; its capital value at stand age  $n$  should therefore be deducted from the value of  $H$ . Instead of this, without prejudice to the correctness of the result, greater conformity with the previous and following formulae can be achieved by deducting the thinnings still

to come in the rest of the first rotation compounded to the end of the rotation ( $= r'D'$ ) from the total thinnings, also compounded to the end ( $= rD$ ), discounting this difference ( $= rD - r'D'$ ) to the present

$$\left( = \frac{rD - r'D'}{(1 \cdot 0p)^{u-n}} \right),$$

and finally deducting this from the value of  $H$ .

Thus:

$$H = (E + rD) \left[ \frac{(1 \cdot 0p)^n - 1}{(1 \cdot 0p)^u - 1} \right] + \frac{C[(1 \cdot 0p)^u - (1 \cdot 0p)^n]}{(1 \cdot 0p)^u - 1} - \frac{rD - r'D'}{(1 \cdot 0p)^{u-n}}$$

This is the most general form of the formula for the stand value of fully stocked stands.

(2) Imagine all incomes and expenditures during the first rotation period ( $u$ ) converted into an annual rent for  $u$  years; then the forest owner can determine his position as follows: "The  $n$ -years' rent income is represented in my  $n$ -years old stand. However, as I have not yet received this rent, I must be compensated by its present capital value, after allowance (in the form of negative rents) for expenditures already paid out. I cannot require compensation for the previous annual expenditure on administration, etc., but the plantation-costs expenditure occurred once and for all at the start of the rotation, and therefore all annual rents corresponding to this expenditure were paid out in advance for the whole rotation. It is evident that I must be compensated for these advance rents in respect of the remaining years of the rotation ( $u - n$ ). If thinnings have already taken place, then their present capital value

$$\left( \frac{rD - r'D'}{(1 \cdot 0p)^{u-n}} \right)$$

should be deducted." Accordingly, if the annual rent corresponding to the income ( $E + rD$ ) is called  $x$ , and the annual rent corresponding to the plantation costs ( $C$ ) is called  $y$ , then, using the method of calculation in section I, A, (1):

$$x = (E + rD) \frac{0 \cdot 0p}{(1 \cdot 0p)^u - 1}$$

$$y = C(1 \cdot 0p)^u \frac{0 \cdot 0p}{(1 \cdot 0p)^u - 1}$$

At the end of the  $n$ th year, the capital value  $K$  of an annual rent  $x$  owed for  $n$  years is:

$$\begin{aligned} K &= x(1 \cdot 0p)^{n-1} + x(1 \cdot 0p)^{n-2} + \dots + x(1 \cdot 0p) + x \\ &= \frac{x[(1 \cdot 0p)^n - 1]}{0 \cdot 0p} = (E + rD) \left[ \frac{(1 \cdot 0p)^n - 1}{(1 \cdot 0p)^u - 1} \right] \end{aligned}$$

when the expression for  $x$  is inserted.

The capital value  $K'$  of an expenditure rent  $y$  paid for  $(u - n)$  years at the start of the  $(u - n)$ th year is :

$$\begin{aligned} K' &= \frac{y}{1 \cdot 0p} + \frac{y}{(1 \cdot 0p)^2} + \dots + \frac{y}{(1 \cdot 0p)^{u-n-1}} + \frac{y}{(1 \cdot 0p)^{u-n}} \\ &= \frac{y}{1 \cdot 0p} \left[ \frac{\left( \frac{1}{1 \cdot 0p} \right)^{u-n} - 1}{\frac{1}{1 \cdot 0p} - 1} \right] = \frac{y[(1 \cdot 0p)^u - (1 \cdot 0p)^n]}{(1 \cdot 0p)^u \times 0 \cdot 0p} \\ &= \frac{C[(1 \cdot 0p)^u - (1 \cdot 0p)^n]}{(1 \cdot 0p)^u - 1} \end{aligned}$$

when the expression for  $y$  is substituted.

Therefore we get:

$$\begin{aligned} H &= K + K' - \frac{rD - r'D'}{(1 \cdot 0p)^{u-n}} \\ &= (E + rD) \left[ \frac{(1 \cdot 0p)^n - 1}{(1 \cdot 0p)^u - 1} \right] + \frac{C[(1 \cdot 0p)^u - (1 \cdot 0p)^n]}{(1 \cdot 0p)^u - 1} - \frac{rD - r'D'}{(1 \cdot 0p)^{u-n}} \end{aligned}$$

as in the previous case.

(3) The difference between the capital values of all the incomes and expenditures, which occur until infinity in a forest, gives the value of the forest. This forest value, which we shall call  $W$ , comprises the land value and the stand value ( $B$  and  $H$  respectively), viz.  $W = B + H$ , and hence  $H = W - B$ .

Therefore, let us find the forest value  $W$ ; the land value  $B$  is known. We shall first convert all incomes and expenditures occurring to infinity to their capital values at the end of the first rotation, and then go on to discount them to the present.

The incomes outstanding in respect of the rest of the first rotation are worth  $E + r'D'$  at the end of this period ( $r'D'$  is the value of the thinnings that have still to be made, compounded to that time).

Of the expenditures, only the annual ones for administration, etc. ( $A$ ) during  $(u - n)$  years remain to be considered; their value at the end of  $(u - n)$  years is :

$$K = A(1.0p)^{u-n-1} + A(1.0p)^{u-n-2} + \dots + A(1.0p) + A = \frac{A[(1.0p)^u - (1.0p)^n]}{(1.0p)^n \times 0.0p}$$

After this first rotation we have an area of forest land which is bare of trees. To find and take into account the net value which this area possesses at the end of the first rotation, we can proceed in the same way as was used to find the net land value  $B$  in section I, A, (2):

$$B = \frac{E + rD - C(1.0p)^u}{(1.0p)^u - 1} - \frac{A}{0.0p}$$

therefore the value at the end of the present rotation of all the net incomes accruing in the forest is:

$$\begin{aligned} K' &= (E + r'D') - \left[ \frac{A[(1.0p)^u - (1.0p)^n]}{(1.0p)^n \times 0.0p} \right] + \left[ \frac{E + rD - C(1.0p)^u}{(1.0p)^u - 1} - \frac{A}{0.0p} \right] \\ &= \frac{E(1.0p)^u + rD - C(1.0p)^u}{(1.0p)^u - 1} + r'D' - \frac{A(1.0p)^{u-n}}{0.0p} \end{aligned}$$

If we discount this value of  $K'$  to the present, by dividing by  $(1 \cdot 0p)^{u-n}$ , then we obtain the forest value:

$$W = \frac{K'}{(1 \cdot 0p)^{u-n}} = \frac{E(1 \cdot 0p)^n + rD(1 \cdot 0p)^{n-u} - C(1 \cdot 0p)^n}{(1 \cdot 0p)^u - 1} + \frac{r'D'}{(1 \cdot 0p)^{u-n}} - \frac{A}{0 \cdot 0p}$$

However:

$$B = \frac{E + rD - C(1 \cdot 0p)^u}{(1 \cdot 0p)^u - 1} - \frac{A}{0 \cdot 0p} \quad \text{and } H = W - B;$$

therefore, by subtracting and splitting the resulting term

$$\frac{rD[(1 \cdot 0p)^{n-u} - 1]}{(1 \cdot 0p)^u - 1} \quad \text{into} \quad \frac{rD[(1 \cdot 0p)^n - 1]}{(1 \cdot 0p)^u - 1} \quad \text{and} \quad -\frac{rD}{(1 \cdot 0p)^{u-n}},$$

then as in the previous case :

$$H = (E + rD) \left[ \frac{(1 \cdot 0p)^n - 1}{(1 \cdot 0p)^u - 1} \right] + \frac{C[(1 \cdot 0p)^u - (1 \cdot 0p)^n]}{(1 \cdot 0p)^u - 1} - \frac{rD - r'D'}{(1 \cdot 0p)^{u-n}}$$

These three approaches all lead to the same result. The third approach also shows us that the land value remains the same whether the area is stocked or not, because the equation  $H + B = W$  and its components are correct, and accord with the value of  $B$  found previously. Also, the age of the stand has no effect, because  $n$  can have any positive actual value. Let us finally test its correctness by a discussion of the equation.

(a) *What is the value of a stand at the rotation age?* We know this already; it is equal to  $E$ , as at this age the stand is mature and therefore its economic value equals its market value. If the formula is correct, then in the present case  $H$  must equal  $E$ . We have the stand age  $n = u$ , and there are no more thinnings; thus  $r'D' = 0$ . By substitution we get:

$$\begin{aligned}
 H &= (E + rD) \left[ \frac{(1 \cdot 0p)^u - 1}{(1 \cdot 0p)^u - 1} \right] + \frac{C[(1 \cdot 0p)^u - (1 \cdot 0p)^n]}{(1 \cdot 0p)^u - 1} - \frac{rD}{(1 \cdot 0p)^{u-n}} \\
 &= E + rD - \frac{rD}{(1 \cdot 0p)^0} = E + rD - rD = E
 \end{aligned}$$

as we set out to prove.

(b) *What is the value of a stand that has just been established, i.e. at the start of the first year of the rotation when its age = 0?* This value is obviously equal to the plantation costs, provided that if the plantation has been destroyed and its value is to be compensated, replanting can be carried out in the same year. Accordingly the formula should show the value  $H = C$  when  $n = 0$  and  $r'D' = rD$  (because all the thinnings have yet to be made). Then:

$$\begin{aligned}
 H &= (E + rD) \left[ \frac{(1 \cdot 0p)^0 - 1}{(1 \cdot 0p)^u - 1} \right] + \frac{C[(1 \cdot 0p)^u - (1 \cdot 0p)^0]}{(1 \cdot 0p)^u - 1} - \frac{rD - rD}{(1 \cdot 0p)^{u-n}} \\
 &= (E + rD) \left[ \frac{1 - 1}{(1 \cdot 0p)^u - 1} \right] + \frac{C[(1 \cdot 0p)^u - 1]}{(1 \cdot 0p)^u - 1} = C
 \end{aligned}$$

as we set out to prove.

By applying this equation, several other interesting forestry truths and principles can also be derived, but this would lead us too far away from our subject.

Let us now calculate the value ( $H$ ) of a ten-year-old stand using the existing numerical example. Thinnings have not yet been made; therefore  $rD = r'D'$ ; also

$$\begin{array}{rcl}
 rD & = & 42,140; \\
 E & = & 42,379; \\
 \hline
 E + rD & = & 84,519;
 \end{array}$$

$C = 540$  and  $n = 10$ ; therefore

$$\begin{aligned}
H &= 84,519 \times \frac{(1.04)^{10} - 1}{(1.04)^{80} - 1} + 540 \times \frac{(1.04)^{80} - (1.04)^{10}}{(1.04)^{80} - 1} \\
&= [(84,519 \times 0.48024) + (540 \times 21.56955)] 0.04535 \\
&= 52,256 \times 0.04535 \\
&= 2369 \text{ Pf.} = 6 \text{ Thl. } 17 \text{ Sgr. } 5 \text{ Pf.} = 11 \text{ fl. } 31 \text{ kr.} \\
&= \text{the stand value.}
\end{aligned}$$

In order to calculate the value of a 65-year stand, we have to insert into the formula for  $H$  the values  $r'D' = 2433$   $(1.04)^{10} = 3601$ ,  $n = 65$ , the values of the other elements remaining as before; thus

$$\begin{aligned}
H &= (42,379 + 42,140) \times \frac{(1.04)^{65} - 1}{(1.04)^{80} - 1} + \frac{540 \left[ \frac{(1.04)^{80} - (1.04)^{65}}{(1.04)^{80} - 1} \right]}{\frac{42,140 - 3601}{(1.04)^{80-65}}} \\
&= [(84,519 \times 11.79873) + (540 \times 10.25106)] \times 0.04535 \\
&\quad - (38,539 \times 0.55526) = 45,475 - 21,389 \\
&= 24,086 \text{ Pf.} = 66 \text{ Thl. } 27 \text{ Sgr. } 2 \text{ Pf.} = 117 \text{ fl. } 5 \text{ kr.} \\
&= \text{the stand value.}
\end{aligned}$$

Now if forest land carrying a 10- or 65-year-old stand is sold then the buyer should compensate the vendor for the amount by which the sum obtained from the cleared stand falls short of 6 Thlr. 17 Sgr. 5 Pf. or 66 Thlr. 27 Sgr. 2 Pf., i.e. by  $H - P$  if we put the sale value of the stand =  $P$ . However, any extra proceeds occurring when  $P > H$  would give the forest owner a clear indication that his management was at fault or the rotation was too high. For example, if the average annual proceeds from felling, viz.  $\frac{42,739}{80} = 530$  Pf., are applied at age 65, then the sale value of the 65-year stand is  $P = 530 \times 65 = 34,450$  Pf. = 95 Thlr. 20 Sgr. 10 Pf. = 167 fl. 28 kr. The sale value of this stand exceeds its economic value by 28 Thlr. 23 Sgr. 8 Pf. = 50 fl. 23 kr., which shows that the



rotation of 80 years is at least 15 years too high and that the forest owner can achieve a temporary advantage by clearing the stand, i.e. he can claim no compensation for prematurely felling it.

The value ( $w$ ) of a stand that is abnormal (i.e. understocked) is calculated by the same principles as a fully stocked stand. We will take particular account of the depressed yields during the first rotation and imagine that the normal state is created after that. Therefore, let us put the last final yield =  $e$  and the reduced thinning yield =  $\bar{r}d$  and let us choose the third approach above: then, at the end of the first rotation, the net value of all incomes occurring from then on to infinity equals the land value  $B$ , and the incomes  $(e + \bar{r}d)$  and the expenditures

$$\frac{A[(1 \cdot 0p)^u - (1 \cdot 0p)^n]}{(1 \cdot 0p)^n \times 0 \cdot 0p}$$

from the first rotation are derived as in the normally stocked stand.

These values must be discounted from the end of the first rotation to the present, in order to obtain the forest value  $w$ . If we take account here of the signs of these two values, then:

$$w = \frac{e + \bar{r}d + B}{(1 \cdot 0p)^{u-n}} - \frac{A}{0 \cdot 0p} \times \frac{(1 \cdot 0p)^{u-n} - 1}{(1 \cdot 0p)^{u-n}}$$

Finally, in order to find the stand value we must deduct the land value from this forest value. The land value, however, is the same as we calculated when imagining normal yields, because one need only fell and regenerate the present stand in order to create fully stocked conditions on the land immediately — i.e. the currently understocked forest stand only influences the value of the present stand. Accordingly:

$$\begin{aligned}
 h &= \frac{e + \bar{r}d + B}{(1 \cdot 0p)^{u-n}} - B - \frac{A}{0 \cdot 0p} \times \frac{(1 \cdot 0p)^{u-n} - 1}{(1 \cdot 0p)^{u-n}} \\
 &= \frac{e + \bar{r}d}{(1 \cdot 0p)^{u-n}} - \frac{B[(1 \cdot 0p)^{u-n} - 1]}{(1 \cdot 0p)^{u-n}} - \frac{A}{0 \cdot 0p} \times \frac{(1 \cdot 0p)^{u-n} - 1}{(1 \cdot 0p)^{u-n}}
 \end{aligned}$$

If we now substitute the value found above :

$$B = \frac{E + rD - C(1 \cdot 0p)^u}{(1 \cdot 0p)^u - 1} - \frac{A}{0 \cdot 0p}$$

then

$$\begin{aligned}
 h &= \frac{e + \bar{r}d}{(1 \cdot 0p)^{u-n}} - \frac{[E + rD - C(1 \cdot 0p)^u]}{(1 \cdot 0p)^u - 1} \times \frac{(1 \cdot 0p)^{u-n} - 1}{(1 \cdot 0p)^{u-n}} \\
 &+ \frac{A}{0 \cdot 0p} \times \frac{(1 \cdot 0p)^{u-n} - 1}{(1 \cdot 0p)^{u-n}} - \frac{A}{0 \cdot 0p} \times \frac{(1 \cdot 0p)^u - (1 \cdot 0p)^n}{(1 \cdot 0p)^u}
 \end{aligned}$$

or

$$h = \frac{e + \bar{r}d}{(1 \cdot 0p)^{u-n}} - \frac{[E + rD - C(1 \cdot 0p)^u]}{(1 \cdot 0p)^u - 1} \times \frac{(1 \cdot 0p)^{u-n} - 1}{(1 \cdot 0p)^{u-n}}$$

For example, if we had an understocked stand aged 65, which in its 70th year will give a thinning worth 1500 Pf. instead of 2433 Pf. and in its 80th year a final yield of 30,000 Pf. instead of 42,379 Pf., then we should insert into the above formula :

$$\begin{aligned}
 e &= 30,000; \quad \bar{r}d = 1500 \times (1 \cdot 04)^{10} = 2220; \\
 E &= 42,379; \quad rD = 42,140; \quad C = 540; \\
 u &= 80, \quad n = 65 \text{ and } p = 4;
 \end{aligned}$$

then

$$\begin{aligned}
 h &= \frac{30,000 + 2200}{(1.04)^{15}} - \left[ 42,379 + 42,140 - 540(1.04)^{80} \right] \\
 &\quad \times \frac{(1.04)^{80} - (1.04)^{65}}{(1.04)^{80} [(1.04)^{80} - 1]} \\
 &= 32,220 \times 0.55526 - 72,072 \times 0.02017 = 17,890 - 1454 \\
 &= 16,436 \text{ Pf.} = 45 \text{ Thlr. } 19 \text{ Sgr. } 8 \text{ Pf.} = 79 \text{ fl. } 54 \text{ kr.} \\
 &= \text{the stand value.}
 \end{aligned}$$

Accordingly, the stand value of the normal stand exceeds that of the understocked stand by 21 Thlr. 7 Sgr. 6 Pf. or 37 fl. 10 kr.

Besides the formulae for land rent, land value and stand value, we have obtained the following important result: *the land value remains the same, whether the area carries a stand or not, whatever the age of the stand, and no matter whether it is fully stocked or abnormal; the difference [in the value of the forest] is attributable solely to differences in the stand value.*

We can only develop our views as to the extent to which the growing stock capital should be taken into account in intermittent management, and whether an addition should be made to the calculated land value to allow for conversion to sustained management, after we have made a mathematical analysis of the latter.

## II. SUSTAINED MANAGEMENT

The method of calculation used by Herr von Gehren cannot be considered correct. He himself saw its inaccuracy, though he only concluded that he got a negative land value. However, the main thing is that Herr von Gehren proposed nothing better in its place, but (again wrongly) just let the forest value serve as the land value. From the forest value ( $W = 44$  Thlr. 25 Sgr. 10 Pf.) found by capitalisation of the net sustained income, he first subtracted the sale value of the growing stock ( $P = 45$  Thlr. 20 Sgr. 3 Pf.), and regarded this difference ( $W - P = B$ ) as the land value. As  $B$  came out negative ( $= -24$  Sgr. 5 Pf.), Herr von Gehren explained that this calculation was wrong, and without further justifica-

tion took  $W = 44 \text{ Thlr. } 25 \text{ Sgr. } 10 \text{ Pf.}$  as the correct land value. It is obvious that  $W$  can never be the correct land value when the land is bare, but only when the land is stocked with trees having a normal ageclass structure. We shall demonstrate the error in the equation  $B = W - P$  when the correct approach is determined.

If we investigate the nature of sustained management we find that it rests on a basis of intermittent management. In the strictest type of sustained management, which we are now considering, annual coupes occur, equal in number to the number of years in the rotation, and stocked with stands of all ages from the youngest to the oldest. Obviously, however, each individual annual coupe, considered by itself, is in intermittent management, for each coupe produces a final yield every  $u$  years and a thinning yield at intervals. Only because  $u$  intermittently managed stands arranged in a normal series of age classes are brought together, does an annual yield occur which is equal to the timber cut from *one* stand during the whole rotation. Sustained management results because, over the whole area of the felling series every year, the same age classes are represented as occur in sequence in an individual annual coupe during the course of a whole rotation. Therefore, whether one takes the intermittent yields from each individual one of the  $u$  coupes or the sustained yield from the sum of their areas, one is bound to get the same results for the land value and the stand value of the whole felling series; thus, the outcome is the same whether one imagines intermittent management of  $u$  individual stands (in a normal age series) or sustained management on the total of these  $u$  felling areas. However, as we saw previously that the land value does not depend on the presence of a stand, or on its age, and that the value of a stand of any age is expressed in the above formula for  $H$ , then it is obvious from the mathematical principle 'the whole is equal to the sum of its parts', that it is correct to say that the land value in sustained management is the same as in intermittent management and to calculate the stand value by the same formula in both cases. Now let us show this with a strict mathematical proof, in which we distinguish between the area when it forms a whole working section and when it is a part of one. To start with we shall consider land carrying a stand.

### A. Land currently carrying a stand

In the following, for the sake of simplicity, we shall ignore thinnings at first. This will not impair the validity of our deductions because what is valid for the final yields must also be valid for the intermediate yields.

We now put forward the claim that *the land value in sustained management is equal to that in intermittent management*, and select the following form of proof. Let us assume a normal forest condition and a forest area which is  $u$  times greater than the area under intermittent management (e.g. = 80 acres), so that the annual cutting area equals the size of the whole area under intermittent management. If the annual yield is  $E$ , and the annual plantation and administration costs etc. are  $C$  and  $uA$  respectively, then the sustained annual net yield =  $E - C - uA$ . By capitalisation of this income, as a perpetual equal annual rent, one finds the economic value of a forest in normal sustained management:

$$W' = \frac{E - C - uA}{0.0p}$$

No one will dispute the correctness of this value, nor that it is equal to the values of the growing stock and of the land. Now let us proceed using the principle that values of the stand and the land are solely determined by the size (allowing for time) of the net yields obtainable from them, and applying this principle to intermittent management. Then the net yields on the individual annual coupes of the sustained management series occur just as was imagined in the case of intermittent management: thus the land value applying here ( $B' = u \times B$ ) and the value of the total growing stock ( $H'$ ) must be equal to the sum of values of the  $u$  individual stands which successively possess all ages from 0 to  $(u - 1)$  years, i.e.  $H' = h + h_1 + h_2 + \dots + h_{(u-2)} + h_{(u-1)}$ . The latter values can be obtained from the formula for  $H$ , if one substitutes these different ages successively instead of  $n$ . If this calculation is correct, then by addition of  $B' + H'$  one must again obtain  $W'$  or  $\frac{E - C - uA}{0.0p}$ .

However,  $u \times B$  is :

$$B' = \frac{u[E - C(1 \cdot 0p)^u]}{(1 \cdot 0p)^u - 1} - \frac{uA}{0 \cdot 0p}$$

Also, in the formula:

$$H = \frac{E[(1 \cdot 0p)^n - 1]}{(1 \cdot 0p)^n - 1} + \frac{C[(1 \cdot 0p)^u - (1 \cdot 0p)^n]}{(1 \cdot 0p)^u - 1}$$

if we separate the constant components from the variable ones, we get:

$$H = \frac{C(1 \cdot 0p)^u - E}{(1 \cdot 0p)^u - 1} + \frac{E - C}{(1 \cdot 0p)^u - 1} \times (1 \cdot 0p)^n$$

from which, by substituting  $T$  for the constant component and  $U$  for the constant factor of the variable component, we obtain  $H = T + U(1 \cdot 0p)^n$ .

If now, we introduce the successive ages in the felling series instead of  $n$ , then we obtain the stand values of the individual annual coupes, and by summing these we can obtain the value of the total growing stock. Accordingly we get:

$$\begin{array}{rcl} h & = & T + U(1 \cdot 0p)^0 \\ h_1 & = & T + U(1 \cdot 0p)^1 \\ h_2 & = & T + U(1 \cdot 0p)^2 \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ h_{(u-2)} & = & T + U(1 \cdot 0p)^{u-2} \\ h_{(u-1)} & = & T + U(1 \cdot 0p)^{u-1} \end{array}$$

Therefore:

$$h + h_1 + h_2 + \dots + h_{(u-2)} + h_{(u-1)} \\ = uT + U \left[ 1 + (1 \cdot 0p)^1 + (1 \cdot 0p)^2 + \dots + (1 \cdot 0p)^{u-2} + (1 \cdot 0p)^{u-1} \right]$$

or

$$H' = uT + \frac{U \left[ (1 \cdot 0p)^u - 1 \right]}{0 \cdot 0p}$$

When the values of  $T$  and  $U$  are reintroduced, then :

$$H' = \frac{u \left[ C(1 \cdot 0p)^u - E \right]}{(1 \cdot 0p)^u - 1} + \frac{E - C}{0 \cdot 0p}$$

The sum of  $B' + H'$  gives :

$$B' + H' = \frac{u \left[ E - C(1 \cdot 0p)^u \right]}{(1 \cdot 0p)^u - 1} - \frac{uA}{0 \cdot 0p} + \frac{u \left[ C(1 \cdot 0p)^u - E \right]}{(1 \cdot 0p)^u - 1} + \frac{E - C}{0 \cdot 0p} \\ = \frac{u \left[ E - C(1 \cdot 0p)^u \right]}{(1 \cdot 0p)^u - 1} - \frac{u \left[ E - C(1 \cdot 0p)^u \right]}{(1 \cdot 0p)^u - 1} + \frac{E - C}{0 \cdot 0p} - \frac{uA}{0 \cdot 0p} \\ = \frac{E - C - uA}{0 \cdot 0p}$$

as we sought to prove.

Since our assertions are correct, it is an easy matter to calculate the land value and stand value in sustained management, *taking account of the thinning yields*. In sustained management the same thinnings occur annually as are made during the whole rotation in an intermittently managed stand; the annual yield in that case is therefore  $E + D$ , and so the forest value is:

$$W' = \frac{E + D - C - uA}{0 \cdot 0p}$$

In intermittent management the full land value is:

$$B = \frac{E + rD - C(1 \cdot 0p)^u}{(1 \cdot 0p)^u - 1} - \frac{A}{0 \cdot 0p}$$

therefore in sustained management

$$B' = u \times B = \frac{u[E + rD - C(1 \cdot 0p)^u]}{(1 \cdot 0p)^u - 1} - \frac{A}{0 \cdot 0p}$$

However, as the difference between forest value and land value gives the value of the growing stock,

$$H' = W' - B' = \frac{E + rD - C - uA}{0 \cdot 0p} - \frac{u[E + rD - C(1 \cdot 0p)^u]}{(1 \cdot 0p)^u - 1} + \frac{uA}{0 \cdot 0p}$$

viz.

$$H' = \frac{u[C(1 \cdot 0p)^u - E - rD]}{(1 \cdot 0p)^u - 1} + \frac{E + D - C}{0 \cdot 0p}$$

Finally, if one takes the size of each intermittently managed area as a unit of area (one acre, etc.), then these expressions become:

$$W' = \frac{E + D - C}{u(0 \cdot 0p)} - \frac{A}{0 \cdot 0p}; \quad B' = \frac{E + rD - C(1 \cdot 0p)^u}{(1 \cdot 0p)^u - 1} - \frac{A}{0 \cdot 0p}$$

and

$$H' = \frac{E + D - C}{u(0 \cdot 0p)} - \frac{E + rD - C(1 \cdot 0p)^u}{(1 \cdot 0p)^u - 1}.$$

Let us now apply these formulae to Herr von Gehren's example. This is again



$$u = 80; \frac{1}{0.0p} = 25$$

$$p = 4; \frac{1}{u(0.0p)} = 0.3125$$

$$E = 42,379; \frac{1}{(1.0p)^{u-1}} = 0.04535$$

$$D = 9808; rD = 42,140$$

$$C = 540; C(1.0p)^u = 12,447$$

$$A = 48; \text{ i.e. per acre:}$$

$$\begin{aligned} W' &= (42,379 + 9808 - 540) 0.3125 - 48 \times 25 \\ &= 16,140 - 1200 = 14,940 \text{ Pf.} \end{aligned}$$

$$\begin{aligned} B' &= (42,379 + 42,140 - 12,447) 0.04535 - 48 \times 25 \\ &= 3268 - 1200 = 2068 \text{ Pf.}^3 \end{aligned}$$

$$\begin{aligned} H' &= -(42,379 + 42,140 - 12,447) 0.04535 \\ &\quad + (42,379 + 9808 - 540) 0.3125 = -3268 + 16,140 \\ &= 12,872 \text{ Pf.} \end{aligned}$$

Per acre, this is:

$$W' = 41 \text{ Thl. } 15 \text{ Sgr. } - \text{ Pf. } = 72 \text{ fl. } 37 \text{ kr.}$$

$$B' = 5 \text{ Thl. } 22 \text{ Sgr. } 4 \text{ Pf. } = 10 \text{ fl. } 3 \text{ kr.}$$

$$H' = 35 \text{ Thl. } 22 \text{ Sgr. } 8 \text{ Pf. } = 62 \text{ fl. } 34 \text{ kr.}$$

By addition of  $B'$  and  $H'$  one in fact obtains the value of  $W'$ . Herr von Gehren found the value of the growing stock per acre to be 45 Thlr. 20 Sgr. 3 Pf., i.e. 9 Thlr, 27 Sgr. 7 Pf. = 17 fl. 21 kr. more than the figure above. However, we have calculated the *economic* value of the growing stock, and Herr von Gehren used its immediate *sale* value; here lies the error in Herr von Gehren's calculation.

Herr von Gehren was quite right in putting the forest value equal to the value of the land plus the value of the growing stock, and in seeking the land value in the difference between the former and the latter. But the error in the

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<sup>3</sup> Herr von Gehren obtained a smaller value, because he did not take into account the annual costs for administration, etc. (=A).

method of calculation was that, in finding the value of the growing stock, he suddenly changed his standpoint. Herr von Gehren calculated both the value of the forest in sustained management and the value of the land in intermittent management by their worth for forestry, but took the growing stock at its immediate sale value; i.e. in the latter case he abandoned the economic approach. According to the theory, the economic value of a stand which has not yet reached rotation age must be greater than its market value, i.e. the outcome of the calculation must be an even greater negative land value than Herr von Gehren obtained. However, this cannot be the case. In the present example the market value of the growing stock is greater (by 17 fl. 21 kr. per acre) than its economic value, which is to say that the envisaged management, i.e. rotation, is disadvantageous, providing the assumed yields, etc., are correct. Moreover, the calculation of the land value can also be approached from another standpoint, as chosen both by us and Herr von Gehren in the intermittent management case, but ignored by him for sustained management. We will use it for sustained management in the following discussion.

### *B. Forest land carrying no timber*

It is clear that here too, as in intermittent management, one can envisage bare forest land and, similarly, can find its value by discounting all net incomes to the present while imagining that (according to forestry convention) a stand structure will be created which leads to normal sustained yield as soon as possible. It is easy to see that, in this calculation, even if  $E$ ,  $D$  and  $A$  remain unchanged, very different results can emerge, caused by possible differences during the transition to the normal state. However there must be one which is the most advantageous or optimum way. First let us make two economic suppositions, then calculate the land value on this basis, and finally see whether the basis is acceptable. In doing this we shall develop the economic principles by analysis, and investigate whether they correspond to the recognised forestry principles. The test here is that the economic conditions should not conflict with sound forestry and also that the land values found should be maxima.

(1) A state of normality and sustained yield can be created after the first rotation, *by planting up only 1 acre each year* during the first rotation; then from the second rotation on, one would have a normal age gradation. This hypothetical form of husbandry, though it accords with sound forestry principles, is clearly less advantageous than intermittent management because a large part of the area must remain unused in the first rotation. Actually the calculation also gives a smaller land value.

On each individual acre of the area under consideration, the fellings, etc., fall due just as in the case of intermittent management, except that they do not occur simultaneously on all the annual coupes; instead they are made gradually by successive fellings from the first year onwards. Felling only occurs immediately on the first coupe and all the others are cut, coupe by coupe, at yearly intervals. Therefore, the land value ( $B$ ), as calculated for intermittent management, is attributable to the individual felling areas in the same time sequence (i.e. from the time that each is planted), and must therefore be discounted from then to the present in order to obtain the total present value of the land ( $B''$ ). Accordingly this is:

$$\begin{aligned} B'' &= B + \frac{B}{(1.0p)^1} + \frac{B}{(1.0p)^2} + \dots + \frac{B}{(1.0p)^{u-2}} + \frac{B}{(1.0p)^{u-1}} \\ &= B \times \frac{(1.0p)^u - 1}{0.0p(1.0p)^{u-1}} \end{aligned}$$

However, if the whole working section was planted up at the same time, and managed intermittently, its land value would be  $uB$ . Provided  $uB > B''$  then the difference

$$uB - B'' = B \left[ u - \frac{(1.0p)^u - 1}{0.0p(1.0p)^{u-1}} \right]$$

and must be positive. It is obvious that this is the case for all positive values of  $u$ ; for the difference = 0 when  $u = 1$ , increases positively with positive increases of  $u$ , and finally becomes infinitely large when  $u = \infty$ . If one again assumes the unit of area chosen previously and calls its land value  $B'$ , so that  $B'' = uB'$  and  $uB - B'' = u(B - B')$ , then:

$$B' = B \left[ \frac{(1 \cdot 0p)^u - 1}{u(0 \cdot 0p)(1 \cdot 0p)^{u-1}} \right], \text{ and}$$

$$B - B' = B \left[ 1 - \frac{(1 \cdot 0p)^u - 1}{u(0 \cdot 0p)(1 \cdot 0p)^{u-1}} \right]$$

In our example, in which  $u = 80$ ,

$B$	$= 5$ Thl.	$22$ Sgr.	$4$ Pf.
$B' = 0.28739 B$	$= 1$ Thl.	$19$ Sgr.	$6$ Pf.
$\text{so } B - B'$	$= 4$ Thl.	$2$ Sgr.	$10$ Pf.

This way of calculating the land value is certainly a bad one, as it always gives a smaller figure for sustained management than for intermittent management — almost four times smaller with an 80-year rotation.

(2) Some valuation officers claim that in a forest with a uniform growing stock of half the rotation age which is producing increment normally, one can get the normal annual increment (i.e. the normal yield) sustained annually. Therefore, for our calculation let us assume that on  $u$  unit areas such a growing stock is created after  $\frac{u}{2}$  years; i.e. from then on the normal net yields occur annually in a sustained way, but until then plantation and administration costs and thinnings occur intermittently. To obtain the land value we calculate the net capital value which these receipts and payments amount to. For this purpose we distinguish the series of the first  $\frac{u}{2}$  years from the whole of the rest of the time. The values at the end of the year  $\frac{u}{2}$  of the following incomes due *from* the first  $\frac{u}{2}$  years are:

$$\text{Thinnings} = u \times \bar{r}D$$

$$\text{Plantation costs} = -uC(1 \cdot 0p)^{\frac{u}{2}}$$

$$\text{Administration costs etc.} = -\frac{uA \left[ (1 \cdot 0p)^{\frac{u}{2}} - 1 \right]}{0 \cdot 0p}$$

In respect of subsequent years:

$$\text{the capitalised annual net yield} = \frac{E + D - C - uA}{0 \cdot 0p}$$

By adding these together, discounting to the present, and dividing by  $u$  in order to obtain the sum per unit of area we get:

$$\begin{aligned} B' &= \left[ \bar{r}D - C(1 \cdot 0p)^{\frac{u}{2}} - \frac{A(1 \cdot 0p)^{\frac{u}{2}}}{0 \cdot 0p} + \frac{A}{0 \cdot 0p} + \frac{E + D - C}{u(0 \cdot 0p)} - \frac{A}{0 \cdot 0p} \right] \div (1 \cdot 0p)^{\frac{u}{2}} \\ &= \frac{E + D - C}{u(0 \cdot 0p)(1 \cdot 0p)^{\frac{u}{2}}} + \frac{\bar{r}D}{(1 \cdot 0p)^{\frac{u}{2}}} - \frac{A}{0 \cdot 0p} - C \end{aligned}$$

In our example, in which  $\frac{u}{2} = 40$ ,

$$\frac{E + D - C}{u(0 \cdot 0p)(1 \cdot 0p)^{\frac{u}{2}}} = 3361$$

$$\frac{\bar{r}D}{(1 \cdot 0p)^{\frac{u}{2}}} = \frac{1000}{(1 \cdot 04)^{20}} + \frac{1880}{(1 \cdot 04)^{30}} + \frac{1380}{(1 \cdot 04)^{40}} = 1323$$

$$\frac{A}{0 \cdot 0p} = 1200 ; \quad C = 540 ; \quad \text{therefore:}$$

$$B' = 3361 + 1323 - 1200 - 540 = 4684 - 1740 = 2944 \text{ Pf.}$$

or  $B' = 8 \text{ Thl. } 5 \text{ Sgr. } 4 \text{ Pf. } = 14 \text{ fl. } 19 \text{ kr.}$

Therefore, by this hypothesis, the land value comes to 2 Thlr. 13 Sgr. = 4 fl. 16 kr. per acre more than was calculated for intermittent management. But the assumption that one can practise sustained management with a growing stock all of which has reached only half the rotation age, as if the felling series was normal, must arouse justifiable doubt. Indeed, some well known valuation officers have questioned it. One can easily see, from examples, that such a growing stock (plus its increment) will only last out for roughly a rotation if the mean annual increment is normal half-way through the rotation and persists at the same level

for half a rotation. The validity of the hypothesis has not been confirmed by experience, and the fact that one cannot expect it is obvious from the start because of its inherent absurdity. Therefore, the hypothesis must be contrary to sound forestry, and the land value calculated from it can be rejected as false. Moreover, it should be noted that this value works out smaller if one takes into account the lower price obtained for the immature wood which is felled; for any rotation less than 80 years, in fact, a smaller value emerges and near the end of this rotation a zeropoint occurs, i.e. there is no difference between the two land values.

(3) Now let us try to calculate the age at which felling the normal annual sustained yield can commence in an even-aged forest stand. At this age the economic value of the stand must equal the economic value of a normal felling series, viz.  $H = H'$ . Using this equation we shall seek the age  $n$  of the even-aged stand whose value is set equal to  $H$ . We shall ignore thinnings but their omission will not give rise to error because they occur on both sides of the equation in approximately equal amounts. In section II, A we found previously:

$$H = \frac{C(1.0p)^u - E}{(1.0p)^u - 1} = \frac{E - C}{(1.0p)^u - 1} \times (1.0p)^n$$

and for the same area:

$$H' = \frac{C(1.0p)^u - E}{(1.0p)^u - 1} = \frac{E - C}{u(0.0p)}$$

As  $H = H'$ , then

$$\frac{E - C}{(1.0p)^u - 1} \times (1.0p)^n = \frac{E - C}{u(0.0p)}, \quad \text{therefore}$$

$$(1.0p)^n = \frac{(1.0p)^u - 1}{u(0.0p)} \quad \text{and} \quad n = \log \frac{(1.0p)^u - 1}{u(0.0p)} \div \log(1.0p)$$

which is *the stand age sought*. As this expression shows ( $p$  being constant),  $n$  is dependent only on the duration of the rotation.

The same expression can be obtained if we use the land value calculation in (2) above, by taking the stand age as the unknown =  $n$  instead of  $\frac{u}{2}$ , equating this land value with that found for intermittent management, and finding  $n$  from the resulting expression. We will do this, again ignoring thinnings for the reason given previously. According to section I, A, (2):

$$B = \frac{E - C(1 \cdot 0p)^u}{(1 \cdot 0p)^u - 1} - \frac{A}{0 \cdot 0p}$$

and according to section II, B, (2):

$$B' = \frac{E - C}{u(0 \cdot 0p)(1 \cdot 0p)^n} - \frac{A}{0 \cdot 0p} - C$$

As  $B = B'$ , so:

$$\begin{aligned} \frac{E - C(1 \cdot 0p)^u}{(1 \cdot 0p)^u - 1} &= \frac{E - C}{u(0 \cdot 0p)(1 \cdot 0p)^n} - C \\ E - C(1 \cdot 0p)^u &= \frac{(E - C)[(1 \cdot 0p)^u - 1]}{u(0 \cdot 0p)(1 \cdot 0p)^n} - C(1 \cdot 0p)^u + C \\ E - C(1 \cdot 0p)^n &= \frac{(E - C)[(1 \cdot 0p)^u - 1]}{u(0 \cdot 0p)} \end{aligned}$$

Finally

$$(1 \cdot 0p)^n = \frac{(1 \cdot 0p)^u - 1}{u(0 \cdot 0p)} \quad \text{and} \quad n = \log \frac{(1 \cdot 0p)^u - 1}{u(0 \cdot 0p)} \div \log(1 \cdot 0p)$$

as above.

Now, for the rate  $p = 4$  per cent, we can work out this stand age for various rotations and see whether these calculated ages still correspond with forestry practice, like the half rotation age assumed above. However, in determining

$n$  we have arranged the calculation so that the first sustained final yield can be taken at the end of stand age  $(n + 1)$ .

*Calculation of the age  $(n + 1)$  at which, for different rotations  $(u)$  in a standard even-aged stand, removal of the normal sustained annual yield can begin.*

$u = 1;$	$(1 \cdot 04)^n = \frac{1 \cdot 04 - 1}{1 \times 0 \cdot 04} = 1;$	$n = \frac{0}{0 \cdot 01703} = 0;$	$n + 1 = 1 \text{ year};$	$\frac{n + 1}{u} = 1 \cdot 00.$
$u = 2;$	$(1 \cdot 04)^n = \frac{1 \cdot 08160 - 1}{2 \times 0 \cdot 04} = 1 \cdot 02;$	$n = \frac{0 \cdot 00860}{0 \cdot 01703} = 0 \cdot 5;$	$n + 1 = 1 \cdot 5 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 75.$
$u = 3;$	$(1 \cdot 04)^n = \frac{1 \cdot 12486 - 1}{3 \times 0 \cdot 04} = 1 \cdot 04;$	$n = \frac{0 \cdot 01703}{0 \cdot 01703} = 1;$	$n + 1 = 2 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 66.$
$u = 4;$	$(1 \cdot 04)^n = \frac{1 \cdot 16985 - 1}{4 \times 0 \cdot 04} = 1 \cdot 06;$	$n = \frac{0 \cdot 02531}{0 \cdot 01703} = 1 \cdot 5;$	$n + 1 = 2 \cdot 5 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 62.$
$u = 5;$	$(1 \cdot 04)^n = \frac{1 \cdot 21665 - 1}{5 \times 0 \cdot 04} = 1 \cdot 083;$	$n = \frac{0 \cdot 03463}{0 \cdot 01703} = 2;$	$n + 1 = 3 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 60.$
$u = 10;$	$(1 \cdot 04)^n = \frac{1 \cdot 48024 - 1}{10 \times 0 \cdot 04} = 1 \cdot 2;$	$n = \frac{0 \cdot 07918}{0 \cdot 01703} = 4 \cdot 5;$	$n + 1 = 5 \cdot 5 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 55.$
$u = 20;$	$(1 \cdot 04)^n = \frac{2 \cdot 19112 - 1}{20 \times 0 \cdot 04} = 1 \cdot 489;$	$n = \frac{0 \cdot 17289}{0 \cdot 01703} = 10 \cdot 1;$	$n + 1 = 11 \cdot 1 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 55.$
$u = 30;$	$(1 \cdot 04)^n = \frac{3 \cdot 23339 - 1}{30 \times 0 \cdot 04} = 1 \cdot 861;$	$n = \frac{0 \cdot 26975}{0 \cdot 01703} = 15 \cdot 8;$	$n + 1 = 16 \cdot 8 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 56.$
$u = 40;$	$(1 \cdot 04)^n = \frac{4 \cdot 80102 - 1}{40 \times 0 \cdot 04} = 2 \cdot 376;$	$n = \frac{0 \cdot 37585}{0 \cdot 01703} = 22;$	$n + 1 = 23 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 57.$
$u = 50;$	$(1 \cdot 04)^n = \frac{7 \cdot 10668 - 1}{50 \times 0 \cdot 04} = 3 \cdot 053;$	$n = \frac{0 \cdot 48473}{0 \cdot 01703} = 28 \cdot 4;$	$n + 1 = 29 \cdot 4 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 58.$
$u = 60;$	$(1 \cdot 04)^n = \frac{10 \cdot 51962 - 1}{60 \times 0 \cdot 04} = 3 \cdot 966;$	$n = \frac{0 \cdot 59835}{0 \cdot 01703} = 35 \cdot 1;$	$n + 1 = 36 \cdot 1 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 60.$
$u = 70;$	$(1 \cdot 04)^n = \frac{15 \cdot 57161 - 1}{70 \times 0 \cdot 04} = 5 \cdot 204;$	$n = \frac{0 \cdot 71634}{0 \cdot 01703} = 42;$	$n + 1 = 43 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 61.$
$u = 80;$	$(1 \cdot 04)^n = \frac{23 \cdot 04979 - 1}{80 \times 0 \cdot 04} = 6 \cdot 89;$	$n = \frac{0 \cdot 83822}{0 \cdot 01703} = 49 \cdot 2;$	$n + 1 = 50 \cdot 2 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 62.$
$u = 90;$	$(1 \cdot 04)^n = \frac{34 \cdot 11933 - 1}{90 \times 0 \cdot 04} = 9 \cdot 2;$	$n = \frac{0 \cdot 96379}{0 \cdot 01703} = 56 \cdot 5;$	$n + 1 = 57 \cdot 5 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 64.$
$u = 100;$	$(1 \cdot 04)^n = \frac{50 \cdot 50494 - 1}{100 \times 0 \cdot 04} = 12 \cdot 37;$	$n = \frac{1 \cdot 09237}{0 \cdot 01703} = 64 \cdot 1;$	$n + 1 = 65 \cdot 1 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 65.$
$u = 120;$	$(1 \cdot 04)^n = \frac{110 \cdot 66255 - 1}{120 \times 0 \cdot 04} = 22 \cdot 84;$	$n = \frac{1 \cdot 35870}{0 \cdot 01703} = 79 \cdot 7;$	$n + 1 = 80 \cdot 7 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 67.$
$u = 140;$	$(1 \cdot 04)^n = \frac{242 \cdot 47529 - 1}{140 \times 0 \cdot 04} = 43 \cdot 12;$	$n = \frac{1 \cdot 63468}{0 \cdot 01703} = 95 \cdot 9;$	$n + 1 = 96 \cdot 9 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 69.$
$u = 160;$	$(1 \cdot 04)^n = \frac{531 \cdot 29304 - 1}{160 \times 0 \cdot 04} = 82 \cdot 85;$	$n = \frac{1 \cdot 91829}{0 \cdot 01703} = 112 \cdot 6;$	$n + 1 = 113 \cdot 6 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 71.$
$u = 180;$	$(1 \cdot 04)^n = \frac{1164 \cdot 1280 - 1}{180 \times 0 \cdot 04} = 161 \cdot 5;$	$n = \frac{2 \cdot 20817}{0 \cdot 01703} = 129 \cdot 6;$	$n + 1 = 130 \cdot 6 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 72.$
$u = 200;$	$(1 \cdot 04)^n = \frac{2550 \cdot 7468 - 1}{200 \times 0 \cdot 04} = 318 \cdot 7;$	$n = \frac{2 \cdot 50338}{0 \cdot 01703} = 147 \cdot 6;$	$n + 1 = 148 \cdot 6 \text{ yrs};$	$\frac{n + 1}{u} = 0 \cdot 74.$



In the case we are considering, when  $u = 80$ ,  $n = 49$ .

During the first 49 years,  $\frac{\bar{r}D}{(1.04)^{49}} = 1323$ ;  $C = 540$ , and  $\frac{A}{0.04} - \frac{A}{0.04(1.04)^{49}} = 1024$ ; so the net present value per acre  $= 1332 - 540 - 1024 = 1323 - 1564 = -241$ . During the rest of the time,  $\frac{E + D - C - uA}{u \times 0.04} = 14,940$  which has a present value  $= 2186$ . Accordingly, the land value  $B' = -241 + 2186 = 1944$  Pf., or  $10\frac{1}{4}$  Sgr. less than was found for intermittent management.

The algebraic expression for  $n$  also gives  $n + 1 = 1$  when  $u = 1$ , which is correct because in a 1-year rotation the intermittent system coincides with the sustained system. The ratio of the age at which cutting starts to the rotation is smallest in value when the rotation length lies between 10 and 20 years. From there it increases gradually, almost in arithmetic progression on both sides, until it reaches a maximum of one when  $u = 1$  and  $u = \infty$ . This variable proportion rule seems to us to be more reasonable than the general assumption of  $n = \frac{u}{2}$ , because, if one starts removing a normal sustained yield from a stand aged  $\frac{u}{2}$ , with the shortest and the longest rotations too much increment is felled during the first rotation. Therefore, a growing stock  $\frac{u}{2}$  years old plus its increment during the rotation does not produce  $u$  times the normal annual yield. This can only be compensated for by providing a greater growing stock for this initial rotation, i.e. by not starting the sustained yield felling until a greater age has been reached.

The foregoing also shows that when a bare forest area is brought to a sustained state, *the land value under sustained management is not greater than under intermittent management*; i.e. both systems furnish exactly equal pecuniary advantages. Also, we cannot concede that an 'anachronism' has occurred in these calculations, as Herr von Gehren suggests.

One merely takes things as they are. If the land is bare, then by using one of the previous yield calculations we can measure the pecuniary benefit of a forest enterprise without having to ascribe a part of the value of the future growing stock to the land. The economic value of the crop can only be attributed to the land if the area is actually *stocked*. Also, by our proofs, when an intermittent system is converted into a sustained one, no special value should be added to the value of the land calculated for the intermittent system. The difference between the forest value of the sustained enterprise (44 Thlr. 25 Sgr. 10 Pf. according to Herr von Gehren) and the land value of the intermittent one (19 Thlr. 10 Sgr. 8 Pf.) is 25.5 Thlr. This amount, which Herr von Gehren discounted from 80 years to the present and added to the latter land value as a possible improvement value, is nothing other than the economic value of the growing stock necessary in the sustained enterprise, i.e.  $W - B = H$ . If one approaches things in a different way from Herr von Gehren, then it appears as if the existing growing stock should be deducted, not added, under the intermittent system. This view disappears, however, with closer scrutiny of the circumstances.

If one compares the growing stock occurring in the two management systems during a whole rotation, then they are found to be absolutely identical in size. If we imagine  $u$  (= rotation) acres with a normal standage sequence, then, year by year throughout the rotation, this area will contain:

Year 1. 1 acre aged 0 + 1 acre aged 1 etc. . + 1 acre aged  $(u - 2)$  + 1 acre aged  $(u - 1)$ .

Year 2. 1 acre aged 0 + 1 acre aged 1 etc. . + 1 acre aged  $(u - 2)$  + 1 acre aged  $(u - 1)$ .

etc.

Year  $(u - 1)$ . 1 acre aged 0 + 1 acre aged 1 etc. . + 1 acre aged  $(u - 2)$  + 1 acre aged  $(u - 1)$ .

Year  $u$ . 1 acre aged 0 + 1 acre aged 1 etc. . + 1 acre aged  $(u - 2)$  + 1 acre aged  $(u - 1)$ .

If we add the vertical columns of this summary, the growing stock during a rotation amounts to:

$u \times 1$  acres aged 0 +  $u \times 1$  acres aged 1 +  $u \times 1$  acres aged 2 + . . .  
 . . . . . +  $u \times 1$  acres aged ( $u - 2$ ) +  $u \times 1$  acres aged ( $u - 1$ ).

An equal area of  $u$  acres under the intermittent system during the rotation contains :

Year 1.  $u$  acres aged 0 years  
 Year 2.  $u$  acres aged 1 year  
 Year 3.  $u$  acres aged 2 years  
 etc.  
 Year ( $u - 1$ ).  $u$  acres aged ( $u - 2$ ) years  
 Year  $u$ .  $u$  acres aged ( $u - 1$ ) years

Thus, by addition of this series we find a standing volume of:  $u$  acres aged 0 +  $u$  acres aged 1 +  $u$  acres aged 2 + . . . +  $u$  acres aged ( $u - 2$ ) +  $u$  acres aged ( $u - 1$ ), just as in the sustained system.

However, comparison of the two systems reveals the following. In sustained management, the forest value is obtained by capitalisation of the forest's net yield. From this value, the economic value of the perpetually present normal growing stock must be deducted in order to get the land value. The same occurs in the intermittent system when a stand is present; here too,  $W - H$  gives the value of  $B$ . However, if in the latter system one starts from bare land, then  $H = 0$ , and the capitalised forest net yield gives the land value directly. Under both management systems the method of calculation is the same in this respect. Only in the sustained system does the stand value have importance as working capital because it is always present in the same amount and possesses the same economic value as the existing stands. This value has already been taken into account in the intermittent system in calculating  $B$ , only not in the form of a working capital because the size of the growing stock does not remain constant from year to year but is smaller each year in the first half of the rotation and larger in the second half. Therefore the value of the growing stock cannot be calculated as a distinct working capital in the intermittent system.

Let us approach the subject from yet another angle. If, in the calculation, one starts with bare land under the intermittent system, then this appears to produce all the forest net yields; no other working capital has been laid out

in forestry apart from the value of the land. In turn, the latter is conditioned by those forest net yields and must be found inversely by their capitalisation. Consequently the economic values of the succeeding stands have the same significance as interest on the land capital. However (with bare land) if one tries to put into the calculation a working capital which is equal to the sum of the values of all the annual stands, then that is just as absurd as if someone had lent a sum of money to accumulate with compound interest for 80 years, and said that he had a working capital equal to the sum of the 79 instalments of interest due on his loaned-out capital. The interest payments are a pure product of the capital, and by getting the capital back in the 80th year with 80 years' compound interest (according to existing concepts) he is completely compensated for his original capital and all expenses, just like the forest owner at the end of an 80-year rotation who gets back his original bare land capital plus the interest on it in the form of thinning and final-crop yields. Therefore, in the intermittent system the net yields from the forest crop are like the interest on the land capital and by capitalising them one can find the land value directly.

On page 255 of his 'Forstabschätzung' (Forest evaluation) of 1826, Hundeshagen says that the *loss of interest* associated with the intermittent receipt of incomes in non-sustained management represents the interest from the growing stock capital in sustained management; i.e. that the loss of interest when capitalised is equal to the capital value of the growing stock. From the formula one is easily convinced that this claim is correct, and the proof of it is apparent from the fact that in calculating *B* nothing further need be considered (other than the factors we have included) because the loss of interest was allowed for from the start.

In the previous discussion we chose an area that formed a working section *by itself*; let us now examine the case of an area that forms *a part of* an existing working section.

An age class which forms part of the area of a working section cannot have a value greater than the sum of the values of the land and the stand, as found for intermittent management, because each annual part represents an intermittently managed addition to the sustained yield se-

ries. If a forest owner sells such a part, he gets the same income from money capital as he previously obtained from his stands. If he buys an area of forest which just fills a gap in an existing working section, then the reverse applies, i.e. he now gets a rent from the stand, which he previously obtained from the money capital. What we maintain is true for a whole section must naturally also apply to a part of it. Accordingly it is wrong to attribute to a part of a working section a greater value than it possesses in intermittent management, as Herr von Gehren and others have done.

Even less can an area have a greater value than this (whether it is carrying a stand or not) when it is added to an existing, complete, normal working section. Admittedly, as we know very well, some writers on evaluation claim that one can make immediate use of the future increment from the area, even when it is blank, by combination with the growing stock of the working section. From this they conclude that the value of such an inserted area is equal to its capitalised *annual* final and thinning yields, i.e. it is greater by the loss of interest, which occurs in non-sustained management due to the intermittent arrival of the incomes. But the claim is false, and therefore the conclusion is also false. It is correct only for quite small inserted blanks; but only because the error then becomes infinitesimally small. In principle, the claim is erroneous in all circumstances. Consider an inserted area and a working section of equal size, and calculate the yield equal to the increment of this doubled area, which can be produced by the existing growing stock and the blank. Then one will see that this is not adequate to cover the whole rotation and that at the start of the second rotation a smaller growing stock must be present than at the start of the first. It cannot be otherwise, because the stands do not reach maturity, therefore a greater yield must be taken than the actual increment produces, and the sustained removal of this inevitably reduces the growing stock which is present. Therefore the assumption of a higher value for the inserted area must also be false. However, we can show that the area possesses a smaller value than in the intermittent system. For if the existing working section contains a normal age series, then by inserting an extra area of forest land the area of the annual coupe will be increased. However, as a result, only the oldest coupe will be felled at its rota-

tion age; all the others will be partly or completely cut at a lesser age, except in the case of an inserted area which is blank. This will be entirely cut at the end of the rotation if it is as large as or smaller than an annual coupe; otherwise, if its area exceeds an annual coupe, it will be partly cut at a lesser age. Now if the working section was normal and its rotation the most advantageous, then by the reduction of the felling age, the value of receipts from sales must be reduced by comparison with the value if the working section and the inserted area are managed individually. This deficit is due to the insertion, and would therefore have to be deducted from the value of the inserted area as calculated for the intermittent system. Therefore we do not put too high a value on the inserted area if we calculate its value under intermittent management.

Finally, if both the working section and the inserted area are bare of trees, then, least of all in this case should a higher value be given to the inserted area, as Herr von Gehren did. There is no reason here for considering both areas separately; together they form *one* bare area which is to be managed as one working section and so should be considered in the same category as the discussion of independent working sections.

In the foregoing pages we have shown the way in which one should calculate land and stand values in forestry. In so doing we have found many new principles and laid the basis for discovering many more. The method used and the results obtained are presented for assessment by experts. To start with, we beg Herr von Gehren, who has a basic mathematical education, to contradict anything with which he does not agree. In particular we stress the principle we have found: *that the forestry value of the land remains unchanged, whether one assumes intermittent or sustained management, normal or abnormal growing stock, and whether the area is independent or forms a part of another area.*

Darmstadt, October 1849